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KLOE-2

Muon g−2 theory: The hadronic part

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Abstract. I present a status report of the hadronic vacuum polarization effects for the muon $g-2$, to be considered as an update of [1]. The update concerns recent new inclusive R measurements from KEDR in the energy range 1.84 to 3.72 GeV. For the leading order contributions I find $a_\mu^{\rm had(1)} = (688.07\pm4.14)[688.77\pm3.38]\times10^{-10}$ based on e^+e^-data [incl. τ data], $a_\mu^{\rm had(2)} = (-9.93\pm0.07)\times10^{-10}$ (NLO) and $a_\mu^{\rm had(3)} = (1.22\pm0.01)\times10^{-10}$ (NNLO). Collecting recent progress in the hadronic light-by-light scattering I adopt π^0,η,η' [95 ± 12] + axial–vector [8 ± 3] + scalar [−6 ± 1] + π, K loops [−20 ± 5] + quark loops [22 ± 4] + tensor [1 ± 0] + NLO [3 ± 2] which yields $a_\mu^{(6)}(\rm lbl, had) = (103 \pm 29) \times 10^{-11}$. With these updates I find $a_\mu^{\rm exp} - a_\mu^{\rm the} = (31.3 \pm 7.7) \times 10^{-10}$ a 4.1 σ deviation. Recent lattice QCD results and future prospects to improve hadronic contributions are discussed.

1 Overview: hadronic effects in $g-2$.

This review of the hadronic vacuum polarization (HVP) contributions to the muon $g-2$ is to be considered as a complement to the theory reviews by Marc Knecht and Massimiliano Procura which focus on the hadronic light-by-light (HLbL) part and the reviews on hadronic cross sections by Graziano Venanzoni, Simon Eidelman and Achim Denig in these Proceedings.

The present experimental muon $g-2$ result from Brookhaven (BNL) $a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$ [2] soon will be improved by the new muon $g-2$ experiments at Fermilab and J-PARC. The Fermilab experiment will be able to reduce the error by a factor 4, the J-PARC experiment will provide an important cross check with a very different technique [3]. It means that the new muon $g-2$ experiments are expected to establish a possible new physics contribution at the level $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the} = 6.7\,\sigma$ provided theory remains as it is today and the central value does not move significantly. If we achieve a reduction of the hadronic uncertainty by factor 2 we would arrive at $\Delta a_\mu = 11.6\,\sigma$. That's what we hope to achieve. Figure 1 illustrates the present status and what has been achieved so far. The present results $a_\mu^{\rm HVP\ LO} = (6888 \pm 34) \times 10^{-11}$ amounts to +59.09 ±0.30 ppm, which poses the major challenge. The subleading results $a_\mu^{\rm HVP\ NLO} = (-99.3 \pm 0.7) \times 10^{-11}$ and $a_\mu^{\rm HVP\ NNLO} = (12.2 \pm 0.1) \times 10^{-11}$ although relevant will be known well enough. These number also compare with the well established weak $a_\mu^{\rm EW} = (154 \pm 1) \times 10^{-11}$ at 1.3 ± 0.0 ppm and the problematic HLbL estimated to contribute $a_\mu^{\rm HLbL} = (103 \pm 29\ [105 \pm 26]) \times 10^{-11}$, which is representing a +0.90 ±0.25 ppm effect.

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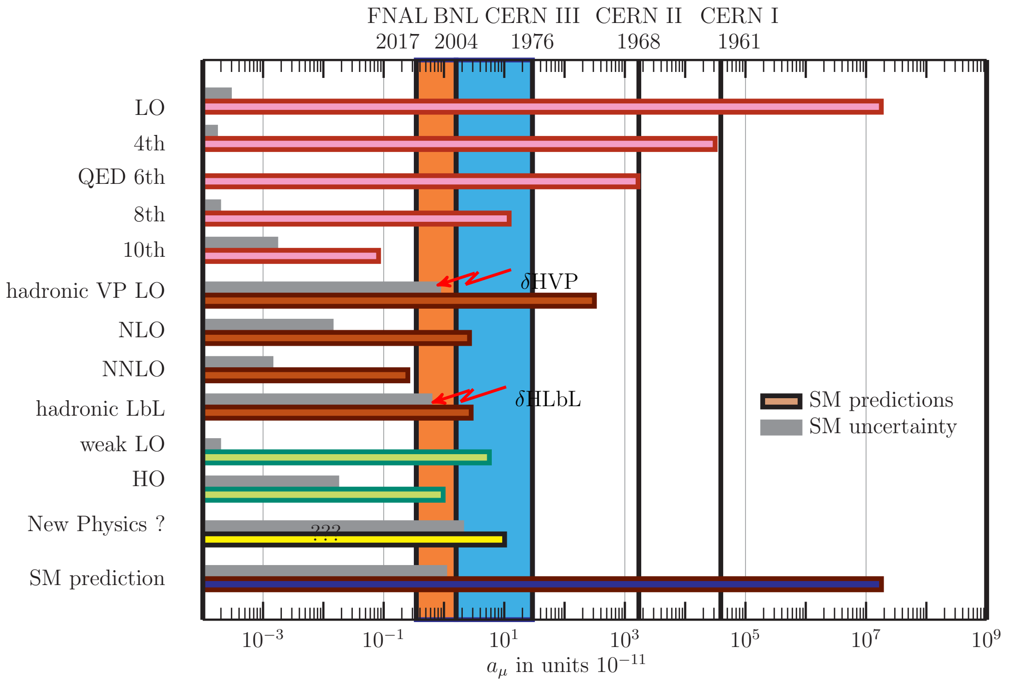


Figure 1. Past and future $g-2$ experiments testing various contributions. As New Physics ? we display the deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$. Arrows point to what is limiting theory precision presently: the Hadronic Vacuum Polarization (HVP) and Hadronic Light-by-Light (HLbL) contributions.

Virtual effect form low energy hadronic excitations are the standard problem in electroweak precision physics. At a certain level of precision predictions are hampered by non-perturbative effects, which technically are not under desirable control on the theory side. For the muon $g-2$ the leading hadronic effects are related to the diagrams in figure 2 and concern

(a) Hadronic vacuum polarization (HVP) of order $O(\alpha^2)$, $O(\alpha^3)$,

(b) Hadronic light-by-light scattering (HLbL) of order $O(\alpha^3)$,

(c) Hadronic effects in 2-loop hadronic electroweak (HEW) corrections of order sub–$O(\alpha G_F m_\mu^2)$.

Light quark loops appear as non-perturbative hadronic "blobs". The evaluation of the corresponding non-perturbative effects relies on hadron production data in conjunction with Dispersion Relations (DR), or on low energy effective modeling by the Resonance Lagrangian Approach (RLA), specifically by the Hidden Local Symmetry (HLS) model [4], or the Extended Nambu–Jona-Lasinio (ENJL) model [5], large–N_c QCD inspired methods [6] and on lattice QCD. Different strategies apply for the different kinds of contributions:

(a) HVP one evaluates via a dispersion integral over $e^+e^- \to$ hadrons data. Here 1 independent amplitude is to be determined by one specific data set. Global fits based on the RLA (like HLS) allow to improve the data-driven evaluations [7]. Lattice QCD is the ultimate tool to get QCD predictions in future.

