

## Multiphoton amplitude in a constant background field

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**Abstract.** In this contribution, we present our recent compact master formulas for the multiphoton amplitudes of a scalar propagator in a constant background field using the worldline formulation of quantum field theory. The constant field has been included non-perturbatively, which is crucial for strong external fields. A possible application is the scattering of photons by electrons in a strong magnetic field, a process that has been a subject of great interest since the discovery of astrophysical objects like radio pulsars, which provide evidence that magnetic fields of the order of  $10^{12}\text{G}$  are present in nature. The presence of a strong external field leads to a strong deviation from the classical scattering amplitudes. We explicitly work out the Compton scattering amplitude in a magnetic field, which is a process of potential relevance for astrophysics. Our final result is compact and suitable for numerical integration.

### 1 Introduction

The “worldline” or “Feynman-Schwinger” [1] representation of the one-loop effective action in scalar QED is

$$\Gamma[A] = - \int_0^\infty \frac{dT}{T} e^{-m^2 T} \int_P Dx(\tau) e^{-\int_0^T d\tau [\frac{1}{4}\dot{x}^2 + ie\dot{x}^\mu A_\mu(x(\tau))]} . \quad (1)$$

Here  $m$  and  $T$  denote the mass and proper-time of the loop scalar, and  $\int_P Dx(\tau)$  the path integral over closed loops in (Euclidean) spacetime with periodicity  $T$  in the proper-time. The above path integral

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can be converted into the following master formula for the N-photon (off-shell) amplitudes [2, 3]

$$\Gamma(k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) = -(-ie)^N (2\pi)^D \delta^D\left(\sum_{i=1}^N k_i\right) \int_0^\infty \frac{dT}{T} (4\pi T)^{-\frac{D}{2}} e^{-m^2 T} \prod_{i=1}^N \int_0^T d\tau_i \times \exp\left\{\left[\sum_{i,j=1}^N \left[\frac{1}{2} G_{Bij} k_i \cdot k_j - i \dot{G}_{Bij} \varepsilon_i \cdot k_j + \frac{1}{2} \ddot{G}_{Bij} \varepsilon_i \cdot \varepsilon_j\right]\right]\right\}_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}. \quad (2)$$

Here  $G_{Bij}$ ,  $\dot{G}_{Bij}$  and  $\ddot{G}_{Bij}$  are the ‘‘bosonic’’ worldline Green’s function and its first and second derivatives with respect to the first variable ( $G_{Bij} \equiv G_B(\tau_i - \tau_j)$  etc):

$$G_{Bij} = |\tau_i - \tau_j| - \frac{(\tau_i - \tau_j)^2}{T}, \quad \dot{G}_{Bij} = \text{sign}(\tau_i - \tau_j) - 2\frac{\tau_i - \tau_j}{T}, \quad \ddot{G}_{Bij} = 2\delta(\tau_i - \tau_j) - \frac{2}{T}. \quad (3)$$

$G_{Bij}$  is the Green’s function for the operator  $\frac{d^2}{d\tau^2}$  with the periodic boundary condition, also obeying the ‘‘string-inspired’’ (SI) boundary conditions:  $\int_0^T d\tau_i G_{Bij} = \int_0^T d\tau_j G_{Bij} = 0$ . The notation  $\Big|_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}$  means taking linear terms in each polarization vector after expanding the exponential. This master formula is extremely compact and well-organized with respect to gauge invariance [2–4]. See [3] for its generalization to the spinor QED case, [6, 7] for multi-loop generalizations, and [5] for a review on the worldline formalism. For scalar QED in the presence of a constant background field  $F_{\mu\nu}$ , a similar ‘‘Bern-Kosower type’’ master formula was obtained in [8, 9]:

$$\Gamma(k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) = -(-ie)^N (2\pi)^D \delta^D\left(\sum_{i=1}^N k_i\right) \int_0^\infty \frac{dT}{T} (4\pi T)^{-\frac{D}{2}} e^{-m^2 T} \det^{-\frac{1}{2}}\left[\frac{\sin(\mathcal{Z})}{\mathcal{Z}}\right] \times \prod_{i=1}^N \int_0^T d\tau_i \exp\left\{\sum_{i,j=1}^N \left[\frac{1}{2} k_i \cdot \mathcal{G}_{Bij} \cdot k_j - i \varepsilon_i \cdot \dot{\mathcal{G}}_{Bij} \cdot k_j + \frac{1}{2} \varepsilon_i \cdot \ddot{\mathcal{G}}_{Bij} \cdot \varepsilon_j\right]\right\}_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}, \quad (4)$$

