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Why do Electrons with “Anomalous Energies” appear in High-Pressure Gas Discharges?

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Abstract. Experimental studies connected with runaway electron beams generation convincingly shows the existence of electrons with energies above the maximum voltage applied to the discharge gap. Such electrons are also known as electrons with “anomalous energies”. We explain the presence of runaway electrons having so-called “anomalous energies” according to physical kinetics principles, namely, we describe the total ensemble of electrons with the distribution function. Its evolution obeys Boltzmann kinetic equation. The dynamics of self-consistent electromagnetic field is taken into the account by adding complete Maxwell’s equation set to the resulting system of equations. The electrodynamic mechanism of the interaction of electrons with a travelling-wave electric field is analyzed in details. It is responsible for the appearance of electrons with high energies in real discharges.

1 Introduction

This paper deals with the phenomenon of runaway electrons generation in fast gas discharges under high pressures. The generation of runaway electrons in high-pressure pulsed discharges is a fundamental physical phenomenon. The possibility of runaway electrons appearing in the Earth atmosphere was predicted at first in 1925 [1], and much later the prediction was successfully proven in laboratory experiments [2].

At present the fact of fast (runaway) electrons detection can be firmly established at the initial stage of high-pressure gas breakdown in discharge gaps with strongly non-uniform electric field. At the same time, various researchers obtain fast electron current pulses with largely spread parameters: amplitudes from 0.1 up to tens of amperes with durations from tens picoseconds to nanoseconds [3, 4]. As the number of runaway electrons is strongly depends on several critical parameters (gas type, geometric enhancement of the electric field near the sharp edges of electrodes, scales of field-enhancement regions, time-resolution of the experimental equipment), so experimental results of fast electrons detection will also differ considerably.

Owing to modern experimental equipment and novel methods of experimental data analysis, it was shown that the runaway electron beams have a broadband spectrum containing several groups of electrons with different mean energies [5, 6]. The main amount of runaway electrons constitutes the group of particles with mean energy corresponding to the voltage amplitude at the gap. However, it was discovered that power spectrum also includes a small group of electrons with mean energy much greater than the applied values. Such electrons represent a group of electrons with so-called “anomalous energies”.

For high-pressure discharges, it was shown that number of electrons with “anomalous” energies usually does not exceed ten percent of the total number of runaway electrons. On the other hand, electrons with “anomalous energies” do not yield to the equilibrium Maxwellian distribution, i.e. pointed out to the nontrivial mechanism of the formation of a runaway electron beam. These experimental results showed the need to formulate new theoretical models allow explaining the appearance of the “anomalous” runaway electron beam component self-consistently.

Essential non-stationarity and spatial three-dimensionality in real experiments represent a great challenge for the theoretical modeling. So simple zero-dimensional and one-dimensional plate theoretical models allowing to understand the mechanism of runaway electron beam formation, do not provide ideal correlations with the existing experimental data [7, 8].

A more realistic one-dimensional coaxial model provides better agreement between the theoretical and experimental data [9, 10]. Simulation show that the breakdown of the coaxial gas-filled gap occurs due to the rapid propagation of the ionization wave front between inner electrode (cathode) and outer electrode (anode). Namely, the necessary conditions for the generation of runaway electrons with "anomalous energies" are formed at the front of the ionization wave.

The aim of this paper is to describe in details the physical mechanism of electron acceleration to "anomalous energies".

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2 The pilot problem of electron acceleration by travelling electric field pulse

2.1 Statement of the model problem

We consider the one-dimensional interaction of a travelling-wave longitudinal electric field pulse with a single electron (initially at rest), as shown in Fig. 1. For the simplicity reason, it is convenient to consider quite simple rectangular shape of field strength profile. Let the amplitude of the pulse is equal to E_0 and the pulse width is equal to L.

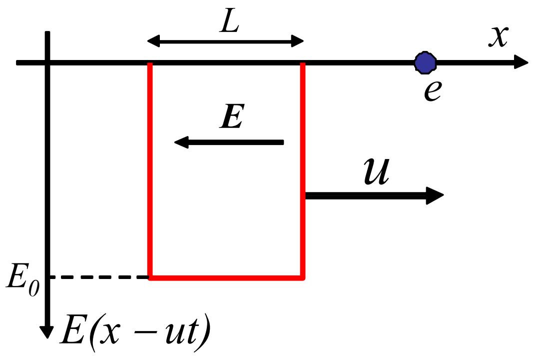

Fig. 1. Schematics of the model problem.

