Scalar Dyon Production In Near Extremal Kerr-Newman Black Holes

Chiang-Mei Chen[1,⋆], *Sang Pyo* Kim[2,⋆⋆], *Jia-Rui* Sun[3,⋆⋆⋆], and *Fu-Yi* Tang[1,⋆⋆⋆⋆]

[1] *Department of Physics, National Central University, Chungli 32001, Taiwan*
[2] *Department of Physics, Kunsan National University, Kunsan 54150, Korea*
[3] *School of Physics and Astronomy, Sun Yat-Sen University, Guangzhou 510275, China*

Abstract. The pair production of charged scalar dyons is analytically studied in near-extremal Kerr-Newman (KN) dyonic black holes. The pair production rate and its thermal interpretation are given. Moreover, the absorption cross section ratio has been compared with the two-point function of the conformal field theories (CFTs) holographically dual to the near horizon geometry, namely warped AdS_3, of the near extremal Kerr-Newman black holes to verify the threefold dyonic KN/CFTs correspondence.

1 Introduction

Two independent processes, the Schwinger mechanism [1] and the Hawking radiation [2] are mixed for the spontaneous pair production in charged black holes. The leading contribution is expected to happen in the near horizon region. In particular, the results of production can be analytically obtained for the near extremal black holes [3–5]. Thus we investigate the pair production of scalar dyons in the spacetime of the near-horizon region of the near extremal dyonic Kerr-Newman (KN) black holes [6] without considering the back-reactions. The spacetime has the geometric structure of a warped AdS_3 plus an electromagnetic field. In such a background we can analytically solve the Klein-Gorden (KG) equation for the probe scalar dyons, and obtain the exact solutions in terms of the well-known hypergeometric functions. Further, by imposing the particle viewpoint of boundary condition on the solutions, the physical quantities such as the vacuum persistence amplitude, the mean number of produced pairs and the absorption cross section ratio are obtained. The corresponding thermal interpretation [7–9] has been discussed. In addition, the physical quantities calculated from the gravity side are shown to match well with those results of the scalar dyon operator in the dual boundary conformal field theories (CFTs) side, based on the KN/CFTs correspondence [10, 11].

⋆e-mail: cmchen@phy.ncu.edu.tw
⋆⋆e-mail: sangkim@kunsan.ac.kr
⋆⋆⋆e-mail: sunjiarui@sysu.edu.cn
⋆⋆⋆⋆e-mail: foue.tang@gmail.com

2 The near horizon near extreme dyonic KN black holes

The geometry of the near horizon of a near extreme dyonic KN black hole has the structure of a warped AdS_3 as [6]

$$ds^2 = \Gamma(\theta)\left[-(\rho^2 - B^2)d\tau^2 + \frac{d\rho^2}{\rho^2 - B^2} + d\theta^2\right] + \gamma(\theta)(d\varphi + b\rho d\tau)^2, \tag{1}$$

$$A_{[1]} = -\frac{Q(r_0^2 - a^2\cos^2\theta) - 2Pr_0 a\cos\theta}{\Gamma(\theta)}\rho d\tau - \frac{Qr_0 a\sin^2\theta - P(r_0^2 + a^2)\cos\theta \pm P\Gamma(\theta)}{\Gamma(\theta)}d\varphi, \tag{2}$$

$$\bar{A}_{[1]} = -\frac{P(r_0^2 - a^2\cos^2\theta) + 2Qr_0 a\cos\theta}{\Gamma(\theta)}\rho d\tau - \frac{Pr_0 a\sin^2\theta + Q(r_0^2 + a^2)\cos\theta \mp Q\Gamma(\theta)}{\Gamma(\theta)}d\varphi, \tag{3}$$

where

$$\Gamma(\theta) = r_0^2 + a^2\cos^2\theta, \quad \gamma(\theta) = \frac{(r_0^2 + a^2)^2\sin^2\theta}{r_0^2 + a^2\cos^2\theta}, \quad b = \frac{2ar_0}{r_0^2 + a^2}, \quad r_0 = \sqrt{Q^2 + P^2 + a^2}. \tag{4}$$