where  $\mathcal{Z} = eFT$ . The main differences between this master formula and the vacuum one in Eq. (2) is the additional factor  $\det^{-1}\left[\frac{\sin \mathcal{Z}}{\mathcal{Z}}\right]$ , and the replacement of the  $G_{Bij}$  Green’s function with a new one  $\mathcal{G}_{Bij}$  that is a function of the external field:

$$\mathcal{G}_{Bij} = \frac{T}{2} \mathcal{Z}^2 \left( \frac{\mathcal{Z}}{\sin(\mathcal{Z})} e^{-i\mathcal{Z} G_{Bij}} + i\mathcal{Z} \dot{G}_{Bij} - 1 \right). \quad (5)$$

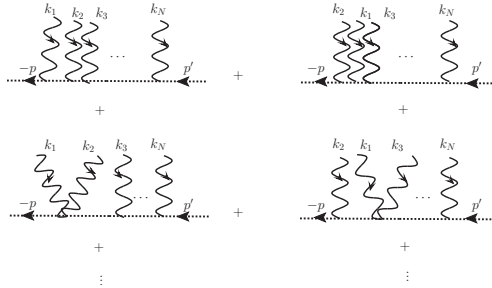
This Green’s function also follows the SI boundary conditions. This master formula, and particularly its extension to spinor QED [5, 10], are generally more efficient for constant field calculations in QED than the standard method based on Feynman diagrams. They have already been applied to photonic processes in a constant field like vacuum polarization [5, 10, 11], photon splitting (in a magnetic field) [5, 12] and the two loop Euler-Heisenberg Lagrangian [5, 9]. Much less has been done for the corresponding amplitudes with an open line (propagator) instead of the closed loop, either in vacuum or in an external field. In 1996 Daikouji et al. [13] obtained the following ‘‘Bern-Kosower type’’ master formula for the N-photon dressed scalar propagator in vacuum (see also [14] for a recent rederivation):

$$D^{pp'}(k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) = (-ie)^N (2\pi)^D \delta^D(p + p' + \sum_{i=1}^N k_i) \int_0^\infty dT e^{-m^2 T} \times \prod_{i=1}^N \int_0^T d\tau_i e^{-T\left[p + \frac{1}{T} \sum_{i=1}^N (k_i \tau_i - i\varepsilon_i)\right]^2 + \sum_{i,j=1}^N [\Delta_{ij} k_i \cdot k_j - 2i^* \Delta_{ij} \varepsilon_i \cdot k_j - i^* \Delta_{ij} \varepsilon_i \cdot \varepsilon_j]} \Big|_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}, \quad (6)$$

where  $\Delta_{ij}$  is the open line Green's function adopted to Dirichlet boundary conditions (DBC)

$$\Delta(\tau_i, \tau_j) = \frac{\tau_i \tau_j}{T} + \frac{|\tau_i - \tau_j|}{2} - \frac{\tau_i + \tau_j}{2}, \quad \Delta(0, \tau_j) = \Delta(T, \tau_j) = \Delta(\tau_i, 0) = \Delta(\tau_i, T) = 0. \quad (7)$$

For this Green's function, we need to distinguish between derivatives with respect to first and second



**Figure 1.** Multi-photon Compton-scattering diagram.

argument, since the DBC break the translation invariance in proper-time. A convenient notation is [15] to use left and right dots to indicate derivatives with respect to the first and the second argument, respectively:

$$\begin{aligned} \bullet\Delta_{ij} &= \frac{\tau_j}{T} + \frac{1}{2}\text{sign}(\tau_i - \tau_j) - \frac{1}{2}, & \bullet\Delta_{ij}^{\bullet} &= \frac{1}{T} - \delta(\tau_i - \tau_j), \\ \Delta_{ij}^{\bullet} &= \frac{\tau_i}{T} - \frac{1}{2}\text{sign}(\tau_i - \tau_j) - \frac{1}{2}, & 2\Delta_{ij} &= G_{Bij} - G_{Bj0} - G_{B0j}, \end{aligned} \quad (8)$$

where the last formula gives a relation between  $\Delta_{ij}$  and  $G_{Bij}$  [16]. The organization of the paper is as follows. In section 2 we include the constant field into the Green's function of a scalar propagator, then dress it with N photons to obtain master formulas in configuration and momentum space. In section 3 we work the momentum space formula out for  $N = 2$  to find a compact representation of the Compton scattering cross section in a general constant field. In section 4 we summarize our discussion.