(b) HLbL so far has been evaluated by modeling via the Resonance Lagrangian Approach (RLA) (chiral perturbation theory (CHPT) extended by vector meson dominance (VMD) in accord with chi-

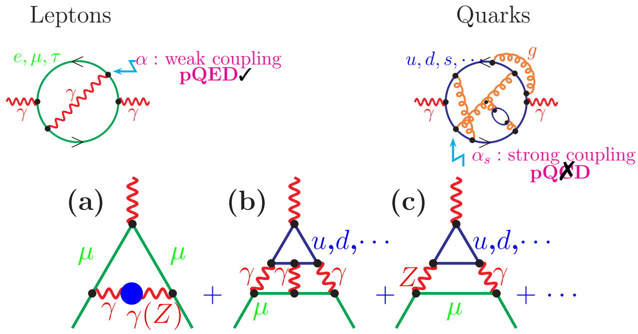
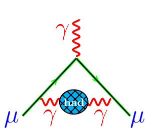


Figure 2. In the upper panel we compare leptonic with hadronic vacuum polarization effects. The lower panel illustrates the three classes of $g-2$ contributions exhibiting substantial hadronic corrections.

ral structure of QCD) or by large–N_c inspired methods and operator product expansions (OPE). A data driven approach based on dispersion relations [8] is attempting to exploit $\gamma\gamma \to$ hadrons – data systematically (here 28 independent amplitudes are to be determined by as many independent data sets, fortunately not all are equally important numerically). Also in this case lattice QCD for me is the ultimate approach, although tough to be achieved with limited computing resources.

(c) HEW corrections due to quark triangle diagrams: since triple vector amplitudes vanish $VVV = 0$ by Furry's theorem only VVA (of $f\bar{f}Z$ -vertex) contributes. Thus it is ruled by the ABJ anomaly, which is perturbative and non-perturbative simultaneously, i.e. the leading effects are calculable. The anomaly cancellation condition intimately relates quark and lepton contributions and the potentially large leading corrections cancel [9–11] such that hadronic corrections are well under control.

2 Evaluation of the leading order $a_\mu^{\rm had}$



The hadronic contribution to the vacuum polarization can be evaluated, with the help of dispersion relations, from the energy scan of the ratio $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ which can be measured up to some energy $E_{\rm cut}$ above which we can safely use perturbative QCD (pQCD) thanks to asymptotic freedom of QCD. We apply pQCD from 5.2 GeV to 9.46 GeV and above 11.5 GeV (see figure 5 below). Note that the DR requires the undressed (bare) cross–section $\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons}) = \sigma(e^+e^- \to \gamma^* \to \text{hadrons})\,|\alpha(0)/\alpha(s)|^2$. The lowest order (LO) VP contribution is given by

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{m_{\pi^0}^2}^{E_{\rm cut}^2} ds\, \frac{R_\gamma^{\rm data}(s)\,\hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\,\hat{K}(s)}{s^2} \right), \qquad (1)$$

where $\hat{K}(s)$ is a known kernel function growing form $0.63\cdots$ at the $2m_\pi$ threshold to 1 as $s \to \infty$. The integral is dominated by the ρ resonance peak shown in figure 3. The experimental errors imply the dominating theoretical uncertainties. As a result I obtain

$$a_\mu^{\rm had(1)} = (688.07 \pm 4.14)[688.77 \pm 3.38]\; 10^{-10}\;;\quad e^+e^- - \text{data based [incl. } \tau] \,. \qquad (2)$$

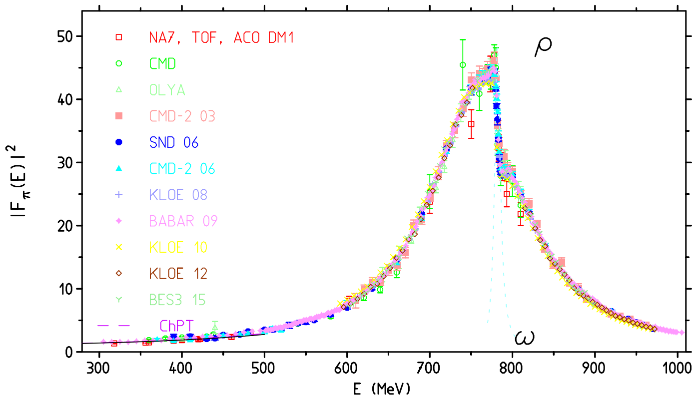


Figure 3. A compilation of the modulus square of the pion form factor in the ρ meson region, which yields about 75% of $a_\mu^{\rm had}$. The corresponding $R(s)$ is $R(s) = \frac{1}{4}\beta_\pi^3\,|F_\pi^{(0)}(s)|^2$, $\beta_\pi = \sqrt{1 = 4m_\pi^2/s}$ is the pion velocity.

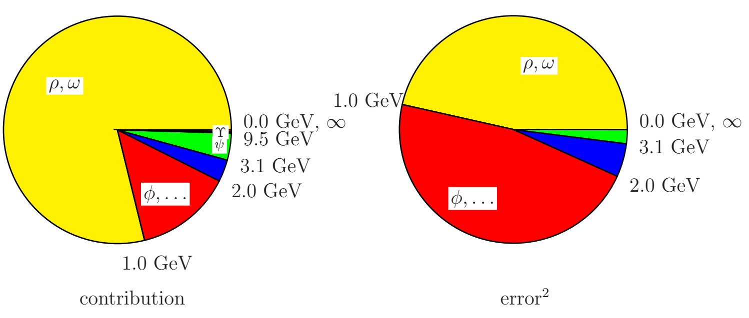


Figure 4. Distribution of contributions and error squares from different energy regions.

Figure 4 shows the distribution of contributions and errors between different energy ranges. One of the main issues is $R_\gamma(s)$ in the region from 1.2 GeV to 2.0 GeV, where more than 30 exclusive channels must be measured and although it contributes about 20% only of the total it contributes about 50% of the uncertainty. In the low energy region, which is particularly important for the dispersive evaluation of the hadronic contribution to the muon $g-2$, data have improved dramatically in the past decade for the dominant $e^+e^- \to \pi^+\pi^-$ channel (CMD-2 [12], SND/Novosibirsk [13], KLOE/Frascati [14–16], BaBar/SLAC [17], BES-III/Beijing [18]) and the statistical errors are a minor problem now. Similarly, the important region between 1.2 GeV to 2.4 GeV has been improved a lot by the BaBar exclusive channel measurements in the ISR mode [19–22]. Recent data sets collected are: $e^+e^- \to 3(\pi^+\pi^-)$, $e^+e^- \to \bar{p}p$ and $e^+e^- \to K_S^0K_L^0, K^+K^-$ from CMD-3 [23, 24], and $e^+e^- \to \bar{n}n$, $e^+e^- \to \eta\pi^+\pi^-$, $e^+e^- \to \pi^0\gamma$, $e^+e^- \to \omega\eta\pi^0$, $e^+e^- \to \omega\eta$, $e^+e^- \to K^+K^-$ and $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND [25–27].

