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Experiments on Frequency Dependence of the Deflection of Light in Yang-Mills Gravity

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Abstract. In Yang-Mills gravity based on flat space-time, the eikonal equation for a light ray is derived from the modified Maxwell's wave equations in the geometric-optics limit. One obtains a Hamilton-Jacobi type equation, $G_L^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = 0$ with an effective Riemannian metric tensor $G_L^{\mu\nu}$. According to Yang-Mills gravity, light rays (and macroscopic objects) move as if they were in an effective curved space-time with a metric tensor. The deflection angle of a light ray by the sun is about $1.53''$ for experiments with optical frequencies $\approx 10^{14}$Hz. It is roughly 12% smaller than the usual value $1.75''$. However, the experimental data in the past 100 years for the deflection of light by the sun in optical frequencies have uncertainties of (10-20)% due to large systematic errors. If one does not take the geometric-optics limit, one has the equation, $G_L^{\mu\nu}[\partial_\mu\Psi\partial_\nu\Psi cos\Psi + (\partial_\mu\partial_\nu\Psi)sin\Psi] = 0$, which suggests that the deflection angle could be frequency-dependent, according to Yang-Mills gravity. Nowadays, one has very accurate data in the radio frequencies $\approx 10^9$Hz with uncertainties less than 0.1%. Thus, one can test this suggestion by using frequencies $\approx 10^{12}$ Hz, which could have a small uncertainty 0.1% due to the absence of systematic errors in the very long baseline interferometry.

1 Introduction

Yang-Mills gravity is a gauge theory based on the 4-dimensional translational group T_4 in flat space-time. Such a theory can be quantized to obtain Feynman-Dyson rules for quantum Yang-Mills gravity and is consistent with known experiments.[1–4] A new interesting property is that the classical objects such as light rays and macroscopic objects move as if they were in an effective curved space-time. The reason is that their equations of motion are derived from the quantum wave equations in flat space-time, which involves T_4 gauge covariant derivative $\Delta_\mu = \partial_\mu - ig\phi_{\mu\nu}p^\nu$ where $p_\mu = i\partial_\mu$ is the generator of the T_4 group, and $\phi_{\mu\nu}$ is the symmetric tensor gauge field associated with space-time translational gauge symmetry. In the geometric-optics (i.e., classical) limit, quantum wave equations reduce to the form of relativistic Hamilton-Jacobi type equations with an effective Riemannian metric tensor for classical objects such as light rays, $G_L^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = 0$. The emergence of an effective metric tensor in the classical limit of quantum equation is a new and key property of the Yang-Mills gravity for its consistence with experiments.[3, 4] Therefore, Yang-Mills gravity reveals that the apparent curvature

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of space-time appears to be simply a manifestation of the flat space-time translational gauge symmetry for the motion of quantum particles in the classical limit.

2 Modified Maxwell's equations and eikonal equations

Let us consider Yang-Mills gravity in an inertial frame $F(w, x, y, z)$ with a broad 4-dimensional symmetry.[5] The Lagrangian for the electromagnetic potential field A_μ in the presence of gravity is obtained by replacing the partial space-time derivatives ∂_μ in the Lagrangian by the T_4 gauge covariant derivative Δ_μ. Thus, we have[1]

$$L_{em} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \Delta_\mu A_\nu - \Delta_\nu A_\mu, \tag{1}$$

$$\Delta_\mu = \partial_\mu - ig\phi_{\mu\nu}p^\nu \equiv J_{\mu\nu}\partial^\nu, \qquad J_{\mu\nu} = (\eta_{\mu\nu} + g\phi_{\mu\nu}),$$

$$\eta_{\mu\nu} = (1, -1, -1, -1), \qquad g = \sqrt{8\pi G_N}\,, \qquad c = \hbar = 1.$$

Since we are interested in the classical light rays, it is not necessary to consider the gauge-fixing terms in the Lagrangian (1). One may carry out variation of the Lagrangian (1) to arrive at modified Maxwell's equations in the presence of gravity. The modified wave equations are

$$\partial_\mu(J^\mu_\alpha F^{\alpha\nu}) = 0. \tag{2}$$

To derive the equation of a light ray, we use the usual limiting expression [6]

$$A_\mu(x) = a_\mu e^{i\Psi(x)}, \qquad a_\mu = \epsilon_\mu(k, \lambda). \tag{3}$$

where the eikonal Ψ and the wave 4-vector $\partial_\mu\Psi = k_\mu$ are large in the geometric-optics limit.