The ρ is the radial coordinate of the warped AdS_3 section, B labels a deviation from the extreme limit and acts as the new horizon radius in this geometry, a is the angular parameter and Q, P are the electric and magnetic charges of the original dyonic KN black holes. The magnetic monopole induces a string-like singularity causing different choices of the gauge potential: the upper sign is regular in the northern hemisphere ($0 \leq \theta < \pi/2$), and the lower sign in the southern hemisphere ($\pi/2 < \theta \leq \pi$). Moreover, the dual gauge potential, $\bar{A}_{[1]}$, is defined as ($\star$ denoting the Hodge dual)

$$\bar{F}_{[2]} = d\bar{A}_{[1]} \quad \text{such that} \quad \bar{F}_{[2]} = {}^{\star}F_{[2]}. \tag{5}$$

The black hole's thermodynamical quantities, such as the Hawking temperature, entropy, angular velocity and chemical potentials in the new coordinates reduce to

$$T_H = \frac{B}{2\pi}, \qquad S_{BH} = \pi(r_0^2 + a^2 + 2Br_0), \qquad \Omega_H = -\frac{2ar_0 B}{r_0^2 + a^2},$$
$$\Phi_H = \frac{Q(Q^2 + P^2)B}{r_0^2 + a^2}, \qquad \bar{\Phi}_H = \frac{P(Q^2 + P^2)B}{r_0^2 + a^2}. \tag{6}$$

3 Particle creation

3.1 Boundary conditions

There is a potential barrier due to the electromagnetic and gravitational forces in the near horizon region, against which the pair production becomes a tunneling process by considering the fluxes in the scattering matrix theory. Two boundary conditions can be imposed (see the Fig. in [12]):

The first one is the outer boundary condition (particle viewpoint), which restricts no incoming fluxes at the asymptotic outer boundary in the new coordinates (1). In the Stückelberg-Feynman picture, the outgoing (transmitted) flux at the asymptotic region represents the spontaneous produced *particle*, the outgoing (incident) flux at the horizon represents the total particles created by vacuum fluctuations, and the incoming (reflected) flux represents the re-annihilated pairs.

The second one is the inner boundary condition (antiparticle viewpoint) that restricts no outgoing fluxes at the inner boundary, which means that the incoming (transmitted) flux at the horizon represents the spontaneous produced *antiparticle*, the incoming (incident) and the outgoing (reflected) fluxes at the asymptotic region represents the total created antiparticles and the re-annihilated pairs. Since particles and anti-particles are always produced in pairs due to the charge conservation and/or the energy-momentum conservation, these two boundary conditions are actually equivalent [3].

3.2 Physical quantities

For bosonic particles, the flux conservation [13]

$$|D_{\text{incident}}| = |D_{\text{reflected}}| + |D_{\text{transmitted}}|, \tag{7}$$

is related to the Bogoliubov relation

$$|\mathcal{A}|^2 - |\mathcal{B}|^2 = 1, \tag{8}$$

where the vacuum persistence amplitude $|\mathcal{A}|^2$ and the mean number of produced pairs $|\mathcal{B}|^2$ are given by the ratio of the flux components in the Coulomb gauge

$$|\mathcal{A}|^2 \equiv \frac{D_{\text{incident}}}{D_{\text{reflected}}}, \qquad |\mathcal{B}|^2 \equiv \frac{D_{\text{transmitted}}}{D_{\text{reflected}}}. \tag{9}$$

Moreover, from the viewpoint of scattering of an incident flux from the asymptotic boundary, we can define the absorption cross section ratio as

$$\sigma_{\text{abs}} \equiv \frac{D_{\text{transmitted}}}{D_{\text{incident}}} = \frac{|\mathcal{B}|^2}{|\mathcal{A}|^2}. \tag{10}$$

4 Scalar dyons production

The EoM of the probe scalar field Φ with mass m, electric and magnetic charges q, p propagating in the near horizon geometry (1) is

$$D_\alpha D^\alpha \Phi - m^2 \Phi = 0, \tag{11}$$

where $D_\alpha \equiv \nabla_\alpha - iqA_\alpha - ip\bar{A}_\alpha$ with ∇_α being the usual covariant derivative. The corresponding radial flux is

$$D = i\sqrt{-g}g^{\rho\rho}(\Phi D_\rho \Phi^* - \Phi^* D_\rho \Phi). \tag{12}$$