The field pulse moves from left to right with a constant velocity, u, and at some time point overlaps the rest electron. Electron starts to move accelerated if the electric field acts on it. In Fig. 2 the area where the electron interacts with travelling-wave field is shown with shaded region at the coordinate-time plane.

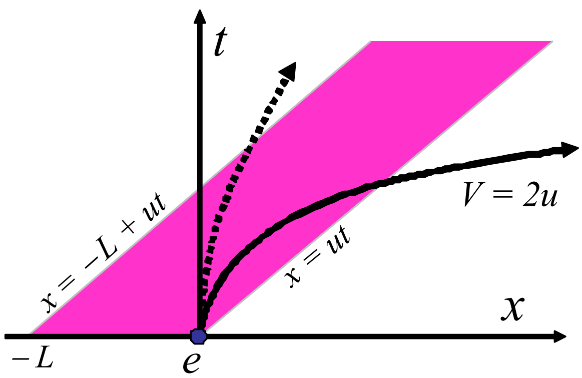

Fig. 2. Two modes of interaction of a field pulse and an electron. The solid line describes the “reflecting mode”, the dashed line corresponds to the “escaping mode”.

2.2 Mathematical description of the rectangular field pulse profile

For simplicity, let us consider the non-relativistic case $u \ll c$ (c is a speed of light). The generalization to the case of relativistic velocities is straightforward, it is only slightly complicates the mathematical formulation. While the electron is in the scope of travelling-wave, it moves with the constant acceleration (e is elementary charge, and m is mass of electron):

$$x(t) = \left(eE_0 / 2m\right)t^2. \quad (1)$$

As can be seen in Fig. 2, there are two fundamentally different scenarios of such interaction.

2.2.1 Escaping mode of interaction

If the rear edge of the electric field pulse advances the electron, the latter gains lower velocity. This mode is shown in Fig. 2 with the dashed curve. The phase trajectories of the electron and the trailing edge of the field pulse will intersect if the quadratic equation has a real root (when its discriminant D is positive):

$$\begin{gathered} x(t) = \left(eE_0 / 2m\right)t^2 = -L + ut, \\ D = u^2 - \left(2eE_0 L / m\right) > 0, \\ \frac{mu^2}{2} > eE_0 L. \end{gathered} \quad (2)$$

If the last condition of (2) is satisfied, then the final velocity of the electron remains less than the velocity of the field pulse, u. In this interaction mode, the electron escapes through the traveling electric field area. Therefore, it can be called the “escaping mode”. This mode takes place at a low voltage drop across the pulse, $eU_0 = eE_0L$, as compared to the “relative kinetic energy” of the electron, $mu^2/2$.

2.2.2 Reflecting mode of interaction

To understand the cause of the appearance electrons with anomalous energies more important is another mode of interaction, when the condition (2) is not satisfied. In this case, the accelerating electron eventually outpaces the field pulse and leaves ahead (solid curve in Fig. 2). We can say that the electron is reflected from the incoming high-voltage pulse, and this mode can be called the “reflecting mode”.

It is easy to show that the velocity of the electron that leaves forward is exactly twice the velocity of the field pulse. Indeed, the interaction time t_0 is determined by the equation: $x(t) = \left(eE_0 / 2m\right)t_0^2 = ut_0$. Therefore, the final velocity of the electron will be equal to $V = \left(eE_0 / m\right)t_0 = 2u$. The kinetic energy of the electron, ε, is appropriate to be written together with the condition of the reflecting mode:

$$\varepsilon = \frac{mV^2}{2} = 2mu^2, \quad \frac{mu^2}{2} < eU_0. \quad (3)$$

In accordance with (3), the kinetic energy of the electron will be limited from above:

$$\varepsilon = 2mu^2 = 4\frac{mu^2}{2} \le 4eU_0. \quad (4)$$

The last inequality shows that the electron passed forward may have kinetic energy exceeding the value of eU_0.