The modified electromagnetic wave equation (2) with the limiting expression (3) lead to

$$\Delta_\mu(\Delta^\mu A^\lambda - \Delta^\lambda A^\mu) + (\partial_\alpha J^\alpha_\mu)(\Delta^\mu A^\lambda - \Delta^\lambda A^\mu) = 0. \tag{4}$$

Using the gauge condition $\partial_\mu A^\mu = 0$, the second term $-\Delta_\mu\Delta^\lambda A^\mu$ can be simplified,

$$-\Delta_\mu\Delta^\lambda A^\mu = -J^\sigma_\mu\partial_\sigma(J^{\lambda\alpha}\partial_\alpha A^\mu) = -(\delta^\sigma_\mu + g\phi^\sigma_\mu)\partial_\sigma(J^{\lambda\alpha}\partial_\alpha A^\mu)$$

$$\approx -g\phi^\sigma_\mu J^{\lambda\alpha}\partial_\alpha\partial_\mu A^\mu - g\phi^\sigma_\mu J^{\lambda\alpha}\partial_\sigma\partial_\alpha A^\mu = -g\phi^\sigma_\mu J^{\lambda\alpha}\partial_\sigma\partial_\alpha A^\mu. \tag{5}$$

Note that the terms involving $(\partial_\alpha J^\alpha_\mu)$ are small and the terms with $\partial_\alpha\partial_\sigma A^\mu$ and $\partial_\alpha\Psi\partial_\sigma\Psi = k_\alpha k_\sigma$ are large. The eikeonal equation (4) can be written as

$$\delta^\lambda_\mu J^\alpha_\beta J^{\beta\sigma}\partial_\alpha\partial_\sigma A^\mu - gJ^{\lambda\sigma}\phi^\alpha_\mu\partial_\alpha\partial_\sigma A^\mu = Z^\lambda_\mu\epsilon^\mu = 0, \tag{6}$$

where

$$Z^\lambda_\mu = (\delta^\lambda_\mu G^{\alpha\sigma}\partial_\alpha\Psi\partial_\sigma\Psi - gJ^{\lambda\sigma}\phi^\alpha_\mu\partial_\alpha\Psi\partial_\sigma\Psi), \qquad G^{\alpha\sigma} = J^\alpha_\beta J^{\beta\sigma},$$

Since we are interested in the law for the propagation of light rays and the further simplification of (4), we use the relation for the polarization vector $\sum_\lambda \epsilon^\mu(k, \lambda)\epsilon^\nu(k, \lambda) \to -\eta^{\mu\nu}$.[7] Multiplying $Z^\lambda_\mu\epsilon^\mu$ in (4) by $\epsilon^\nu\eta_{\nu\lambda}$ and summing over all polarizations, we obtain

$$\sum_\lambda Z^\lambda_\mu\epsilon^\mu\epsilon^\nu\eta_{\nu\lambda} = -\delta^\mu_\lambda Z^\lambda_\mu = 0. \tag{7}$$

After some straightforward calculations, we obtain a new eikonal equation in Yang-Mills gravity that involves an effective metric tensor $G_L^{\mu\nu}$,

$$G_L^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = 0, \tag{8}$$

$$G_L^{\mu\nu} = J^\mu_\lambda J^{\lambda\nu} - \frac{g}{4}\phi^\mu_\lambda J^{\lambda\nu},$$

in the geometric-optics (i.e., classical) limit. It leads to the deflection angle of a light ray by the sun to be about 1.53″.[3, 4] It is stressed that we have the effective Riemannian metric tensor $G_L^{\mu\nu}$ and eikonal equation (8) only in this classical limit.

In order to see the frequency dependence of the eikonal equation in Yang-Mills gravity, let us consider the eikonal equation involving both dominate term $\partial_\alpha\Psi\partial_\beta\Psi$ and non-dominate term $\partial_\alpha\partial_\beta\Psi$. We calculate the real part of (2),

$$Re[\partial_\mu(J^\mu_\alpha F^{\alpha\nu})] = (\epsilon^\nu J^\alpha_\mu J^{\mu\sigma} - \epsilon^\mu g\phi^\alpha_\mu J^{\nu\sigma})[\partial_\alpha\Psi(\partial_\sigma\Psi)cos\Psi + (\partial_\alpha\partial\sigma\Psi)sin\Psi] \tag{9}$$