## 2 Scalar propagator in a constant field

In the previous section we have presented a master formula for an open line in vacuum, in the following we consider a constant external field to be included in this master formula. We first start with a free propagator in a constant field.

### 2.1 Free propagator

A constant field in the Fock-Schwinger gauge is written as

$$A_{\mu}(y) = -\frac{1}{2}F_{\mu\nu}(y - x')^{\nu}, \quad A(y = x') = 0 \quad (x' \text{ is the reference point of the potential}). \quad (9)$$

After decomposing the arbitrary trajectory  $x(\tau)$  into a straight-line part and a fluctuation part

$$x(\tau) = x' + \frac{\tau}{T}(x - x') + q(\tau), \quad q(T) = q(0) = 0 \quad (\text{DBC}), \quad (10)$$

and using (9) and  $Q^\mu = \int_0^T d\tau q^\mu(\tau)$ , with some manipulations the free scalar propagator can be written as

$$\begin{aligned} D^{xx'}[A] &= \int_0^\infty dT e^{-m^2 T} \int_{x(0)=x'}^{x(T)=x} Dx e^{-\int_0^T d\tau [\frac{1}{4}\dot{x}^2 + ie\dot{x}\cdot A(x)]} \\ &= \int_0^\infty dT e^{-m^2 T - \frac{(x-x')^2}{4T}} \int Dq(\tau) e^{-\int_0^T d\tau [\frac{1}{4}q^2 + 2ieF\frac{d}{d\tau}q + \frac{ie}{T}(x-x')FQ]}, \end{aligned} \quad (11)$$

which is gaussian. Now besides the free path integral normalization  $\int Dq(\tau) e^{-\int_0^T d\tau q(-\frac{1}{4}\frac{d^2}{d\tau^2})q} = (4\pi T)^{-\frac{D}{2}}$  [5] we have to take into account the ratio of the field-dependent and free path integral normalization:

$$\frac{\text{Det}'_P^{-\frac{1}{2}}\left(-\frac{1}{4}\frac{d^2}{d\tau^2} + \frac{1}{2}ieF\frac{d}{d\tau}\right)}{\text{Det}'_P^{-\frac{1}{2}}\left(-\frac{1}{4}\frac{d^2}{d\tau^2}\right)} = \text{Det}'_P^{-\frac{1}{2}}\left(\mathbb{1} - 2ieF\left(\frac{d}{d\tau}\right)^{-1}\right) = \det^{-\frac{1}{2}}\left[\frac{\sin(\mathcal{Z})}{\mathcal{Z}}\right], \quad (12)$$

where we have eliminated the zero mode which exists in the path integral by putting a 'prime'. Now we have to introduce a new Green's function which is again related to the one with SI boundary conditions in (5) for a constant field:

$$\Delta_{ij} \equiv \langle \tau_i | \left(\frac{d^2}{d\tau^2} - 2ieF\frac{d}{d\tau}\right)^{-1} | \tau_j \rangle_{\text{DBC}} = \frac{1}{2}(\mathcal{G}_{Bij} - \mathcal{G}_{Bi0} - \mathcal{G}_{B0j} + \mathcal{G}_{B00}). \quad (13)$$

After using this Green's function in the usual completing-the-square procedure, we get the following well-known representation of the scalar propagator in a constant external field in  $x$ -space:

$$\begin{aligned} D^{xx'}(F) &= \int_0^\infty dT e^{-m^2 T} e^{-\frac{(x-x')^2}{4T}} (4\pi T)^{-\frac{D}{2}} \det^{-\frac{1}{2}}\left[\frac{\sin(eFT)}{eFT}\right] e^{\int_0^T d\tau \int_0^T d\tau' \frac{ie}{T}(x-x')F\Delta(\tau,\tau')\frac{ie}{T}F(x-x')} \\ &= \int_0^\infty dT e^{-m^2 T} (4\pi T)^{-\frac{D}{2}} \det^{\frac{1}{2}}\left[\frac{\mathcal{Z}}{\sin \mathcal{Z}}\right] e^{-\frac{1}{4T}(x-x')\mathcal{Z} \cot \mathcal{Z}(x-x')}. \end{aligned} \quad (14)$$