Using the following ansatz

$$\Phi(\tau, \rho, \theta, \varphi) = \mathrm{e}^{-i\omega\tau + i[n \mp (qP - pQ)]\varphi} R(\rho) S(\theta), \tag{13}$$

the KG equation can be completely separated into the angular part, see [6], and the radial part

$$\partial_\rho \left[(\rho^2 - B^2)\partial_\rho R\right] + \left(\frac{\left[\omega(r_0^2 + a^2) - (qQ + pP)(Q^2 + P^2)\rho + 2nar_0\rho\right]^2}{(r_0^2 + a^2)^2(\rho^2 - B^2)} - m^2(r_0^2 + a^2) - \lambda_l \right) R = 0, \tag{14}$$

where λ_l is the separation constant. The radial equation resembles the EoM of a scalar field with an effective mass

$$m_{\text{eff}}^2 = m^2 - \frac{[2nar_0 - (qQ + pP)(Q^2 + P^2)]^2}{(r_0^2 + a^2)^3} + \frac{\lambda_l}{r_0^2 + a^2}, \tag{15}$$

propagating in the AdS_2 space with the radius $L_{\text{AdS}} = r_0^2 + a^2$. The instability in the AdS_2 space occurs if the Breitenlohner-Freedman (BF) bound [14, 15] is violated, i.e., $m_{\text{eff}}^2 < -\frac{1}{4L_{\text{AdS}}^2}$, whose relation becomes

$$m_{\text{eff}}^2 < -\frac{1}{4(r_0^2 + a^2)} \quad \Rightarrow \quad \frac{[2nar_0 - (qQ + pP)(Q^2 + P^2)]^2}{(r_0^2 + a^2)^2} - m^2(r_0^2 + a^2) - \left(\lambda_l + \frac{1}{4}\right) > 0. \tag{16}$$

Now Eq. (14) can be exactly solved and leads to the following solution

$$\begin{aligned} R(\rho) &= c_1(\rho - B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)}(\rho + B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} F\left(\frac{1}{2} + i\tilde{\kappa} + i\mu, \frac{1}{2} + i\tilde{\kappa} - i\mu; 1 + i\tilde{\kappa} - i\kappa; \frac{1}{2} - \frac{\rho}{2B}\right) \\ &+ c_2(\rho - B)^{-\frac{i}{2}(\tilde{\kappa}-\kappa)}(\rho + B)^{\frac{i}{2}(\tilde{\kappa}+\kappa)} F\left(\frac{1}{2} + i\kappa + i\mu, \frac{1}{2} + i\kappa - i\mu; 1 + i\kappa - i\tilde{\kappa}; \frac{1}{2} - \frac{\rho}{2B}\right), \end{aligned} \quad (17)$$

with three essential parameters

$$\tilde{\kappa} = \frac{\omega}{B}, \qquad \kappa = \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{r_0^2 + a^2}, \qquad \mu = \sqrt{\kappa^2 - m^2(r_0^2 + a^2) - \lambda_l - \frac{1}{4}}, \quad (18)$$

in which μ^2 is positive due to the BF bound violation in Eq. (16). Then the Bogoliubov coefficients and the absorption cross section ratio in the particle viewpoint can be obtained as [6]

$$|\mathcal{A}|^2 = \frac{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}, \quad (19)$$

$$|\mathcal{B}|^2 = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa + \pi\mu)\cosh(\pi\tilde{\kappa} - \pi\mu)}, \quad (20)$$

$$\sigma_{\rm abs} = \frac{|\mathcal{B}|^2}{|\mathcal{A}|^2} = \frac{\sinh(2\pi\mu)\sinh(\pi\tilde{\kappa} - \pi\kappa)}{\cosh(\pi\kappa - \pi\mu)\cosh(\pi\tilde{\kappa} + \pi\mu)}. \quad (21)$$

