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Baryonic Force for Accelerated Cosmic Expansion and Generalized U_{1b} Gauge Symmetry in Particle-Cosmology

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Abstract. Based on baryon charge conservation and a generalized Yang-Mills symmetry for Abelian (and non-Abelian) groups, we discuss a new baryonic gauge field and its linear potential for two point-like baryon charges. The force between two point-like baryons is repulsive, extremely weak and independent of distance. However, for two extended baryonic systems, we have a dominant linear force $\propto r$. Thus, only in the later stage of the cosmic evolution, when two baryonic galaxies are separated by an extremely large distance, the new repulsive baryonic force can overcome the gravitational attractive force. Such a model provides a gauge-field-theoretic understanding of the late-time accelerated cosmic expansion. The baryonic force can be tested by measuring the accelerated Wu-Doppler frequency shifts of supernovae at different distances.

1 Introduction

In particle-cosmology, apart from the well-known forces in quantum chromodynamics, the electroweak theory, and gravity, there are other possible forces related to quark confinement and the accelerated cosmic expansion. The quark confinement force could be related to a linear potential. The force responsible for the accelerated cosmic expansion should be extremely weak and repulsive between baryon matter. Collecting these together, a total-unified group G_{tu} in particle cosmology [1], consistent with established conservation laws, might be $G_{tu} = T_4 \times [SU_3]_{color} \times \{SU_2 \times U_1\} \times U_{1b} \times U_{1\ell}$, where U_{1b} and $U_{1\ell}$ are associated to baryon and lepton charges (or numbers) respectively. For simplicity, we ignore $U_{1\ell}$ here, for it can be included without difficulty.

Let us concentrate on the baryon sector U_{1b} of the particle-cosmology model based on the following two postulates:

(i) a ‘generalized’ gauge symmetry [2, 3] for conserved baryon charges with the U_{1b} group,

(ii) the invariant Lagrangian $\propto \partial^\mu B_{\mu\lambda} \partial_\nu B^{\nu\lambda}$ for the baryon dynamics with the gauge fields B_μ.

This particle-cosmology model could provide an understanding of the late-time accelerated cosmic expansion based on the U_{1b} gauge fields in flat space-time rather than the unknown dark energy. Such a particle-cosmology model does not require 68% of dark energy to explain the accelerated expansion. Its prediction of the specific acceleration could be tested in the future.

The generalization of gauge symmetry with exact conservation law has been discussed by introducing vector gauge functions [2, 3] and Hamilton’s characteristic (phase) functions [4, 5] into the new

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gauge transformations. It differs from the usual gauge transformation, which involves scalar gauge function and ordinary phase function. They lead to new massless gauge fields which satisfy 4th-order filed equations. These 4th-order field equations in the static limit give us a linear static potential and a constant repulsive force between two point-like baryon charges.

In the special case, when the vector gauge functions $\omega_\mu(x)$ can be expressed in terms of the space-time derivative of scalar functions, $\omega_\mu(x) = \partial_\mu\omega(x)$, the generalized gauge symmetry reduces to the usual gauge symmetry [2]. As a result, one has the usual second-order differential equation for massless fields, which has been discussed by Lee and Yang in 1955 [6]. This usual Lee-Yang U_1 gauge field for conserved baryon charge is identical to that of the electromagnetic theory with the conserved electric charge. E*ö*tv*ö*s experiments gave a stringent limit of the strength of such a 'Lee-Yang force.' The inverse-square force between baryons in nucleus would violate the equivalence principle and, hence, the force should be roughly a million times weaker than the gravitational force. Thus, such an extremely weak Lee-Yang force does not have any observable results in the laboratory or in the cosmos. However, the new 'generalized' gauge symmetry for baryon charges with the U_{1b} group will have important implications in cosmos and may be called 'general Lee-Yang gauge symmetry.'