After Fourier transforming its representation in momentum space gives

$$D^{pp'}(F) = (2\pi)^D \delta(p + p') D(p, F) \quad , \quad D(p, F) = \int_0^\infty dT e^{-m^2 T} \frac{e^{-Tp(\frac{\tan \mathcal{Z}}{\mathcal{Z}})p}}{\det^{\frac{1}{2}}[\cos \mathcal{Z}]}. \quad (15)$$

## 2.2 Dressed propagator

Now, if we dress the scalar propagator with  $N$  photons in addition to the constant field, we write the potential as  $A = A_{\text{ext}} + A_{\text{phot}}$  which the former is the same as in (9) and the later is written as a sum of plane waves, each photon is represented as a vertex operator:

$$A_{\text{phot}}^\mu = \sum_{i=1}^N \varepsilon_i^\mu e^{ik_i \cdot x} \quad , \quad V^A[k, \varepsilon] = \int_0^T d\tau \varepsilon \cdot \dot{x}(\tau) e^{ik \cdot x(\tau)} = \int_0^T d\tau e^{ik \cdot x(\tau) + \varepsilon \cdot \dot{x}(\tau)} \Big|_{\text{lin}(\varepsilon)}. \quad (16)$$

The path integral representation of the scalar propagator in a constant field dressed with  $N$  photons is written as

$$D^{xx'}(F|k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) = (-ie)^N \int_0^\infty dT e^{-m^2 T} \int_P Dx e^{-\int_0^T d\tau [\frac{1}{4}\dot{x}^2 + ie\dot{x}\cdot A_{\text{ext}}(x)]} V^A[k_1, \varepsilon_1] \dots V^A[k_N, \varepsilon_N]. \quad (17)$$

After applying the path decomposition (10) and completing the square we get ( $x_- = x - x'$ )

$$\begin{aligned}
 D^{xx'}(F|k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) &= (-ie)^N \int_0^\infty dT e^{-m^2 T} (4\pi T)^{-\frac{D}{2}} \det^{\frac{1}{2}} \left[ \frac{\mathcal{Z}}{\sin \mathcal{Z}} \right] e^{-\frac{1}{4T} x_- \cdot \mathcal{Z} \cot \mathcal{Z} x_-} \\
 &\times \int_0^T d\tau_1 \dots \int_0^T d\tau_N e^{\sum_{i=1}^N (\varepsilon_i \cdot \frac{x_-}{T} + ik_i \cdot \frac{x_- \tau_i}{T} + ik_i \cdot x')} \\
 &\times \exp \left[ \sum_{i,j=1}^N \left( k_i \underline{\Delta}_{ij} k_j - 2i\varepsilon_i \bullet \underline{\Delta}_{ij} k_j - \varepsilon_i \bullet \underline{\Delta}_{ij} \varepsilon_j \right) + \frac{2e}{T} x_- \cdot \sum_{i=1}^N \left( F \circ \underline{\Delta}_i k_i - iF \circ \underline{\Delta}_i \varepsilon_i \right) \right] \Big|_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}, \quad (18)
 \end{aligned}$$

where a left (right) ‘open circle’ on  $\underline{\Delta}(\tau, \tau')$  denotes an integral  $\int_0^T d\tau$  ( $\int_0^T d\tau'$ ). This  $x$ -space master formula was obtained by McKeon and Sherry for a purely magnetic field [17]. Now we can Fourier transform to momentum space to get

$$\begin{aligned}
 D^{pp'}(F|k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) &= (-ie)^N (2\pi)^D \delta(p + p' + \sum_{i=1}^N k_i) \int_0^\infty dT e^{-m^2 T} \frac{1}{\det^{\frac{1}{2}} [\cos \mathcal{Z}]} \\
 &\times \int_0^T d\tau_1 \dots \int_0^T d\tau_N e^{\sum_{i,j=1}^N (k_i \underline{\Delta}_{ij} k_j - 2i\varepsilon_i \bullet \underline{\Delta}_{ij} k_j - \varepsilon_i \bullet \underline{\Delta}_{ij} \varepsilon_j)} e^{-Tb(\frac{\tan \mathcal{Z}}{\mathcal{Z}})b} \Big|_{\varepsilon_1 \varepsilon_2 \dots \varepsilon_N}, \\
 b &\equiv p + \frac{1}{T} \sum_{i=1}^N \left[ (\tau_i - 2ieF \circ \underline{\Delta}_i) k_i - i \left( 1 - 2ieF \circ \underline{\Delta}_i \right) \varepsilon_i \right]. \quad (19)
 \end{aligned}$$

Diagrammatically, these master formula hold the full information on the set of Feynman diagrams appearing in Fig. 1, with the full scalar propagator in a constant field (usually indicated by a double line).