5 Thermal Interpretation

Following our previous studies [3, 5], the mean number of produced pairs (20) can be reexpressed as

$$\mathcal{N} = |\mathcal{B}|^2 = \left(\frac{e^{-2\pi\kappa+2\pi\mu} - e^{-2\pi\kappa-2\pi\mu}}{1 + e^{-2\pi\kappa-2\pi\mu}}\right)\left(\frac{1 - e^{-2\pi\tilde{\kappa}+2\pi\kappa}}{1 + e^{-2\pi\tilde{\kappa}+2\pi\mu}}\right). \quad (22)$$

Note that the mean number (22) has a similar form except for different quantum numbers as that of charged scalars in a near-extremal RN [3] and KN [5] since the near-horizon geometry is an $AdS_2 \times S^2$ for the near-extremal RN black hole while it is a warped AdS_3 for the near-extremal KN black hole. Following Refs. [7, 8], we introduce an effective temperature and its associated counterpart

$$T_{\rm KN} = \frac{\bar{m}}{2\pi\kappa - 2\pi\mu} = T_U + \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \qquad \bar{T}_{\rm KN} = \frac{\bar{m}}{2\pi\kappa + 2\pi\mu} = T_U - \sqrt{T_U^2 + \frac{\mathcal{R}}{8\pi^2}}, \quad (23)$$

where the effective mass $\bar{m}$ is

$$\bar{m} = \sqrt{m^2 - \frac{\lambda + 1/4}{2}\mathcal{R}}, \quad (24)$$

and the corresponding Unruh temperature T_U and AdS curvature $\mathcal{R}$ are

$$T_U = \frac{\kappa}{2\pi\bar{m}(r_0^2 + a^2)} = \frac{(qQ + pP)(Q^2 + P^2) - 2nar_0}{2\pi\bar{m}(r_0^2 + a^2)^2}, \qquad \mathcal{R} = -\frac{2}{r_0^2 + a^2}. \quad (25)$$

Therefore, we have

$$2\pi\kappa - 2\pi\mu = \frac{\bar{m}}{T_{\rm KN}}, \qquad 2\pi\kappa + 2\pi\mu = \frac{\bar{m}}{\bar{T}_{\rm KN}}, \qquad 2\pi\tilde{\kappa} = \frac{\omega}{T_{\rm H}}, \qquad 2\pi\kappa = \frac{q\Phi_{\rm H} + p\bar{\Phi}_{\rm H} + n\Omega_H}{T_{\rm H}}, \quad (26)$$

and the mean number (22) can be expressed as

$$\mathcal{N} = e^{\frac{\bar{m}}{T_{\rm KN}}} \times \left(\frac{e^{-\frac{\bar{m}}{T_{\rm KN}}} - e^{-\frac{\bar{m}}{T_{\rm KN}}}}{1 + e^{-\frac{\bar{m}}{T_{\rm KN}}}} \right) \times \left\{ \frac{e^{-\frac{\bar{m}}{T_{\rm KN}}} \left(1 - e^{-\frac{\omega - q\Phi_{\rm H} - p\bar{\Phi}_{\rm H} - n\Omega_{\rm H}}{T_{\rm H}}} \right)}{1 + e^{-\frac{\omega - q\Phi_{\rm H} - p\bar{\Phi}_{\rm H} - n\Omega_{\rm H}}{T_{\rm H}}} e^{-\frac{\bar{m}}{T_{\rm KN}}}} \right\}. \tag{27}$$

Now the physical meaning of each term in Eq. (27) becomes clear; we interpret the first parenthesis as the Schwinger effect with the effective temperature $T_{\rm KN}$ in AdS_2 [16] and the second parenthesis as the Schwinger effect in the Rindler space [17], in which the Unruh temperature is given by the Hawking temperature and the charges have the chemical potentials of $\Phi_{\rm H}, \bar{\Phi}_{\rm H}$, and $\Omega_{\rm H}$, while the effective temperature for the Schwinger effect due to the electric field on the horizon is still determined by $T_{\rm KN}$.