Above 2 GeV fairly accurate BES-II data [28] are available. Recently, a new inclusive determination of $R_\gamma(s)$ in the range 1.84 to 3.72 GeV has been obtained with the KEDR detector at Novosibirsk [29] (see figure 5). A big step in improving low energy cross section measurements has been

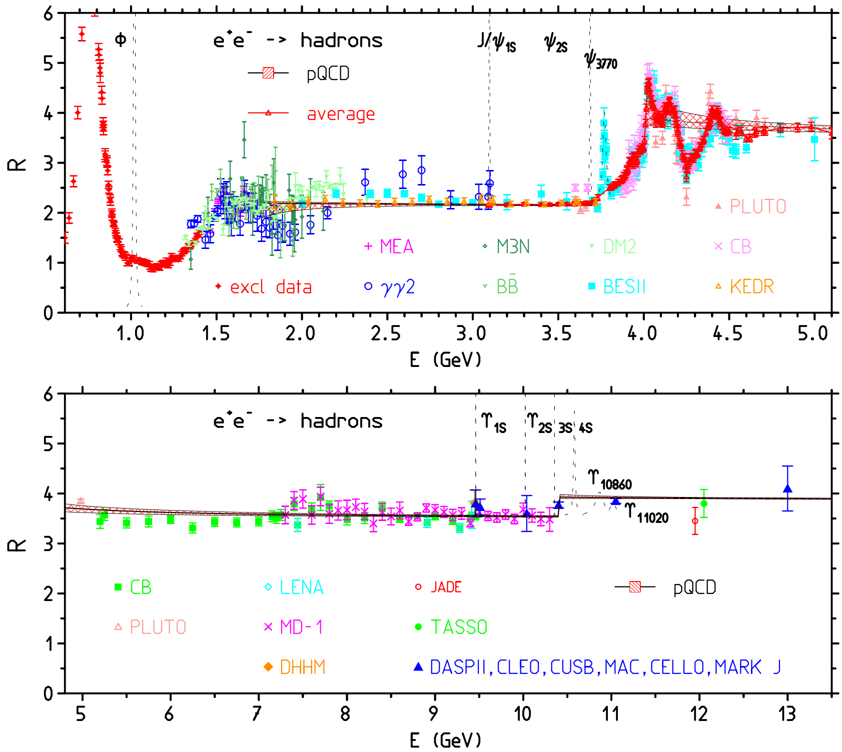


Figure 5. Compilation of the $R(s)$ data. New are the KEDR data between 1.84 and 3.72 GeV. Perturbative QCD predictions are also shown.

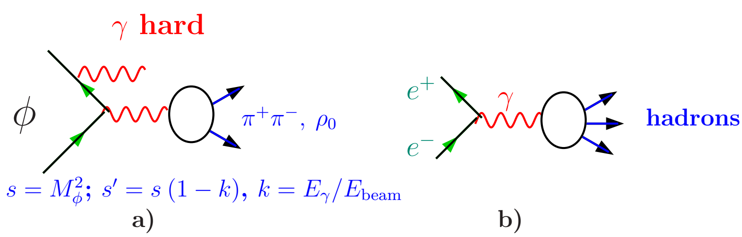


Figure 6. a) Initial state radiation (ISR) on resonance, b) Standard energy scan by tuning the beam energy.

possible with the radiative return or Initial State Radiation (ISR) method figure 6 which has been pioneered by the KLOE Collaboration, followed by BaBar and BES3 experiments. Recent new experimental input for HVP has been obtained by CMD-3 and SND at VEPP-2000 via energy scan and by BESIII at PEPC in the ISR setup (see Contributions by G. Venanzoni, S. Eidelman and A. Denig).

3 NLO and NNLO HVP effects

The next-to-leading order (NLO) HVP is represented by diagrams in figure 7. With kernels from [30],

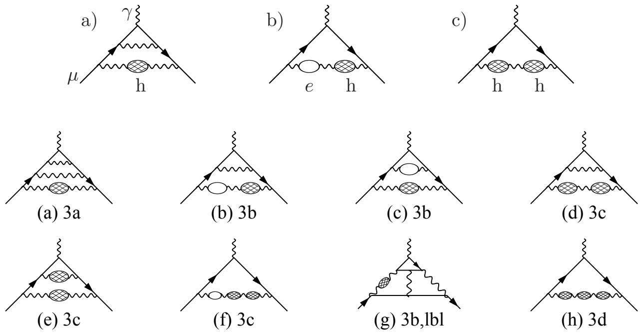


Figure 7. Feynman diagrams with hadronic insertions at NLO (top row) and NNLO.

the results of an updated evaluation are presented in table 1. The next-to-next leading order (NNLO)

Table 1. NLO contributions diagrams a) - c) (in units 10^{-11})

$a_\mu^{(2a)}$	$a_\mu^{(2b)}$	$a_\mu^{(2c)}$	$a_\mu^{\text{had}(2)}$
-206.13(1.30)	103.49(0.63)	3.37(0.05)	-99.27 (0.67)

Table 2. NNLO contributions diagrams (a) - (h) (in units 10^{-11})

$a_\mu^{(3a)}$	$a_\mu^{(3b)}$	$a_\mu^{(3b,\text{lbl})}$	$a_\mu^{(3c)}$	$a_\mu^{(3d)}$	$a_\mu^{\text{had}(3)}$	Ref.
8.0	−4.1	9.1	−0.6	0.005	12.4(1)	[31]
7.834 (61)	−4.033 (28)	9.005 (63)	−0.569 (5)	0.00518 (12)	12.24 (10)	[1]

contributions have been calculated recently [31]. Diagrams are shown in figure 7 and corresponding contributions evaluated with kernels from [31] are listed in table 2.

4 News on VP subtraction

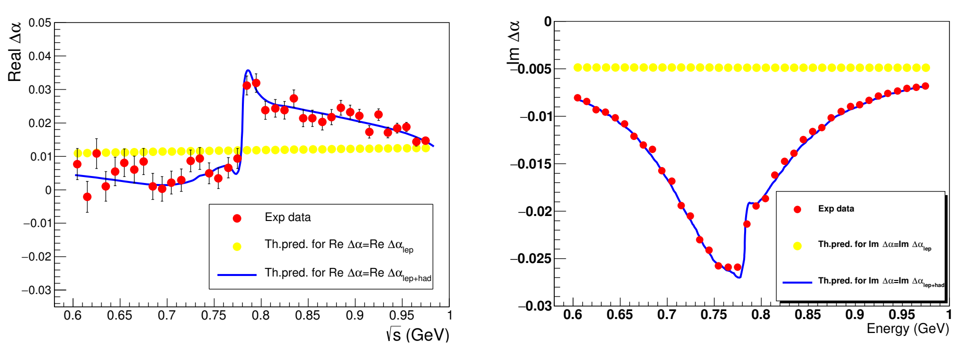


Figure 8. Real and imaginary part of the complex shift $\Delta\alpha$ of the fine structure constant as measured by KLOE. Reprinted from [33] in Phys. Lett. B `doi:10.1016/j.physletb.2016.12.016`

The first direct measurement of the timelike complex VP function in the ρ resonance region by KLOE [33] (see figure 8) nicely confirms dispersion relation calculation and demonstrates the importance of including the imaginary part in vacuum polarization subtraction in obtaining the undressed $\sigma^{(0)}(s)$ version of the physical hadronic cross-sections $\sigma(s)$. The complex running fine structure constant $\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha(s)}$ is defined in terms of the complex shift $\Delta\alpha(s) = -[\Pi'_\gamma(s) - \Pi'_\gamma(0)]$. Measuring $\left|\frac{\alpha(s)}{\alpha(0)}\right|^2 = \frac{\sigma(e^+e^-\to\mu^+\mu^-)}{\sigma(e^+e^-\to\mu^+\mu^-)_{\rm pt}}$ as well as $R(s) = \frac{\sigma(e^+e^-\to\pi^+\pi^-)}{\sigma(e^+e^-\to\mu^+\mu^-)}$, which determines $\mathrm{Im}\,\alpha(\mathrm{s}) = -\frac{\alpha}{3}\,\mathrm{R(s)}$, and knowing the modulus $|\alpha(s)|$ one can extract $\mathrm{Re}\,\alpha(\mathrm{s})$ as well (see G. Venanzoni's Contribution fort details).