2 Generalized U_{1b} gauge symmetry

Let us consider a physical system with a gauge field $B_\mu(x)$ and a quark field $q(x)$ with spin 1/2 . We define the generalized U_{1b} gauge transformations for these two fields [2],

$$B'_\mu(x) = B_\mu(x) + \omega_\mu(x), \qquad \mu = 0, 1, 2, 3, \quad c = \hbar = 1, \tag{1}$$

$$q'(x) = e^{-iP_\omega(x)}q(x), \quad \overline{q}'(x) = \overline{q}(x)e^{iP_\omega(x)}, \quad P_\omega(x) = g_b\left(\int_{x'_o}^{x'_e=x} \omega_\lambda(x')dx'^\lambda\right)_{Le}, \tag{2}$$

where $P_\omega(x)$ is a 'characteristic function'[1] and the subscript *Le* in (2) implies that the vector gauge function $\omega(x)_\mu$ satisfies the Lagrange equation,

$$d\omega_\lambda(x') - \frac{\partial\omega_\mu(x')}{\partial x'^\lambda}dx'^\mu = 0. \tag{3}$$

We can show that the variation of the action functional P_ω is actually a function of the coordinates. The reasons are that the initial point x_o is fixed so that $\delta x'(x'_o) = 0$, the end point is variable, $x'_e = x$, and that the gauge function $\omega_\lambda(x')$ satisfies (3). Furthermore, in order to have the general gauge invariant Lagrangian for the baryon dynamics, the vector gauge functions $\omega_\mu(x)$ are required to satisfy the constraint,

$$\partial^2\omega_\mu(x) - \partial_\mu\partial^\lambda\omega_\lambda(x) = 0, \qquad \partial^2 = \partial_\mu\partial^\nu\eta^{\mu\nu}, \quad \eta^{\mu\nu} = (1, -1, -1, -1). \tag{4}$$

The requirement (3) is necessary for the integral in the variation of $P_\omega(x)$ to vanish, i.e., $+g_b\int_{x'_o}^{x}\left[(\partial\omega_\mu(x')/\partial x'^\lambda)dx'^\mu - d\omega_\lambda(x')\right]\delta x'^\lambda = 0$, so that we have [4, 5]

$$\delta P_\omega(x) = g_b\omega_\mu(x)\delta x^\mu, \quad or \quad \partial_\mu P_\omega = g_b\omega_\mu(x). \tag{5}$$

This equation (5) for the U_{1b} phase 'characteristic function $P_\omega(x)$ is crucial for a Lagrangian to have the general Lee-Yang symmetry. The U_{b1} covariant derivative $\Delta_{b\mu}$ is defined as usual

$$\Delta_{b\mu} = \partial_\mu + ig_bB_\mu. \tag{6}$$

[1] In the literature, it is called Hamilton's characteristic function, which is well-defined, independent of path, and a local function of x, as discussed in refs. 4 and 5.

In the generalized gauge symmetry, we have the vector gauge functions $\omega_\mu(x)$, which have four functions instead of a mere scalar function in the usual gauge symmetry. These four gauge functions are required to satisfy four partial differential equations in (4), therefore the choice of these four vector gauge functions is not completely arbitrary. However, the gauge functions $\omega_\lambda(x)$ in the generalized gauge transformations still involve infinitely many functions.

One can verify that $\overline{\psi}(\partial_\mu + ig_b B_\mu)\psi$ is invariant under the general U_{1b} transformations (1)-(3),

$$\overline{\psi}'(\partial_\mu + ig_b B'_\mu)\psi' = \overline{\psi}(\partial_\mu + ig_b B_\mu)\psi, \tag{7}$$

where we have used equations (1), (2), (3) and (6).