The mean number of produced pairs above and the absorption cross section ratio in the previous section have been obtained using the exact solution in the near horizon geometry of an extremal or near extremal KN black hole. In addition, by applying the phase-integral formula, we can derive the instanton actions from the Hamilton-Jacobi action for the field equation, which lead to the mean number, for details see [6]. This method allows one to understand the physical origin of each term as a consequence of simple poles in the complex plane of space [18] and further connects the interpretation to other physical systems involving the Schwinger effect in curved spacetimes [19].

6 Holographic CFT description

According to the KN/CFTs duality [10, 11], the absorption cross section ratio of scalar field in Eq. (21) corresponds to that of its dual operator in the dual two-dimensional CFT with left- and right-hand sectors

$$\sigma_{\rm abs} \sim T_{\rm L}^{2h_{\rm L}-1} T_{\rm R}^{2h_{\rm R}-1} \sinh\left(\frac{\tilde{\omega}_{\rm L}}{2T_{\rm L}} + \frac{\tilde{\omega}_{\rm R}}{2T_{\rm R}} \right) \left| \Gamma\left(h_{\rm L} + i\frac{\tilde{\omega}_{\rm L}}{2\pi T_{\rm L}} \right) \right|^2 \left| \Gamma\left(h_{\rm R} + i\frac{\tilde{\omega}_{\rm R}}{2\pi T_{\rm R}} \right) \right|^2, \tag{28}$$

where $T_{\rm L}, T_{\rm R}$ are the temperatures, $h_{\rm L}, h_{\rm R}$ are the conformal dimensions of the dual operator, $\tilde{\omega}_{\rm L} = \omega_{\rm L} - q_{\rm L}\Phi_{\rm L}$ and $\tilde{\omega}_{\rm R} = \omega_{\rm R} - q_{\rm R}\Phi_{\rm R}$ are the total excited energy in which $(q_{\rm L}, q_{\rm R})$ and $(\Phi_{\rm L}, \Phi_{\rm R})$ are respectively the charges and chemical potentials (both including the electric and the magnetic contributions for the dyonic KN black hole case) of the dual left and right-hand operators.

From the field/operator duality in the AdS/CFT correspondence, the complex conformal dimensions $(h_{\rm L}, h_{\rm R})$ of the dual operator in the two-dimensional CFT can be determined from the asymptotic expansion of the bulk dyonic charged scalar field (17) at the AdS boundary [6]

$$h_{\rm L} = h_{\rm R} = \frac{1}{2} \pm i\mu. \tag{29}$$

Next, we will analyze the dual CFTs descriptions of the pair production rate and the absorption cross section ratio in the near extremal dyonic KN black hole in the J- and Q- as well as the P-pictures, respectively.

6.1 J-picture

In the J-picture, the left- and right-hand central charges of the dual CFT are determined by the angular momentum [10, 11]

$$c_{\rm L}^J = c_{\rm R}^J = 12J, \tag{30}$$

and the associated left- and right-hand temperatures for the near extremal dyonic KN black hole are

$$T^J_{\rm L} = \frac{r_0^2 + a^2}{4\pi a r_0}, \qquad T^J_{\rm R} = \frac{B}{2\pi a}. \tag{31}$$

The CFT microscopic entropy is calculated from the Cardy formula

$$S_{\rm CFT} = \frac{\pi^2}{3}(c^J_{\rm L} T^J_{\rm L} + c^J_{\rm R} T^J_{\rm R}) = \pi(r_0^2 + a^2 + 2r_0 B), \tag{32}$$

which agrees with the macroscopic entropy (6) of the near extremal KN black hole.

Besides, by matching the first law of black hole thermodynamics with that of the dual CFT, i.e., $\delta S_{\rm BH} = \delta S_{\rm CFT}$, the following relation holds

$$\frac{\delta M - \Omega_{\rm H}\delta J - \Phi_{\rm H}\delta Q - \bar{\Phi}_{\rm H}\delta P}{T_{\rm H}} = \frac{\tilde{\omega}_{\rm L}}{T_{\rm L}} + \frac{\tilde{\omega}_{\rm R}}{T_{\rm R}}, \tag{33}$$