The imaginary parts in the perturbative regions usually are small relative to the leading logarithms which govern the running couplings (renormalization group approach). In the hadronic shift however, resonances are accompanied by imaginary parts which may be huge in particular near resonances which can decay via OZI suppressed channels only (see Sect. 5 of [1]).

5 Low energy effective Lagrangian theory

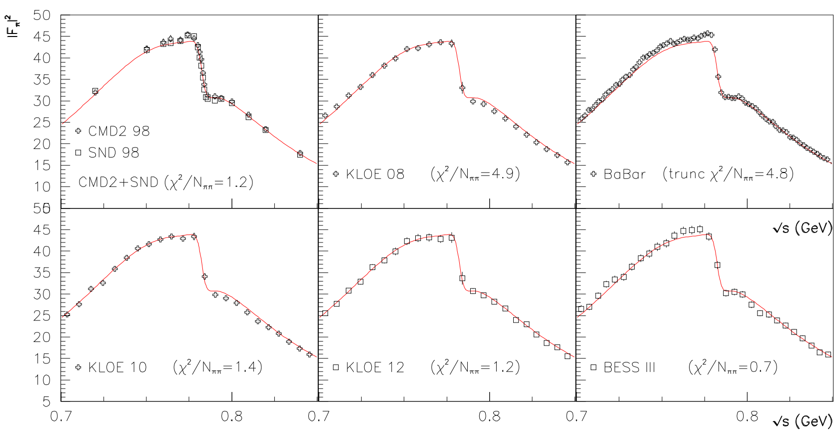


Figure 9. Comparing the τ+PDG prediction (red curve) of the pion form factor in e^+e^- annihilation in the $\rho-\omega$ interference region. Reprinted from [7], with kind permission of The European Physical Journal (EPJ).

A low energy effective field theory description of hadronic data requires an extension of CHPT towards higher energies, which mainly requires to include spin 1 resonances $\rho, \omega, \phi, \cdots$ in accord with the symmetries of QCD. Principles to be included are the chiral structure of QCD, VMD and electromagnetic gauge invariance. The spin 1 resonances are important in the evaluation of HVP as well as of HLbL effects. One possible implementation is the HLS model, which for what concerns HVP can also be seen as a generalized Gounaris-Sakurai model. In the neutral channel $e^+e^- \to$ hadrons $\gamma, \rho^0, \omega, \phi$ mixing makes the channel rather complicated, while in the charged channel of $\tau \to \nu_\tau$hadrons [34–38] is much simpler as the $\rho^\pm$ do not mix with other hadrons. It is thus tempting to start with the isospin rotated $\tau^\pm \to \nu_\tau\pi^\pm\pi^0$ decay spectra and supplement them with appropriate isospin breaking and mixing effects to predict $e^+e^- \to \pi^+\pi^-$, with the result shown in figure 9. It shows that there is no τ vs. e^+e^- conflict and actually simultaneous fits allows one to reduce uncertainties

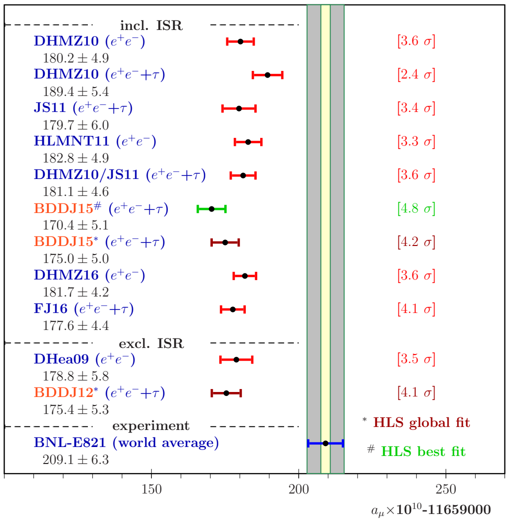


Figure 10. Dependence of a_μ predictions on recent evaluations of $a_\mu^{\mathrm{had,LO}}$. The HLS best fit BDDJ15$^\#$ (NSK+KLOE10+KLOE12) does not include BaBar $\pi\pi$ data [39], while BDDJ15* does. JS11 [40], FJ16 [1] is updated and includes the BES III and KEDR data. Further points are BDDJ12 [7], DHMZ10 [41], DHMZ16 [22, 44], HLMNT11 [43] and DHea09 [42]. The DHMZ10 ($e^+e^-+\tau$) result is not including the $\rho-\gamma$ mixing correction, i.e. it misses important isospin breaking effects. In contrast, DHMZ10/JS11 is obtained by including this correction, which brings the point into much better agreement with standard analyses based on e^+e^- data alone, as for example the DHMZ10 (e^+e^-) result. (see also [21, 45]). Note: results depend on which value has been taken for HLbL. JS11 and BDDJ13 includes $116(39)\times10^{-11}$ [JN] [46], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26)\times10^{-11}$ [PdRV] [47], while FJ16 includes an updated $103(29)\times10^{-11}$.

of HVP by using indirect constraints. The global fit strategy followed in [7] takes into account data below $E_0 = 1.05$GeV (just above the ϕ) to constrain the effective Lagrangian couplings. Used are 45 different data sets, 6 annihilation channels and 10 partial width decays. The effective theory then allows us to predict cross sections for the channels $\pi^+\pi^-, \pi^0\gamma, \eta\gamma, \eta'\gamma, \pi^0\pi^+\pi^-, K^+K^-, K^0\bar{K}^0$, which account for 83.4% of a_μ^{had}. The missing channels $4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi$ and the higher energy tail $E > E_0$ is evaluated using data directly and pQCD for the perturbative region and tail. All mixing effects, as $\gamma\rho$ -mixing, $\rho\omega$ -mixing, $\cdots$, as well as the decay branching fractions are dynamically generated by including self-energy effects of the spin 1 mesons. One thus is taking into account proper phase space, energy dependent widths etc. Such fit strategy is able to shed light on incompatibilities in the data, e.g. KLOE vs BaBar, by comparing the fit qualities, but also reveals the compatibility of τ–decay spectra with e^+e^-–data after accounting for the mixing effects like including $\gamma-\rho^0$ mixing. HLS estimates are included in table 10 together with other recent results.

6 HVP from lattice QCD (following H. Wittig at LATTICE 2016)

The need for ab initio calculation of a_μ^{had} is well motivated: – the problems to determine non-perturbative contributions to the muon $g-2$ from experimental data at sufficient precision persists and is not easy to improve, – a model–independent extension of CHPT to the relevant energies ranges up to 2 GeV is missing, while the new experiments E989 FNAL and E34 J-PARC require an improvement of the hadronic uncertainties by a factor two to four.

The hope is that LQCD can deliver estimates of accuracy

$$\delta a_\mu^{\mathrm{HVP}}/a_\mu^{\mathrm{HVP}} < 0.5\% \;, \quad \delta a_\mu^{\mathrm{HLbL}}/a_\mu^{\mathrm{HLbL}} \lesssim 10\% \tag{3}$$

in the coming years.