The general U_{1b} gauge covariant derivative, $\partial_\mu + ig_b B_\mu$, in (6) has the same form as that in the usual U_1 gauge theory QED. The U_{1b} gauge curvature is, as usual, determined by the commutator of the covariant derivative,

$$[\partial_\mu + ig_b B_\mu, \partial_\nu + ig_b B_\nu] = ig_b B_{\mu\nu}, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{8}$$

Under the general U_{1b} transformation (3), the U_{1b} gauge curvature $B_{\mu\nu}$ turns out not to be invariant, $B'_{\mu\nu} = B_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu \neq B_{\mu\nu}$. However, the divergence of the gauge curvature, $\partial^\mu B_{\mu\nu}$, is invariant,

$$\partial^\mu B'_{\mu\nu} = \partial^\mu B_{\mu\nu}, \tag{9}$$

because the vector functions $\omega_\mu(x)$ satisfy the constraint equation (4). The result (9) enables us to write down the general U_{1b} gauge invariant Lagrangian,

$$L = \frac{L_s^2}{2}\partial^\mu B_{\mu\lambda}\partial_\nu B^{\nu\lambda} + i\overline{q}[\gamma^\mu(\partial_\mu - ig_b B_\mu) - M]q, \tag{10}$$

where L_s denotes a universal scale with the dimension of length (for a physical system with a general Lee-Yang symmetry). The fourth order gauge field equation can be derived from (10),

$$\partial^2\partial^2 B_\mu = \frac{g_b}{L_s^2}\overline{q}\gamma_\mu q, \tag{11}$$

where we have chosen the gauge condition $\partial_\mu B^\mu = 0$. One can replace the source term in (11) for $\mu = 0$ by a point baryon charge $g_b\delta^3(\mathbf{r})/L_s^2$, and find the static potential $B_0(\mathbf{r}) = -g_b r/(8\pi L_s^2)$. This linear potential B_0 leads to a constant force, i.e., independent of distance between two point baryon charges. The four-dimensional symmetry framework for the baryon dynamics can be formulated in a broader view with the coordinate $x^\mu = (w, x, y, z)$, so that it can be generalized to a general frame (inertial and non-inertial) on the basis of the limiting Lorentz-Poincaré invariance [7, 8].

3 Calculations of cosmic force between baryonic systems

Now let us consider a simple model of particle-cosmology to investigate cosmic baryon force in detail. Although the force between two point baryonic charges is distance-independent, one cannot simply assume that the total baryonic charges in a uniform sphere are concentrated at the center of the sphere and that the resultant force is distance-independent. The situation for baryon force differs from that of the well-known inverse-square force.

The cosmic baryon force can be calculated in two steps: First, we approximate the supernova as a point in a huge cluster of galaxies, which is treated as a big sphere of uniform baryon charges with a very large radius R_o.[2] The resultant force is no longer distance-independent, but the sphere of baryon charges can be considered as concentrated at the center of the sphere. Second, we replace the point-like supernova by a sphere of baryon charge with a radius $R_s \ll R_o$. The result could provide a way to test the cosmic baryon force by measuring the distance-dependence of accelerations of supernovae.

We may picture a supernova having a mass m_s and a radius R_s inhabiting in a sphere of, say, approximately 100 billions of galaxies. These baryonic galaxies can be idealized as points uniformly spread out in the entire sphere with a baryon mass density ρ. The mass of this sphere is $M = \rho 4\pi R_o^3/3$. Each nucleon has three quark charges, i.e., $3g_b$. Suppose the point-like supernova has a mass m and

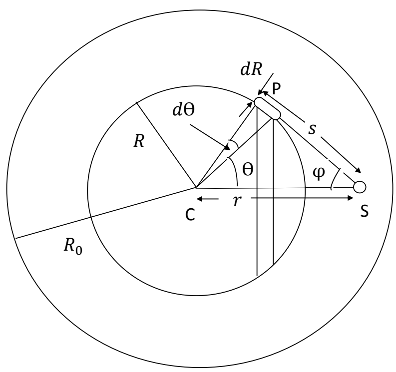

Fig. 1. A schematic diagram for calculations of the cosmic baryon force F_{CS1} between a gigantic uniform sphere of baryon galaxies and a point-like baryon supernova.