where the angular velocity and chemical potentials at $\rho = B$ are given in Eq. (6). To probe the rotation we need to turn off the charges of the probe scalar field and set $T_{\rm L} = T^J_{\rm L}$ and $T_{\rm R} = T^J_{\rm R}$, then for the dyonic KN black hole $\delta M = \omega,\ \delta J = n,\ \delta Q = 0,\ \delta P = 0$. Thus, we have

$$\tilde{\omega}^J_{\rm L} = n \quad \text{and} \quad \tilde{\omega}^J_{\rm R} = \frac{\omega}{a} \quad \Rightarrow \quad \frac{\tilde{\omega}^J_{\rm L}}{2T^J_{\rm L}} = -\pi\kappa \quad \text{and} \quad \frac{\tilde{\omega}^J_{\rm R}}{2T^J_{\rm R}} = \pi\tilde{\kappa}, \tag{34}$$

where q, p are set to zero. Consequently, the agreement between the absorption cross section ratio (21) of the scalar field (with $q = p = 0$) in the near extremal dyonic KN black hole and that of its dual scalar operator in Eq. (28) is found in the J-picture.

6.2 Q-picture

In the Q-picture, the left- and right-hand central charges of the dual CFT are [10, 11]

$$c^Q_{\rm L} = c^Q_{\rm R} = \frac{6Q(Q^2 + P^2)}{\ell}, \tag{35}$$

where the parameter ℓ is the measure of the $U(1)$ bundle formed by the background Maxwell field, which can be interpreted as the radius of the embedded extra circle in the fifth dimension. The associated temperatures of the left- and right-hand sectors for near horizon geometry of the near extremal dyonic KN black hole are

$$T^Q_{\rm L} = \frac{(r_0^2 + a^2)\ell}{2\pi Q(Q^2 + P^2)}, \qquad T^Q_{\rm R} = \frac{r_0 B\ell}{\pi Q(Q^2 + P^2)}. \tag{36}$$

Then, the microscopic entropy of the dual CFT obtained from the Cardy formula

$$S_{\rm CFT} = \frac{\pi^2}{3}(c^Q_{\rm L} T^Q_{\rm L} + c^Q_{\rm R} T^Q_{\rm R}) = \pi(r_0^2 + a^2 + 2r_0 B), \tag{37}$$

again reproduces the area law for the entropy of the near extremal dyonic KN black hole.

In the Q-picture, the modes n and p, which characterize the rotation and magnetic charge of the probe scalar field, should be turned off in order that the electrically charged probe only detects the electric charge of the black hole, i.e., $\delta M = \omega,\ \delta J = 0,\ \delta Q = q,\ \delta P = 0$. Then, using Eq. (33), with $T_{\rm L} = T_{\rm L}^{Q}$ and $T_{\rm R} = T_{\rm R}^{Q}$, we have

$$\tilde{\omega}_{\rm L}^{Q} = -q\ell \quad \text{and} \quad \tilde{\omega}_{\rm R}^{Q} = \frac{2\omega r_0 \ell}{Q(Q^2+P^2)}, \tag{38}$$

and $\frac{\tilde{\omega}_{\rm L}^{Q}}{2T_{\rm L}^{Q}} = -\pi\kappa$ and $\frac{\tilde{\omega}_{\rm R}^{Q}}{2T_{\rm R}^{Q}} = \pi\tilde{\kappa}$ for the dual CFT. Therefore, the absorption cross section ratio of the charged scalar field (with $n = p = 0$) in Eq. (21) also agrees with that of its dual scalar operator in Eq. (28) in the Q-picture.

6.3 P-picture

The left- and right-hand central charges of the P-picture CFT can be obtained from the electromagnetic duality, which are

$$c_{\rm L}^{P} = c_{\rm R}^{P} = \frac{6P(Q^2+P^2)}{\ell}, \tag{39}$$

and the associated left- and right-hand temperatures are

$$T_{\rm L}^{P} = \frac{(r_0^2+a^2)\ell}{2\pi P(Q^2+P^2)}, \qquad T_{\rm R}^{P} = \frac{r_0 B\ell}{\pi P(Q^2+P^2)}. \tag{40}$$