Primary object for getting HVP in LQCD is the e.m. current correlator in configuration space

$$\langle J_\mu(\vec{x},t)\, J_\nu(\vec{0},0)\rangle \;; \quad J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \cdots \tag{4}$$

In principle, a Fourier transform

$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4 x e^{\mathrm{i}\,Qx}\, \langle J_\mu(x)\, J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}\, Q^2\right)\, \Pi(Q^2) \tag{5}$$

yields the vacuum polarization function $\Pi(Q^2)$ needed to calculate

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2\, f(Q^2)\, \left\{\Pi(Q^2) - \Pi(0)\right\} \;. \tag{6}$$

The integration kernel in this representation is

$$f(Q^2) = w(Q^2/m_\mu^2)/Q^2 \;; \quad w(r) = \frac{16}{r^2\left(1+\sqrt{1+4/r}\right)^4\,\sqrt{1+4/r}} \;. \tag{7}$$

As LQCD per se has to work on a lattice in a finite box, momenta are quantized $Q_{\mathrm{min}} = 2\pi/L$, where

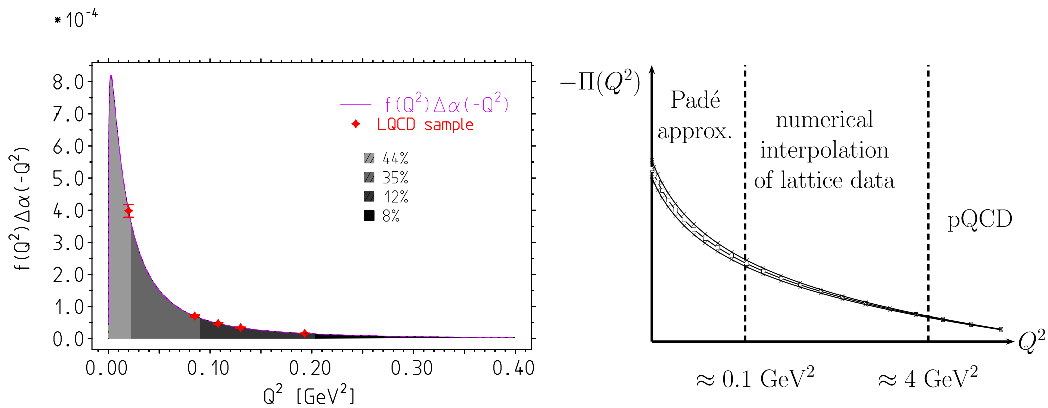


Figure 11. Left: the integrand of (6) as a function of Q^2. Also shown are ranges between Q_i = 0.00, 0.15, 0.30, 0.45 and 1.0GeV and their percent contribution to a_μ^{had} and a possible "LQCD sample". Right: typical ranges for lattice data and their extrapolation to low Q^2.

L is the lattice box length. Therefore, approaching low momenta $Q_{\mathrm{min}} \to 0 \Leftrightarrow L \to \infty$ requires a sufficiently large volume. Present state of the art calculations reach $Q_{\mathrm{min}} = 2\pi/L$ with $m_\pi a L \gtrsim 4$ for $m_\pi \sim 200$MeV, such that $Q_{\mathrm{min}} \sim 314$MeV. This means that about 44% of the low Q contribution to a_μ^{had} is not covered by data yet. Typically, lattice data are available for $Q^2 > (2\pi/L)^2$, which one has to extrapolate to $Q^2 = 0$ by VMD type modeling [48] or via Padé's [49] or analytic continuation [51]. The method requires a reliable estimate of the bare $\Pi(0)$ (see e.g. [50]). In order to reach the required accuracy one needs LQCD data down to $Q^2_{\mathrm{min}} \approx 0.1\mathrm{GeV}^2$. [51–55] Some recent results are collected in figure 12.

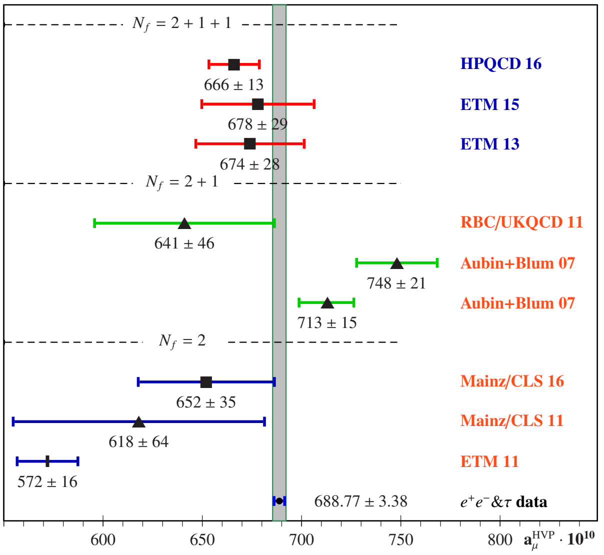


Figure 12. Summary of recent LQCD results for the leading order $a_\mu^{\rm HVP}$, in units 10^{-10}. Labels: ■ marks u, d, s, c, ▲ u, d, s and ❙ u, d contributions. Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%. Results shown are from HPQCD 16 [56], ETM 15 [57], ETM 13 [58], RBC/UKQCD 11 [52], Aubin+Blum 07 [59], Mainz/CLS 16 [60], Mainz/CLS 11 [61] and ETM 11 [48]. The vertical band shows the e^+e^- data driven DR estimate (2).

7 Alternative method to get $a_\mu^{\rm had}$: using $\alpha(t = -Q^2)$ measured via t–channel exchange processes.

A promising alternative method to determine $a_\mu^{\rm had}$ is possible by a dedicated measurement of $\alpha(t)$ at spacelike momentum transfer as advocated in [62] and [63]. Given $\alpha(-Q^2)$ and the fact that the leptonic contribution is well under control in perturbation theory one can extract the hadronic shift

$$\Delta\alpha^{\rm had}(-Q^2) = 1 - \frac{\alpha}{\alpha(-Q^2)} - \Delta\alpha^{\rm lep}(-Q^2) \tag{8}$$

and determine $a_\mu^{\rm had}$ via the representation

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 {\rm dx}\,(1-{\rm x})\,\Delta\alpha^{\rm had}\left(-{\rm Q}^2({\rm x})\right) \tag{9}$$

where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the spacelike square momentum–transfer. In the Euclidean region the integrand is highly peaked around half of the ρ meson mass scale (see figure 13). The method is very different from the standard approach based on (1): radiative corrections are very different (much simpler) as no hadronic final states need to be understood, no VP subtraction is to be performed, no exclusive channel collection etc. So, even a 1% level measurement can provide important independent information. This in view of the problem to get accurate hadronic total cross–section in the range

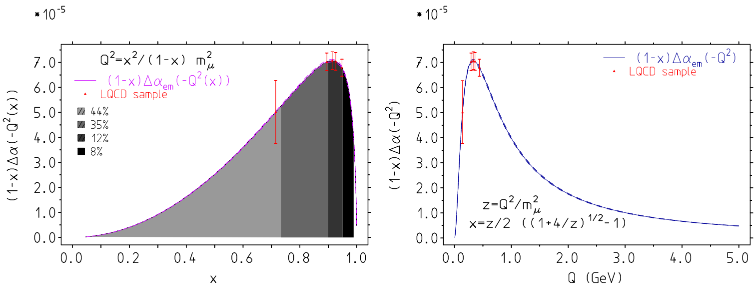


Figure 13. The integrand of the $a_\mu^{\rm had}$ integral (9) as functions of x and Q is strongly peaked at about 330MeV. Legend as in figure 11.

between 1 and 2 GeV and possible unsettled problems (non-convergence of the Dyson resummation near OZI suppressed resonances) in VP subtraction as addressed recently in Sect. 5 of [1].