2.3 Generalization for general cases

After analyzing the simplest problem, we can extend our conclusions to more general cases. In particular, it is possible to extend the expressions (3) to more general profiles of the field pulse and to the relativistic energies.

2.3.1 Extension to more general pulse profiles

The fact that in the reflecting mode the electron acquires an exactly double value of the velocity of the traveling field pulse occurs for any bell-shaped pulse profiles $E(s)$. The restriction applies only to the function $E(s)$ was single-humped and did not change the sign of the field.

For such a pulse profile, it is possible to determine the voltage drop

$$U_0 = \int_{-\infty}^{+\infty} E(s)ds \,. \qquad (5)$$

If we go over to the reference frame in which the pulse profile is stationary, then the electron bumps into the potential barrier with the same velocity u. If the height of the potential barrier will be greater than kinetic energy of the incident electron, it is reflected from it in the opposite direction at the same speed. With the reverse transformation to the laboratory frame, we obtain exactly twice the value of the velocity $V = 2u$.

Thus, expression (3) is completely valid for a more general field-pulse profile. The Fig. 3 shows the final kinetic electron energy from the velocity field pulse.

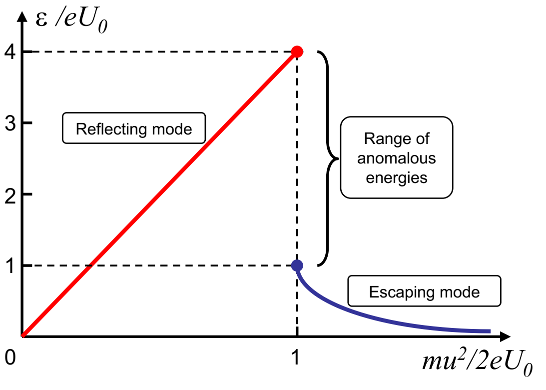

Fig. 3. Dependence of the final kinetic energy of the electron from the speed of motion of the field pulse at $u \ll c$.

2.3.2 Extension to relativistic energies

Finally, it is possible to extend the calculations to the relativistic energy range. In relativistic mechanics, it is customary to express the kinetic energies of particles ε using the relativistic factor γ.

Turning to the reference system, tied to a field pulse, and then back to the laboratory frame of reference, one can obtain the following expression for the velocity and kinetic energy of the electron interaction in the reflectance mode. The condition of the "reflecting mode" (3) is written in the following form:

$$mc^2(\gamma - 1) < eU_0, \quad \gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}. \qquad (6)$$

The electron velocity, V, in the laboratory system after its reflection from the traveling field pulse will be determined by the law of velocity composition:

$$V = \frac{2u}{1 + u^2 / c^2}. \qquad (7)$$

The kinetic energy of the reflected electron, ε, is uniquely determined by the field pulse factor γ:

$$\varepsilon = mc^2\left(\frac{1}{\sqrt{1 - V^2 / c^2}} - 1\right) = mc^2 \cdot 2\left(\gamma^2 - 1\right). \qquad (8)$$

Formulas (5), (6) and (8) generalize two formulas (3) to arbitrary pulse profile and relativistic energies. For low potential barriers $eU_0 \ll mc^2$ from (8) we obtain the classical formulas (4) for the maximum electron energy: $\varepsilon_{max} \approx 4 \cdot eU_0$. But, in the ultra-relativistic limit, when $eU_0 \gg mc^2$, $\gamma \gg 1$ take place, we obtain expression for the maximum electron energy: $\varepsilon_{max} \approx 2\gamma \cdot eU_0$.

3 Kinetic model of high-pressure gas discharge in coaxial gap

Let us show that in the nanosecond breakdown of a gas-filled coaxial gap there are conditions for the generation of electrons with "anomalous energies". The theoretical analysis is based on the model of the kinetic description of the electronic component of gas-discharge plasma [11, 12].

We are going to describe the discharge process by using of the kinetic Boltzmann relativistic equation for electron distribution function (EDF) with the model right-hand part:

$$\gamma\left(\frac{\partial f}{\partial t} + \frac{p}{m\gamma}\frac{\partial f}{\partial r} - eE(r,t)\frac{\partial f}{\partial p}\right) = -Q_- + Q_+ + S_{sc}. \quad (9)$$

Here $f(r, p, t)$ is the EDF depending on time t and radial coordinate r, $p = mv\gamma$ is the relativistic momentum, $\gamma = \sqrt{1 + (p/mc)^2}$ is the relativistic factor, E is the electric field strength .