$$-g\partial_\alpha\phi^\alpha_\mu[-\epsilon^\nu J^{\mu\sigma}\partial_\sigma\Psi + \epsilon^\mu J^{\nu\sigma}\partial_\sigma\Psi]sin\Psi,$$

where the imaginary part is ignored because the electromagnetic wave is a real function. The last term in (9) is small and ignored because the gravitational strength on the surface of the sun is approximately $g\phi^{00} \approx 10^{-6}$. Eq. (9) can be simplified,

$$G_L^{\alpha\sigma}[\partial_\alpha\Psi(\partial_\sigma\Psi)cos\Psi + (\partial_\alpha\partial_\sigma\Psi)sin\Psi] \approx 0, \tag{10}$$

$$G_L^{\alpha\sigma} = (J^\alpha_\mu J^{\mu\sigma} - \frac{g}{4}\phi^\alpha_\mu J^{\mu\sigma}) \tag{11}$$

where we have used $\sum_\lambda \epsilon_\alpha(k,\lambda)\epsilon^\nu(k,\lambda) = -\delta^\nu_\alpha$.

3 Discussions of frequency dependence and tests

The simple eikonal equation (8) describes the trajectories of light rays with optical frequencies ($\approx 10^{14}$Hz) because it is derived in the limit of geometric-optics. This property is reliable and supported by optical experiments. Thus, the eikonal equation (8) may not be a good approximation for the rays with radio frequencies ($\approx 10^9 Hz$). However, the general 'eikonal equation' (10) is obtained without taking the geometric-optics limit, so it should be better in principle for describing the rays with radio frequencies. Although the numerical results are difficult to obtain from the equation (10), it implies that, experimentally, the angle of deflection by the sun depend on frequencies in Yang-Mills gravity. In other words, the eikonal equation takes the simplest form (8) only in the limit of geometric-optics. Suppose one does the deflection of light experiments, Yang-Mills gravity predicts that the angles of deflection for rays with optical frequencies ($\approx 10^{14}$Hz) and for rays with radio frequencies ($\approx 10^9 Hz$) will not be the same because equations (8) and (10) are different.

In Yang-Mills gravity, the deflection angle of a light ray by the sun is found to be about 1.53″ in the geometric-optics limit.[3, 4] This deflection angle is roughly 12% smaller than the usual value 1.75″ in general relativity.[1] The 1919 measurements made during a solar eclipse by Dyson, Eddington and Davidson[8] were very difficult and any repeat would be equally difficult. As a result, the experimental results for the bending of light by the sun with optical frequencies in the past 100 years have large

[1]The situation is quite different for the perihelion shift. In this case, the result for the advance of the perihelion differs from that of general relativity by correction terms which are of the order of 10^{-7} for Mercury.[3] The observational accuracy of the perihelion shift of Mercury is about 1 %.

uncertainties of $(10-20)\%$,[9–11] due to large systematic errors. In the experiments with optical frequencies, the systematic errors completely dominate the analysis. For example, when one measures in the daytime looking at the sun, there are extreme temperature variations and optical distortions within the optics. As a result, the precision of image measurement from space station to avoid earth's atmospheric inhomogeneities does not help much in reducing the uncertainty.

All recent accurate measurements of the light deflection angle by the sun have been performed in radio frequencies near 10^9 Hz (with uncertainties less than 0.1%), which are too far away from optical frequencies to test Yang-Mills gravity with the eikeonal equation (8). However, Yang-Mills gravity could be tested by detecting the frequency-dependence of the deflection angle by the sun with a better accuracy, say, with $(0.1-1)\%$ uncertainties.

We propose carrying out a new experiment using higher frequencies $\approx 10^{12}$ Hz which would have an uncertainty of $\approx 0.1\%$. Nowadays, this is technically possible due to the absence of systematic errors in the very long baseline interferometry. Such an experiment could test the frequency dependence of the angle of light deflection by the sun, as implied by the difference of eikonal equations (8) and (10) in Yang-Mills gravity. The eikonal equation (10) seems too complicated to be solved numerically (due to the presence of the functions $cos\Psi$ and $sin\Psi$). However, the experimental test of Yang-Mills gravity could be done, independent of the solution to (10). One can simplify concentrate on whether there is an experimental difference between the deflection angle measured with the radio frequencies $\approx 10^9 Hz$ and that measured in the frequencies $\approx 10^{12} Hz$. We believe that such an experimental test is feasible by using the very long baseline interferometry and can test the prediction of frequency-dependence of the deflection angle in Yang-Mills gravity.

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