baryon charges $3g_b m/m_p$, where m_p stands for the mass of a baryon (proton or neutron). In Fig. 1, we consider a thin uniform shell of radius R , where R is less than R_o . The thickness on this shell is dR . Here the supernova is outside the shell at a distance $r > R$. A point P is on the ring with radius R. The angle SCP in the diagram is θ . The mass dM_r of the ring is

$$dM_r = \rho 2\pi R^2 dR \ \sin\theta d\theta,$$

where $2\pi R \sin\theta$ is the circumference of the ring. At point P , let us consider an infinitesimal element. The effective repulsive force in between this infinitesimal element and the point-like supernova is dF_p. The horizontal component of the force dF_p is given by $dF_p \cos\phi$. The vertical components of the force dF_p are cancelled completely due to the symmetry involved in the geometry we are considering here. We calculate the total force dF_{CS} in the horizontal direction CS from the entire ring on the point-like supernova at S. Using the equation for the constant force between two point-like baryonic charges, we find

$$dF_{CS} = A \int dR \int_0^\pi \left[R^2 \cos\phi \right] \sin\theta \, d\theta = A \int \int_{R-r}^{R+r} [dF] \frac{s}{Rr} ds, \qquad A = \frac{(3g_b)^2 m\rho}{4L_s^2 m_p^2}, \qquad (12)$$

$$dF \equiv dR \, R^2 \left[\frac{s^2 + r^2 - R^2}{2sr} \right], \qquad s^2 = R^2 + r^2 - 2Rr \ \cos\theta, \qquad R^2 = s^2 + r^2 - 2rs \ \cos\phi.$$

[2]The observable portion of the universe is approximate by a finite and gigantic sphere for our calculations.

The magnitude of the total repulsive force can be obtained [2] from the integrations of ds and dR in (12) for two cases:

$$(i) \qquad F_{CS1} = A \int_0^r \frac{RdR}{2r^2} \int_{r-R}^{r+R} ds(s^2 + r^2 - R^2), \quad r > R, \tag{13}$$

$$(ii) \qquad F_{CS2} = A \int_r^{R_o} \frac{R'dR'}{2r^2} \int_{R'-r}^{R'+r} ds(s^2 + r^2 - R'^2), \quad R_o > R' > r. \tag{14}$$

In the calculation of F_{CS2}, we modify Fig.1 such that the distance CS is smaller than $R'(= R)$. We obtain the distance-dependence of the cosmic baryon force $F_{CS1} + F_{CS2}$ to be $\propto [r/R_o - r^3/(5R_o^3)]$.

Next, we consider an extended supernova with a radius R_s and baryon mass density ρ_s. We may use Fig. 1 with the modifications: (a) C is the center of the supernova and replace R_o by R_s, and (b) move the point S to outside so that $r > R_s$, and consider all baryon charges of the gigantic sphere, $3g_b M/m_p$, as concentrated at S. After some tedious but straightforward calculations, the total effective cosmic baryon force (cbf) of the gigantic sphere with baryon galaxies exerted on a supernova with radius R_s is

$$F_{cbf} = \left(\frac{(3g_b)^2 mM}{8\pi L_s^2 m_p^2}\right)\left[\frac{r}{R_o} - \frac{r^3}{5R_o^3} - \frac{rR_s^3}{5R_o^3}\right], \qquad m = \frac{4\pi R_s^3 \rho_s}{3}, \tag{15}$$

which turns out to be dominated by a linear force because $R_s < r << R_o$.[3] In the limit $R_s \to 0$, the force in (15) reduces to that for a point-like supernova, i.e., $F_{CS1} + F_{CS2}$ in (13)-(14), as it should be. We may remark that (a) we use a simple model corresponds to a finite R_o for simplicity and (b) if one wishes, one may take the limit $R_o \to \infty$, provided that the density ρ also approaches zero such that $R_o^2 \rho$ remains finite. We may remark that the trajectories of two macroscopic baryon objects have been numerically calculated [9]. For the case corresponding to the elliptical orbit in the Kepler problem, one has a 'distorted elliptical orbit' for two baryonic systems.

The dominant linear force in (15) predicts that the measured acceleration of the cosmic expansion should be approximately linear in r because R_o is very much larger than r and R_s. This prediction could be tested by measuring the accelerated frequency shifts[4] of supernovae at different distances.

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