The first term Q_- on the right-hand side of (9) describes the loss of electrons in a given element of the phase space due to inelastic electron-atom collisions, the second term Q_+ describes the production of electrons in an element of the phase space in the same inelastic processes, and the third term S_{sc} describes the effect of scattering, in particular, elastic collisions on the EDF.

In the framework of the kinetic approach, we must write the integral expression for the convective electron current in any cross section of the discharge gap:

$$J_e(r,t) = -\frac{e}{m}\int_{-\infty}^{\infty}(p/\gamma) f(r,p,t)\,dp. \qquad (10)$$

A coaxial diode (cathode radius r_c, anode cylinder radius r_a, discharge tube length L) is chosen as a simplified model of a discharge with nonuniform discharge-gap geometry. An electric pulse $U_0(t)$ with a steep leading edge and an amplitude by far exceeding the static gas breakdown voltage is applied to the gap switched in series with a voltage supply and a ballast resistance R. This equivalent power-supply circuit matches well with the discharge of the pulse forming line with impedance R loaded onto a diode, which is used in most experiments.

Using the Kirchhoff rule for the complete circuit, we can relate the voltage drop across the gap to the value of the total current in the discharge circuit J_{tot}:

$$J_{tot}(t) = -\frac{1}{2\pi rLR}\left(U_0(t) + \int_{r_c}^{r_a} E(r,t)\,dr\right). \qquad (11)$$

Finally, using the conservation law for the total current (convective current of electrons plus Maxwell's displacement current), we can close our system by the evolution equation for the intensity of the electric field in the gap (ε_0 is vacuum permittivity):

$$\varepsilon_0 \frac{\partial E(r,t)}{\partial t} = J_{tot}(t) - J_e(r,t). \qquad (12)$$

The above system of equations (9)-(12) represents mathematical equations of the kinetic model of a fast gas discharge where the electron component of the discharge plasma is described from the first principles in terms of the EDF evolution.

4 Results of the discharge simulation in nitrogen

A discharge in a coaxial diode filled with nitrogen (cathode radius r_c = 1 mm, anode radius r_a = 10 mm, length L = 1 cm) is developed at a pressure of 760 Torr. A voltage pulse with leading edge duration 300 ps, in its level 0.1–0.9 of the amplitude 200 kV, was fed to the diode switched in series with ballast resistance R = 75 Ω.

To simplify the problem, we took into account only two types of electron-atom collisions, namely, impact ionization and elastic scattering collisions. The experimental data for these cross-sections are given in [13] for electron energies up to 1 keV. We extrapolate the dependence of the cross-sections to the high-energy region, using the asymptotic dependences [14]. The dependences of the cross section on the energy of the electron are shown in Fig. 4 (solid lines show experimental values from [13] and dashed lines correspond to high-energy extrapolations).

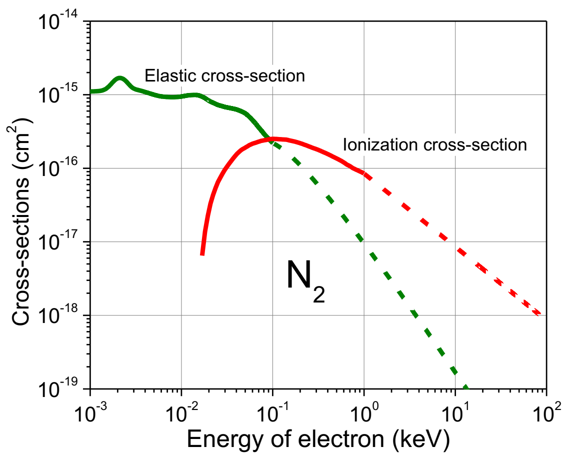

Fig. 4. Dependences of collision cross section on the energy of an electron in nitrogen.

The system of equations (9)-(12) is solved numerically with uniform initial conditions for the EDF and the electric field. We choose the initial Maxwellian EDF, which determines the initial electron density of $n_0 \sim 10^3$ cm^{-3} that homogeneously fills the discharge gap with the thermal spread $kT \sim 5$ eV.