Then, the microscopic and macroscopic entropies are the same:

$$S_{\rm CFT} = \frac{\pi^2}{3}(c_{\rm L}^{P}T_{\rm L}^{P} + c_{\rm R}^{P}T_{\rm R}^{P}) = \pi(r_0^2 + a^2 + 2r_0 B). \tag{41}$$

Along the same line as the Q-picture, we need to turn off the modes of n and q in the P-picture, i.e. $\delta M = \omega,\ \delta J = 0,\ \delta Q = 0,\ \delta P = p$. Then, applying Eq. (33), with $T_{\rm L} = T_{\rm L}^{P}$ and $T_{\rm R} = T_{\rm R}^{P}$, we obtain

$$\tilde{\omega}_{\rm L}^{P} = -p\ell \quad \text{and} \quad \tilde{\omega}_{\rm R}^{P} = \frac{2\omega r_0 \ell}{P(Q^2+P^2)}, \tag{42}$$

and $\frac{\tilde{\omega}_{\rm L}^{P}}{2T_{\rm L}^{P}} = -\pi\kappa$ and $\frac{\tilde{\omega}_{\rm R}^{P}}{2T_{\rm R}^{P}} = \pi\tilde{\kappa}$. Therefore, in the P-picture of the KN/CFTs duality, we find the agreement between the absorption cross section ratio of the charged scalar field (with $n = q = 0$) in Eq. (21) and that of its dual scalar operator in Eq. (28).

7 Conclusions

We study the scalar dyon production for the near extremal dyonic KN black hole without back-reaction. The KG equation is eactly solved in the near horizon region where the geometry includes a warped AdS_3 structure. The near horizon region contains a causal horizon and a dominating electric field which capture both contributions: the Hawking radiation and the Schwinger mechanism. The exact solutions were obtained in terms of the hypergeometric functions. By imposing the particle viewpoint of boundary condition, the physical quantities associated to the pair production can be derived by the ratio of boundary fluxes. In particular, the expressions of the vacuum persistence amplitude, the mean number of pairs and the absorption section ratio were obtained. The existence condition for the pair production corresponds to the instability of the probe fields in the AdS_2, i.e. the violation of the BF bound.

The thermal interpretation of the pair production rate for the dyonic KN black holes includes two contributions: one is the Schwinger effect in the AdS_2 space caused mainly by the electromagnetic field of black holes and the other is the Schwinger effect in the Rindler space due to the Hawking temperature, i.e. the deviation from the extremality. The leading term of the production rate is determined by the effective temperature T_{KN}, which is a kind of Unruh temperature originated from the acceleration due to the electromagnetic force and the AdS_2 curvature. There is duality between the electric charges and magnetic charges of the black hole and produced pairs. The effect of the AdS_2 curvature is to bind pairs and thereby decrease the pair production while the electromagnetic field increases the Unruh temperature for the effective temperature and thus enhances the pair production. The angular momentum, electric and magnetic charges of the produced dyonic particles are expected to couple with the corresponding thermal quantities, i.e. the angular velocity, electric chemical potential, and magnetic chemical potential, of the dyonic KN black hole as the subleading terms to the leading Boltzmann factor.

The twofold dual CFTs descriptions (i.e., J-picture and Q-picture) of the Schwinger pair production of the near extremal KN black hole [5] are generalized into the threefold dual CFTs pictures for the dyonic KN black hole which includes an additional magnetic charge. The presence of the P-picture is associated with the dual gauge potential which can be regarded as a new "magnetic hair" of the dyonic KN black hole and provides the $U(1)$ fiber on the base manifold. It can only be probed by the magnetic charge of the scalar field. Furthermore, based on the threefold dyonic KN/CFTs duality, the dual CFTs descriptions of the absorption cross section ratios and the pair production rate of the dyonic charged scalar field are found in the J-, Q-, and P-pictures, respectively.

Acknowledgments

The work of C. M. C. was were supported by the Ministry of Science and Technology of the R.O.C. under the grants MOST 105-2112-M-008-018 and 106-2112-M-008-010. The work of S. P. K. was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2015R1D1A1A01060626). The work of J. R. S. was supported by the NSFC under Grant No. 11675272 and the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China (No. Y5KF161CJ1) and the Fundamental Research Funds for the Central Universities.