The possible processes to measure $\alpha(t)$ are Bhabha scattering $e^+(p_+)\ e^-(p_-) \to e^+(p'_+)\ e^-(p'_-)$ or muon electron scattering (see figure 14). The Bhabha process has two tree level diagrams a t– and

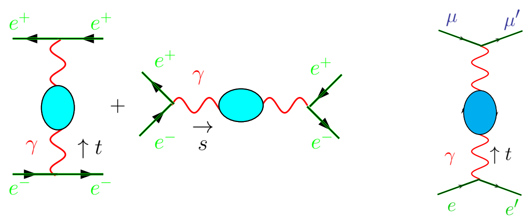


Figure 14. Measuring the running charge in the spacelike regime. Left: VP dressed tree level Bhabha scattering in QED, for small t (small angels) the s channel is suppressed. Right: getting $\alpha(t)$ from the $\mu^- e^- \to \mu^- e^-$ process.

a s–channel one. With the positive c.m. energy square $s = (p^+ + p^-)^2$ and the negative momentum transfer square $t = (p_- - p'_-)^2 = -\frac{1}{2}\,(s - 4m_e^2)\,(1-\cos\theta)$, θ the e^- scattering angle, there are two very different scales involved which helps to isolate the t channel of interest. The VP dressed lowest order cross–section is

$$\frac{{\rm d}\sigma}{{\rm d}\cos\Theta} = \frac{s}{48\pi}\ \sum\nolimits_{ik} |A_{ik}|^2\,, \tag{10}$$

where A_{ik} are tree level helicity amplitudes, i,k =L,R denote left– and right–handed electrons.

The dressed transition amplitudes in the massless limit ($m_e \approx 0$) read

$$|A_{\rm LL,RR}|^2 = \frac{3}{8}\,(1+\cos\theta)^2\,\left|\frac{e^2(s)}{s} + \frac{e^2(t)}{t}\right|^2\ ;\ \ |A_{\rm LR,RL}|^2 = \frac{3}{8}\,(1-\cos\theta)^2\,\left|\frac{e^2(s)}{s} + \frac{e^2(t)}{t}\right|^2\ . \tag{11}$$

Preferably one uses small angle Bhabha scattering (small $|t|$) as a normalizing process which is dominated by the t–channel $\sim 1/t$, however, detecting electrons and positrons along the beam axis often has its technical limitations. Care also is needed concerning the ISR corrections because cuts for the Bhabha process ($e^+e^- \to e^+e^-$) typically are different from the ones applied to $e^+e^- \to$ hadrons. Usually, experiments have included corresponding uncertainties in their systematic errors, if they have not been explicitly accounted for by applying appropriate radiative corrections. For details I refer to [62] and the Contribution by Luca Trentadue.

While the Bhabha process requires to sort out the s channel from the t channel, the pure t channel reaction of $\mu^- e^-$ scattering $\mu^-(p_-)\ e^-(q_-) \to \mu^-(p'_-)\ e^-(q'_-)$ provides a much simpler setup and could be realized as a fixed target experiment [63] at existing facilities. The leading order cross–section in this case has the simple form

$$\frac{\mathrm{d}\sigma^{\mathrm{unpol.}}_{\mu^-\mathrm{e}^-\to\mu^-\mathrm{e}^-}}{\mathrm{dt}} = 4\pi\,\alpha(t)^2\,\frac{1}{\lambda(s,m_e^2,m_\mu^2)}\left\{\frac{\left(s-m_\mu^2-m_e^2\right)^2}{t^2}+\frac{s}{t}+\frac{1}{2}\right\}, \tag{12}$$

exhibiting the effective charge as an overall factor. For details see Luca Trentadue's Contribution. Such an experiment would provide data for the Euclidean electromagnetic current correlator $\Pi'_\gamma(Q^2)-\Pi'_\gamma(0) = -\Delta\alpha^{\mathrm{had}}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\mathrm{lep}}(-Q^2) - 1$ and would allow for a directly check of lattice QCD data. In addition one could determine $\Delta\alpha^{\mathrm{had}}\left(-Q^2\right)$ at $Q \approx 2.5$GeV by this method (one single number!) as the non-perturbative part of $\Delta\alpha^{\mathrm{had}}\left(M_Z^2\right) = \Delta\alpha^{\mathrm{had}}\left(-Q^2\right) + \mathrm{pert.}$ when evaluated in "Adler function" approach advocated in [64].

8 Theory confronts experiment

Table 3. Standard model theory and experiment comparison [in units 10^{-10}].

Contribution	Value	Error	Reference
QED incl. 4-loops+5-loops	11 658 471 . 8851	0 . 036	Remiddi, Kinoshita et al.
Leading hadronic vac. pol.	688 . 77	3 . 38	data-driven $e^+e^- + \tau$
Subleading hadronic vac. pol.	-9 . 927	0 . 072	2016 update
NNLO hadronic vac. pol.	1 . 224	0 . 010	[31]
Hadronic light–by–light	10 . 34	2 . 88	[46, 69]
Weak incl. 2-loops	15 . 36	0 . 11	[11, 70]
Theory	11 659 177 . 6	4 . 4	–
Experiment	11 659 209 . 1	6 . 3	[2] updated
Exp.- The. 4.1 standard deviations	31 . 3	7 . 7	–

Table 3 summarizes the present status of the SM prediction for a_μ in comparison with the experimental value [2]. For a recent update of the weak contribution see [70]. As an estimate based on [4–6, 10, 46, 65, 68, 69] we adopt π^0, η, η' [95 ± 12] + axial–vector [8 ± 3] + scalar [−6 ± 1] + π, K loops [−20 ± 5] + quark loops [22 ± 4] + tensor [1 ± 0] + NLO [3 ± 2] which yields

$$a_\mu^{(6)}(\mathrm{lbl,had}) = (103 \pm 29) \times 10^{-11}\ . \tag{13}$$

The result differs little from the "agreed" value $(105 \pm 26) \times 10^{-11}$ presented in [47] and $(116 \pm 39) \times 10^{-11}$ estimated in [46]. Both included a wrong, too large, Landau-Yang theorem violating axial–vector contribution from [10], correcting for this we obtain our reduced value relative to [46].

The following tabular collects recent new/updated evaluations:

New contribution	Reference			$\Delta a_\mu \cdot 10^{11}$
NNLO HVP	Kurz et al. 2014	12.4	±	0.1
NLO HLbL	Colangelo et al. 2014	3	±	2
New axial exchange HLbL	Pauk, Vanderhaeghen [66], FJ14 [1, 67]	7.55	±	2.71
Tensor exchange HLbL	Pauk, Vanderhaeghen 2014	1.1	±	0.1
New π^0 exchange HLbL	$\pi^0\gamma^*\gamma^*$ constraint from LQCD [68]	64.68	±	12.40
...				...
Old axial exchange HLbL	Melnikov, Vainshtein 2004	22	±	5
Old π^0 exchange HLbL	JN [46]	72	±	12
Total change		−5.6	±	12.85 [← 13]

The uncertainty from these contributions remains unchanged, while the central value is shifted downwards by almost 1 SD.