Calculations show a detailed picture of the breakdown development. The current switching in the gap occurs due to its rapid filling with dense plasma, the front of which moves from the cathode (internal electrode) to the anode. The dynamics of the electric field distribution in the gap during the development of breakdown is shown in Fig. 5. It can be seen the field strength at the ionization front has a local maximum, which moves with increasing speed to the anode.

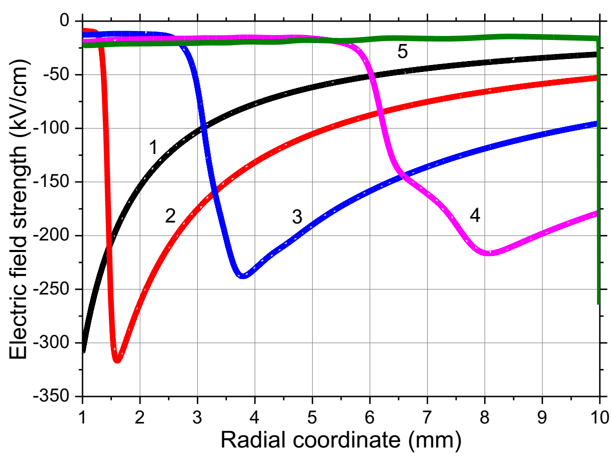

Fig. 5. Electric field strength distribution in the coaxial gap at different time points: curve *1* – 150 ps, *2* – 200, *3* – 250, *4* – 300, *5* – 350.

Runaway electrons are generated at the leading edge of the moving ionization region, where both the field strength and electron number density are high. An increase of the plasma electrons energy in the EDF tail is accompanied by a decrease of the collision cross section, and a group of fast electrons is formed.

This process is clearly shown in the phase portraits shown in Fig. 6. These functions are obtained as a solution to the Boltzmann equation (9). We pay attention to a group of runaway electrons that is separated from the total plasma group under the impact of a strong electric field. Here, some instantaneous EDF in the vicinity of the maximum discharge voltage are shown. The dashed lines show the level of the electron momentum corresponding to the kinetic energy equal to the instantaneous gap voltage.

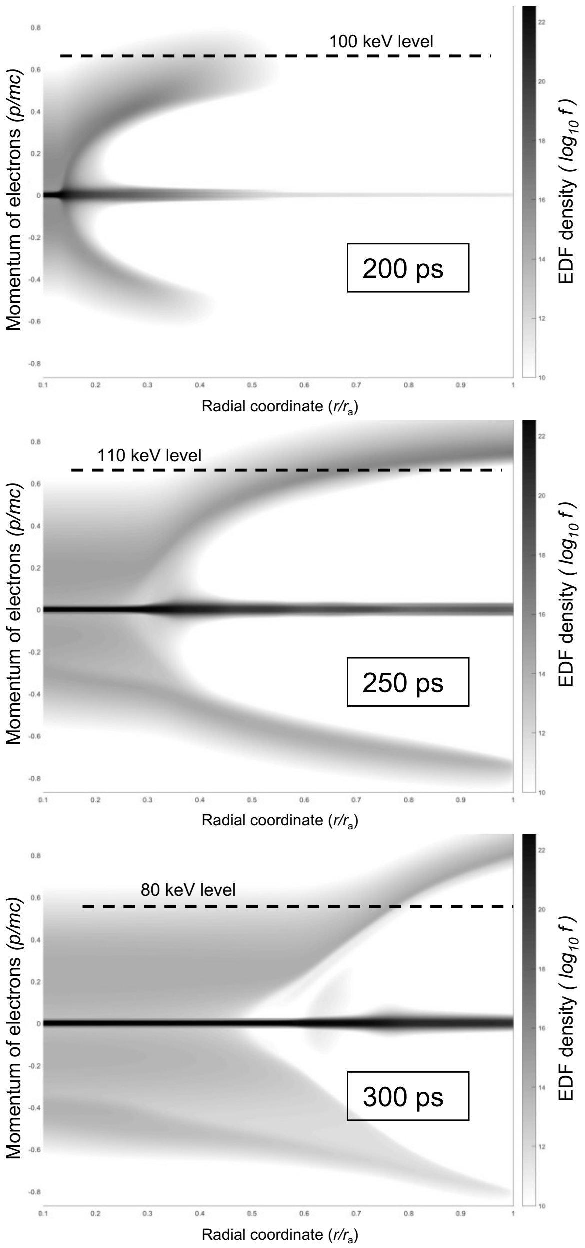

Fig. 6. Phase portraits EDF for various time points of the breakdown in nitrogen (cathode left, right anode). Shaded region represents a logarithmic scale.