References

[1] J. S. Schwinger, "On Gauge Invariance and Vacuum Polarization," Phys. Rev. **82**, 664 (1951).

[2] M. K. Parikh and F. Wilczek, "Hawking Radiation as Tunneling," Phys. Rev. Lett. **85**, 5042 (2000).

[3] C.-M. Chen, S. P. Kim, I.-C. Lin, J.-R. Sun and M.-F. Wu, "Spontaneous Pair Production in Reissner-Nordstrom Black Holes," Phys. Rev. D **85**, 124041 (2012) [arXiv:1202.3224 [hep-th]].

[4] C.-M. Chen, J.-R. Sun, F.-Y. Tang and P.-Y. Tsai, "Spinor Particle Creation in Near Extremal Reissner-Nordstrom Black Holes," Class. Quant. Grav. **32**, 195003 (2015) [arXiv:1412.6876 [hep-th]].

[5] C.-M. Chen, S. P. Kim, J.-R. Sun and F.-Y. Tang, "Pair Production in Near Extremal Kerr-Newman Black Holes," Phys. Rev. D **95**, 044043 (2017) [arXiv:1607.02610 [hep-th]].

[6] C.-M. Chen, S. P. Kim, J.-R. Sun and F.-Y. Tang, "Pair Production of Scalar Dyons in Kerr-Newman Black Holes," arXiv:1705.10629 [hep-th].

[7] S. P. Kim, H. K. Lee and Y. Yoon, "Thermal Interpretation of Schwinger Effect in Near-Extremal RN Black Hole," arXiv:1503.00218 [hep-th].

[8] S. P. Kim, "Schwinger Effect, Hawking Radiation and Gauge-Gravity Relation," Int. J. Mod. Phys. A **30**, 1545017 (2015) [arXiv:1506.03990 [hep-th]].

[9] S. P. Kim, "Schwinger Effect in (A)dS and Charged Black Hole," arXiv:1509.05532 [hep-th].

[10] C.-M. Chen, Y.-M. Huang, J.-R. Sun, M.-F. Wu and S.-J. Zou, "Twofold Hidden Conformal Symmetries of the Kerr-Newman Black Hole," Phys. Rev. D **82**, 066004 (2010) [arXiv:1006.4097 [hep-th]].

[11] C.-M. Chen and J.-R. Sun, "The Kerr-Newman/CFTs Correspondence," Int. J. Mod. Phys. Conf. Ser. **07**, 227 (2012) [arXiv:1201.4040 [hep-th]].

[12] C.-M. Chen, J.-R. Sun and F.-Y. Tang, "Pair Production In Near Extremal Charged Black Holes," arXiv:1510.00141 [hep-th].

[13] S. P. Kim and D. N. Page, "Schwinger Pair Production in Electric and Magnetic Fields," Phys. Rev. D **73**, 065020 (2006) [hep-th/0301132].

[14] P. Breitenlohner and D. Z. Freedman, "Stability in Gauged Extended Supergravity," Annals Phys. **144**, 249 (1982).

[15] P. Breitenlohner and D. Z. Freedman, "Positive Energy in anti-De Sitter Backgrounds and Gauged Extended Supergravity," Phys. Lett. B **115**, 197 (1982).

[16] R. G. Cai and S. P. Kim, "One-Loop Effective Action and Schwinger Effect in (Anti-) de Sitter Space," JHEP **1409**, 072 (2014) [arXiv:1407.4569 [hep-th]].

[17] Cl. Gabriel and Ph. Spindel, "Quantum charged fields in (1+1) Rindler space," Annals Phys. **284**, 263 (2000) [gr-qc/9912016].

[18] S. P. Kim and D. N. Page, "Improved Approximations for Fermion Pair Production in Inhomogeneous Electric Fields," Phys. Rev. D **75**, 045013 (2007) [hep-th/0701047].

[19] S. P. Kim, "Geometric Origin of Stokes Phenomenon for de Sitter Radiation," Phys. Rev. D **88**, 044027 (2013) [arXiv:1307.0590 [hep-th]].