Possible interpretations of the 4 σ deviation: new physics?, a statistical fluctuation?, underestimated uncertainties (experimental, theoretical)? Do experiments measure what theoreticians calculate? The challenge for the future is to keep up with the future experiments, which will improve the experimental accuracy from $\delta a_\mu^{\rm exp} = 63 \times 10^{-11}$ [±0.54 ppm] at present to $\delta a_\mu^{\rm exp} = 16 \times 10^{-11}$ [±0.14 ppm] the next years. Next generation experiments require a factor 4 reduction of the uncertainty optimistically feasible should be a factor 2 we hope.

In view of the upcoming two **complementary** experiments, one at Fermilab working with ultra hot muons and the other at J-PARC operating with ultra cold muons (very different radiation effects), the big challenge is to keep up on the prediction side as much as possible. The deviation between theory and experiment can be scrutinized provided theory and the needed cross section data improves the same as the muon $g-2$ experiments. Primarily we need more/better data and/or progress in non-perturbative QCD, where the main obstacle (data, lattice QCD, RLA) is the hadronic light-by-light scattering contribution. Progress in evaluating HVP also depends on more data (BaBar, Belle, VEPP-2000, BESIII,...) and lattice QCD where recent progress is very promising (see figure 12). In both cases HVP as well as HLbL, lattice QCD will provide answers one day, but also low energy effective RL and DR approaches need be further developed. One has also to keep in mind that progress in calculations of radiative corrections [71–73] is mandatory in precision measurements of hadronic cross sections.

For future improvements of HLbL one urgently needs more information from $\gamma\gamma \to$ hadrons physics [74, 75] in order to have better constraints on modeling hadronic amplitudes (see [76–78] for theoretical studies). Some sample processes are collected in figure 15. Mostly experiments at e^+e^-–

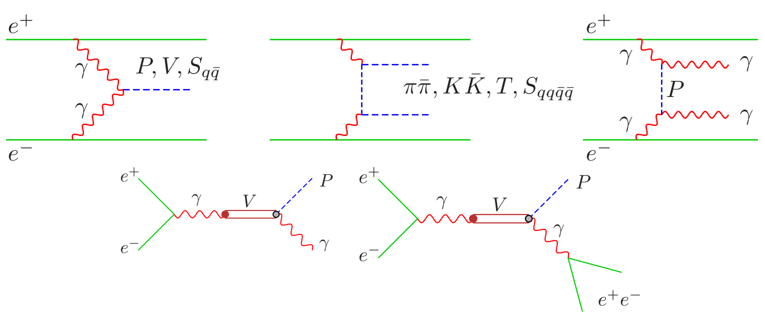


Figure 15. Sample processes relevant for the dispersive approach to HLbL.

facilities investigate single-tag events (higher rates, lower background). New data are expected from KLOE, KEDR exhibiting taggers and from BaBar, Belle, BES III which have high luminosity. More information is also expected from Dalitz–decay studies $\rho, \omega, \phi \to \pi^0(\eta)e^+e^-$ possible at Novosibirsk, CERN NA60, JLab, Mainz, Bonn, Jülich and BES. Unfortunately some of the interesting processes seem to be buried in the background. The background is a general problem in $\gamma\gamma \to$ hadrons physics.

The dispersive approach [8, 66] is able to allow for real progress since contributions which we have treated so far as separate contributions will be treated in an integral manner. An example is the $\gamma\gamma \to \pi\pi$ process which includes contributions attributed to the two–pion channel , the pion–loop, the scalar contribution as well as the tensor contribution. All-in-one can be gotten from the experimental data (see e.g. figure 3 of [79]). This also will settle such issues as the pion polarizability. A lot remains to be done while new $a_\mu^{\rm exp}$ is expected soon. For details see the Contribution by M. Procura.

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References

[1] F. Jegerlehner, EPJ Web Conf. **118**, 01016 (2016)

[2] G. W. Bennett et al. [Muon g-2 Collaboration], Phys. Rev. D **73**, 072003 (2006)

[3] D. W. Hertzog, EPJ Web Conf. **118**, 01015 (2016)

[4] M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. **75**, 790 (1995); Phys. Rev. D **54**, 3137 (1996); M. Hayakawa, T. Kinoshita, Phys. Rev. D **57**, 465 (1998) [Erratum-ibid. D **66**, 019902 (2002)];

[5] J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. **75**, 1447 (1995) [Erratum-ibid. **75**, 3781 (1995)]; Nucl. Phys. B **474**, 379 (1996); [Erratum-ibid. **626**, 410 (2002)];
J. Bijnens, J. Prades, Mod. Phys. Lett. A **22**, 767 (2007)

[6] M. Knecht, A. Nyffeler, Phys. Rev. D **65**, 073034 (2002)

[7] M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, Eur. Phys. J. C **72**, 1848 (2012); **73**, 2453 (2013)

[8] G. Colangelo et al., Phys. Lett. B **738**, 6 (2014)

[9] M. Knecht, S. Peris, M. Perrottet, E. de Rafael, JHEP **0211**, 003 (2002)

[10] K. Melnikov, A. Vainshtein, Phys. Rev. D **70**, 113006 (2004)

[11] A. Czarnecki, W. J. Marciano, A. Vainshtein, Phys. Rev. D **67**, 073006 (2003) [Erratum-ibid. D **73**, 119901 (2006)]

[12] R. R. Akhmetshin et al. [CMD-2 Collab.], Phys. Lett. B **578**, 285 (2004); V. M. Aulchenko et al. [CMD-2 Collab.], JETP Lett. **82**, 743 (2005) [Pisma Zh. Eksp. Teor. Fiz. **82**, 841 (2005)]; R. R. Akhmetshin et al., JETP Lett. **84**, 413 (2006) [Pisma Zh. Eksp. Teor. Fiz. **84**, 491 (2006)]; Phys. Lett. B **648**, 28 (2007)

[13] M. N. Achasov et al. [SND Collab.], J. Exp. Theor. Phys. **103**, 380 (2006) [Zh. Eksp. Teor. Fiz. **130**, 437 (2006)]

[14] A. Aloisio et al. [KLOE Collab.], Phys. Lett. B **606**, 12 (2005); F. Ambrosino et al. [KLOE Collab.], Phys. Lett. B **670**, 285 (2009)

[15] F. Ambrosino et al. [KLOE Collab.], Phys. Lett. B **700**, 102 (2011)

[16] D. Babusci et al. [KLOE Collab.], Phys. Lett. B **720**, 336 (2013)

[17] B. Aubert et al. [BABAR Collab.], Phys. Rev. Lett. **103**, 231801 (2009); J. P. Lees et al., Phys.Rev. **D86**, 032013 (2012)