Here the gap voltage is maximal (~110 kV) at time point 240 ps. The phase diagrams show how an electron beam with “anomalous energy” is formed at the anode. The travelling-wave electric field profile forms in the discharge (see Fig. 5) providing extra acceleration to the electrons beyond the maximum level of the potential drop at the gap. This represents the same situation that was discussed in Section 2.

In order to obtain the fast electrons current, which values could be compared with the experimental data, in the calculations we use an “energy filter” – an $d = 10$ μm Al foil, which partially cuts off slow plasma electrons coming to the anode from the discharge column. The spectral attenuation factor of the foil, $\varphi(d, \varepsilon)$, is calculated using semi-empirical formulas [15]. Thus, the current, $J_{fast}(t)$, and the total spectrum of fast electrons per pulse behind the anode, $N(\varepsilon)$, are given with

$$J_{fast}(t) = 2\pi r_a L \cdot e\int_0^\infty \varphi(d,\varepsilon) f(r_a,\varepsilon,t) v(\varepsilon) d\varepsilon,$$

$$N(\varepsilon) \propto \varphi(d,\varepsilon)\int_0^\infty f(r_a,\varepsilon,t) dt.$$

It should be noted, the bulk of the electron beam has an "anomalous energy" as shown in phase pictures Fig. 6. The integral spectrum of electrons per pulse, shown in Fig. 7, of course, contains electrons with lower energies also.

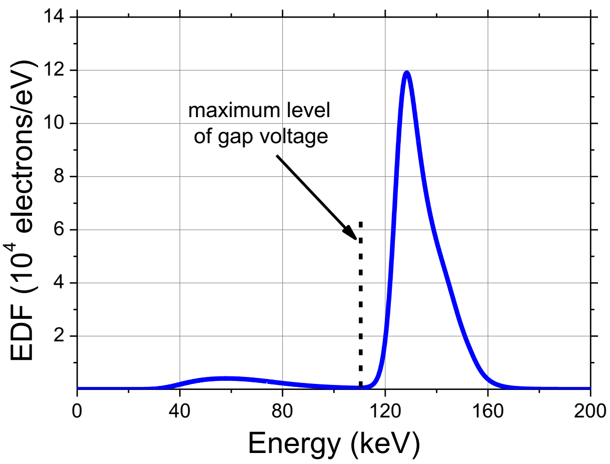

Fig. 7. Integral spectrum of electron beam behind the anode foil.

The amplitude of the runaway electron beam current $J_{fast}(t)$ at 25 A is reached at the 250 ps time point. Current pulse duration at half maximum is equal to 12 ps. Thus, the total yield is estimated to be $2\cdot 10^9$ fast electrons per pulse. This value agrees well with typical experiment data [5, 16, 17].

5 Summary

This work presents an original theoretical model of a non-stationary high-pressure discharge. The physical kinetics of electrons is described using the Boltzmann kinetic equation for the electron momentum distribution function, and the collisions are described on the basis of known sections of elementary processes.

Self-consistent descriptions of both the process of forming total EDF in the discharge, and generate a group of runaway electrons. It does not use any semi-empirical coefficients and the functional dependence of the type: the critical field, the field enhancement factor, the drift velocity, etc.

The physical mechanism responsible for the appearance in the discharge of electrons with “anomalous energies” is convincingly demonstrated.

However, we must mention that in actual discharges of the three-dimensional geometry, the relative fraction of electrons with “anomalous energies” should be significantly lower than our calculations showed. This is clear because in our model of axisymmetric discharge all the electrons generated in the amplified field on the ionization wave front, while in real discharges only a small group of particles participate in this interaction with field wave [5].

We believe that such a mechanism for electron acceleration can also take place in other phenomena, for example, during the expansion of dense plasma initiated by a focused laser beam.

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