### 3 Compton scattering in a constant field

Now, let us apply our momentum space master formula in (19) to work out the Compton scattering cross section, corresponding to  $N = 2$ . Notice that our master formula describes the *untruncated* dressed propagator so in applying it to physical processes one has to truncate the external scalars, we define the matrix element  $\mathcal{T}$  as

$$\mathcal{T}(F|k_1, \varepsilon_1; \dots; k_N, \varepsilon_N) = \frac{D^{pp'}(F|k_1, \varepsilon_1; \dots; k_N, \varepsilon_N)}{D(p, F)D(p', F)}. \quad (20)$$

Moreover, it will be convenient to Wick rotate from Euclidean to Minkowski space (see [18] for our conventions). Expanding out the exponentials in (19) up to terms linear in both polarizations we arrive at

$$D^{pp'}(F|k_1, \varepsilon_1; k_2, \varepsilon_2) = e^2 \int_0^\infty dT \frac{e^{-m^2 T}}{\det^{\frac{1}{2}} [\cos \mathcal{Z}]} \int_0^T d\tau_1 d\tau_2 e^{-Tb_0(\frac{\tan \mathcal{Z}}{\mathcal{Z}})b_0 + \sum_{i,j=1}^2 k_i \underline{\Delta}_{ij} k_j} \varepsilon_1 M_{12} \varepsilon_2, \quad (21)$$

with

$$\begin{aligned}
 b_0 &\equiv p + \frac{1}{T} \sum_{i=1}^2 (\tau_i - 2ieF \circ \underline{\Delta}_i) k_i, \\
 M_{12} &\equiv 2 \bullet \underline{\Delta}_{12} - \frac{2}{T} \left( 1 + 2ie \bullet \underline{\Delta}_1^T F \right) \frac{\tan \mathcal{Z}}{\mathcal{Z}} \left( 1 - 2ieF \circ \underline{\Delta}_2 \right) \\
 &+ 4 \left[ \left( 1 + 2ie \bullet \underline{\Delta}_1^T F \right) \frac{\tan \mathcal{Z}}{\mathcal{Z}} b_0 - \sum_{i=1}^2 \bullet \underline{\Delta}_{1i} k_i \right] \left[ b_0 \frac{\tan \mathcal{Z}}{\mathcal{Z}} \left( 1 - 2ieF \circ \underline{\Delta}_2 \right) - \sum_{i=1}^2 k_i \bullet \underline{\Delta}_{i2} \right]. \quad (22)
 \end{aligned}$$

To obtain the cross section we square and sum over the photon polarizations via  $\sum_{\text{pol}} \varepsilon_i^{\mu} \varepsilon_i^{\nu} \rightarrow g^{\mu\nu}$  which finally leads to the following expression for the Compton cross section:

$$\begin{aligned} \sum_{\text{pol}} \mathcal{T}^* \mathcal{T} &= \frac{e^4}{|D(p, F)|^2 |D(p', F)|^2} \int_0^\infty dT' \frac{e^{-m^2 T'}}{\det^{\frac{1}{2}} [\cos Z']} \int_0^{T'} d\tau'_1 \int_0^{T'} d\tau'_2 e^{-T' b_0 (\frac{\tan Z'}{Z'}) b_0 + \sum_{i,j=1}^2 k_i \Delta'_{ij} k_j} \\ &\times \int_0^\infty dT \frac{e^{-m^2 T}}{\det^{\frac{1}{2}} [\cos Z]} \int_0^T d\tau_1 \int_0^T d\tau_2 e^{-T b_0 (\frac{\tan Z}{Z}) b_0 + \sum_{i,j=1}^2 k_i \Delta_{ij} k_j} \text{tr}(M'_{12}{}^\dagger M_{12}). \end{aligned} \quad (23)$$

## 4 Summary

In this short report we have extended our previous master formulas for multiphoton amplitudes in vacuum in scalar QED [14] to include a constant external field using the worldline formalism, see [18] for more details. Our master formula is valid off-shell, and combines the various orderings of the  $N$  photons along the scalar line. We have worked out the cross section for the case of  $N = 2$ , that is linear Compton scattering in the field, reaching a compact integral representation suitable for numerical evaluation. This cross section is, for the magnetic field case, of importance for astrophysics, but appears to have been studied so far only in the strong-field limit [19].

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