[18] M. Ablikim et al. [BESIII Collab.], Phys. Lett. B **753**, 629 (2016)
[19] B. Aubert et al. [BABAR Collab.], Phys. Rev. D **70**, 072004 (2004); **71**, 052001 (2005); **73**, 012005 (2006); **73**, 052003 (2006); **76**, 012008, ibid. 092005, ibid. 092006, (2007); **77**, 092002 (2008)
[20] J. P. Lees et al. [BaBar Collab.], Phys. Rev. D **85**, 112009 (2012); **86**, 012008 (2012); **87**, 092005 (2013); **88**, 032013 (2013); **89**, 092002 (2014)
[21] M. Davier, Nucl. Part. Phys. Proc. **260**, 102 (2015)
[22] M. Davier, A. Höcker, B. Malaescu, Z. Zhang, Adv. Ser. Direct. High Energy Phys. **26**, 129 (2016)
[23] R. R. Akhmetshin et al. [CMD-3 Collaboration], Phys. Lett. B **723**, 82 (2013); **759**, 634 (2016)
[24] E. A. Kozyrev et al. [CMD-3 Collaboration], Phys. Lett. B **760**, 314 (2016)
[25] M. N. Achasov et al., Phys. Rev. D **90**, no.11, 112007 (2014)
[26] V. M. Aulchenko et al. [SND Collaboration], Phys. Rev. D **91**, no.5, 052013 (2015)
[27] M. N. Achasov et al. [SND Collaboration], Phys. Rev. D **93**, no.9, 092001 (2016); **94**, no.3, 032010 (2016); **94**, no.9, 092002 (2016); **94**, no.11, 112006 (2016); **94**, no.11, 112001 (2016)
[28] J. Z. Bai et al. [BES Collab.], Phys. Rev. Lett. **84**, 594 (2000); Phys. Rev. Lett. **88**, 101802 (2000); M. Ablikim et al., Phys. Lett. B **677**, 239 (2009)
[29] V. V. Anashin et al., Phys. Lett. B **753**, 533 (2016); arXiv:1610.02827 [hep-ex].
[30] B. Krause, Phys. Lett. B **390**, 392 (1997)
[31] A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Phys.Lett. **B734**, 144 (2014)
[32] G. Colangelo et al., Phys.Lett. **B735**, 90 (2014)
[33] A. Anastasi et al. [KLOE-2 Collaboration], doi:10.1016/j.physletb.2016.12.016 arXiv:1609.06631 [hep-ex].
[34] R. Barate et al. [ALEPH Collab.], Z. Phys. C **76**, 15 (1997); Eur. Phys. J. C **4**, 409 (1998); S. Schael et al. [ALEPH Collab.], Phys. Rept. **421**, 191 (2005)
[35] M. Davier et al., Eur.Phys.J. **C74**, 2803 (2014)
[36] K. Ackerstaff et al. [OPAL Collab.], Eur. Phys. J. C **7**, 571 (1999)
[37] S. Anderson et al. [CLEO Collab.], Phys. Rev. D **61**, 112002 (2000)
[38] M. Fujikawa et al. [Belle Collab.], Phys. Rev. D **78**, 072006 (2008)
[39] M. Benayoun, P. David, L. DelBuono, F. Jegerlehner, Eur. Phys. J. C **75**, no.12, 613 (2015)
[40] F. Jegerlehner, R. Szafron, Eur. Phys. J. C **71**, 1632 (2011)
[41] M. Davier, A. Höcker, B. Malaescu, Z. Zhang, Eur. Phys. J. C **71**, 1515 (2011) [Erratum-ibid. C **72**, 1874 (2012)]
[42] M. Davier et al., Eur. Phys. J. C **66**, 127 (2010)
[43] K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, T. Teubner, J. Phys. G G **38**, 085003 (2011)
[44] M. Davier, arXiv:1612.02743 [hep-ph].
[45] Z. Zhang, EPJ Web Conf. **118**, 01036 (2016) doi:10.1051/epjconf/201611801036
[46] F. Jegerlehner, A. Nyffeler, Phys. Rept. **477**, 1 (2009)
[47] J. Prades, E. de Rafael, A. Vainshtein, Adv. Ser. Direct. High Energy Phys. **20**, 303 (2009)
[48] X. Feng, K. Jansen, M. Petschlies, D. B. Renner, Phys. Rev. Lett. **107**, 081802 (2011)
[49] M. Golterman, K. Maltman, S. Peris, Phys. Rev. D **90**, no.7, 074508 (2014)
[50] S. Borsanyi et al., arXiv:1612.02364 [hep-lat].
[51] X. Feng et al., Phys. Rev. D **88**, 034505 (2013)
[52] P. Boyle, L. Del Debbio, E. Kerrane, J. Zanotti, Phys. Rev. D **85**, 074504 (2012)
[53] C. Aubin, T. Blum, M. Golterman, S. Peris, Phys. Rev. D **88**, 074505 (2013)

[54] A. Francis et al., arXiv:1411.3031 [hep-lat]
[55] R. Malak et al. [Budapest-Marseille-Wuppertal Collab.], PoS LATTICE **2014**, 161 (2015)
[56] B. Chakraborty, C. T. H. Davies, P. G. de Oliviera, J. Koponen, G. P. Lepage, arXiv:1601.03071 [hep-lat].
[57] F. Burger, X. Feng, K. Jansen, M. Petschlies, G. Pientka, D. B. Renner, EPJ Web Conf. **118**, 01029 (2016)
[58] F. Burger et al. [ETM Collab.], JHEP **1402**, 099 (2014)
[59] C. Aubin, T. Blum, Phys. Rev. D **75**, 114502 (2007)
[60] M. Della Morte, G. Herdoiza, H. Horch, B. Jäger, H. Meyer, H. Wittig, PoS LATTICE **2015**, 111 (2015) [arXiv:1602.03976 [hep-lat]]
[61] M. Della Morte, B. Jäger, A. Jüttner, H. Wittig, JHEP **1203**, 055 (2012)
[62] C. M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, Phys. Lett. B **746**, 325 (2015)
[63] G. Abbiendi et al., arXiv:1609.08987 [hep-ex]
[64] F. Jegerlehner, Nucl. Phys. Proc. Suppl. **181-182**, 135 (2008); *Hadronic effects in* $(g-2)_\mu$ *and* $\alpha_{\rm QED}(M_Z)$*: Status and perspectives*, In: *Radiative Corrections*, ed by J. Solà (World Scientific, Singapore 1999) pp 75–89 [hep-ph/9901386].
[65] A. Nyffeler, EPJ Web Conf. **118**, 01024 (2016)
[66] V. Pauk, M. Vanderhaeghen, Eur. Phys. J. C **74**, 3008 (2014)
[67] F. Jegerlehner, Talk at the MITP Workshop "Hadronic contributions to the muon anomalous magnetic moment", 1-5 April 2014, Waldthausen Castle near Mainz,
[68] A. Gérardin, H. B. Meyer, A. Nyffeler, arXiv:1607.08174 [hep-lat]
[69] J. Bijnens, J. Relefors, arXiv:1608.01454 [hep-ph].
[70] C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D **88**, 053005 (2013)
[71] S. Actis et al., Eur. Phys. J. C **66**, 585 (2010)
[72] H. Czyż, Int. J. Mod. Phys. Conf. Ser. **35**, 1460402 (2014) doi:10.1142/S2010194514604025
[73] F. Jegerlehner, K. Kołodziej, arXiv:1701.01837 [hep-ph].
[74] D. Babusci et al., Eur. Phys. J. C **72**, 1917 (2012)
[75] A. Nyffeler, PoS CD **12**, 045 (2013) [arXiv:1306.5987 [hep-ph]]
[76] G. Mennessier, Z. Phys. C **16**, 241 (1983)
[77] B. Moussallam, Eur. Phys. J. C **73**, 2539 (2013)
[78] L. Y. Dai, M. R. Pennington, Phys. Rev. D **90**, no.3, 036004 (2014)
[79] F. Jegerlehner, Acta Phys. Polon. B **44**, no.11, 2257 (2013)