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Colliding scalar pulses in the Einstein-Gauss-Bonnet gravity

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Abstract. We numerically investigated how the nonlinear dynamics depends on the dimensionality and on the higher-order curvature corrections in the form of Gauss-Bonnet (GB) terms, with a model of colliding scalar pulses in plane-symmetric space-time. We observed that a collision of large scalar pulses will produce a large-curvature region, of which the magnitude depends on $\alpha_{\rm GB}$. The normal corrections ($\alpha_{\rm GB} > 0$) work for avoiding the appearance of singularity, although it is inevitable.

1 Introduction

The Einstein-Gauss-Bonnet (GB) gravity theory is one of the plausible candidates which describes the early Universe, but so far little is known of its nonlinear dynamical behaviors. We numerically investigated the dynamics in higher-dimensional space-time with and without the GB terms. (Please refer the detail report [1]).

The Einstein-GB gravity is derived from string theory, with additional higher-order-curvature correction terms to GR in the form of the Lagrangian,

$$\mathcal{L}_{\rm GB} = \mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}, \tag{1}$$

where $\mathcal{R}$, $\mathcal{R}_{\mu\nu}$, and $\mathcal{R}_{\mu\nu\rho\sigma}$ are the n-dimensional scalar curvature, the Ricci tensor, and the Riemann curvature, respectively. This particular combination gives us several reasonable properties, as such ghost-free combinations, and a set of equations up to the second derivative in spite of the higher-curvature combinations. The theory is expected to have singularity-avoidance features in the context of gravitational collapses and/or cosmology. However, only a few studies so far have reported on the investigation of nonlinear dynamical features in Einstein-GB gravity (e.g., numerical studies on critical phenomena [2, 3], black hole formation in AdS [4]).

Our investigative model deals with colliding wave packets. Due to the nonlinear features of the theory, in GR, gravitational waves interact with themselves when they pass through each other. Considering a collision of plane gravitational waves is the simplest scenario of this nonlinear interaction problem (see Ref. [5] and references therein). In fact, Penrose [6] pointed out that the future light cone of a plane wave is distorted as it passes through another plane wave. Since these solutions assume a plane-symmetric space-time, this singularity does not have a horizon; it is a “naked" one.

Our attention to this problem focuses on the differences in the growth of curvature, especially the dependences on the dimension and the GB terms.

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2 Field Equations and numerical technique

2.1 Action

The Einstein-GB action in n-dimensional space-time $(\mathcal{M}, g_{\mu\nu})$ is described as

$$S = \int_{\mathcal{M}} d^n x \sqrt{-g}\Big[\frac{1}{2\kappa^2}(\mathcal{R} + \alpha_{\rm GB}\mathcal{L}_{\rm GB}) + \mathcal{L}_{\rm matter}\Big], \tag{2}$$

where $\mathcal{L}_{\rm GB}$ is the GB term [Eq. (1)], κ^2 is the n-dimensional gravitational constant, and $\mathcal{L}_{\rm matter}$ is the matter Lagrangian. This action reproduces the standard n-dimensional Einstein gravity, if we set $\alpha_{\rm GB} = 0$. In the actual simulations, we set $\kappa^2 = 1$ and change $\alpha_{\rm GB}$ as a parameter.

The action (2) gives the gravitational equation as

$$G_{\mu\nu} + \alpha_{\rm GB} H_{\mu\nu} = \kappa^2 T_{\mu\nu}, \tag{3}$$

where $G_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R}$, and

$$H_{\mu\nu} = 2\Big(\mathcal{R}\mathcal{R}_{\mu\nu} - 2\mathcal{R}_{\mu\alpha}\mathcal{R}^{\alpha}{}_{\nu} - 2\mathcal{R}^{\alpha\beta}\mathcal{R}_{\mu\alpha\nu\beta} + \mathcal{R}_{\mu}{}^{\alpha\beta\gamma}\mathcal{R}_{\nu\alpha\beta\gamma}\Big) - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\rm GB}. \tag{4}$$

We assume massless scalar field, ψ, of which energy-momentum tensor is given by

$$T_{\mu\nu} = \partial_\mu\psi\partial_\nu\psi - g_{\mu\nu}\frac{1}{2}(\nabla\psi)^2, \tag{5}$$

which obeys the Klein-Gordon equations, $\Box\psi = 0$.

2.2 Dual-null formulation

We use dual-null formulation for expressing space-time which has planar symmetry. The use of dual-null coordinates simplifies the treatment of horizon dynamics, enables us to approach close to large-curvature regions, and also clarifies radiation propagation in far regions. We implement our dual-null evolution code which was used for four-dimensional GR [7] so as to follow higher-dimensional space-time with the GB terms.

We adopt the line element

$$ds^2 = -2e^{f(x^+,\, x^-)}dx^+\, dx^- + r^2(x^+,\, x^-)\gamma_{ij}dz^i dz^j, \tag{6}$$

where the coordinates (x^+, x^-) are along the null propagation directions, and $\gamma_{ij}dx^i dx^j$ is the metric of the flat $(n-2)$-dimensional space.

For writing down the field equations, we introduce the variables

$$\Omega \equiv \frac{1}{r}, \qquad \vartheta_\pm \equiv (n-2)\partial_\pm r, \qquad \nu_\pm \equiv \partial_\pm f, \qquad \pi_\pm \equiv \partial_\pm\psi \tag{7}$$

where $\partial_\pm \equiv \partial/\partial x^\pm$, and these are the conformal factor, expansions, and inaffinities, respectively. Please refer [1] for a set of equations.

2.3 Numerical integration scheme

The basic idea of numerical integration is as follows. We prepare our numerical integration range as drawn in Fig. 1. We give initial data on a surface Σ_0, where $x^+ = x^- = 0$, and the two null hypersurfaces $\Sigma_\pm$, generated from it, where $x^\mp = 0$ and $x^\pm > 0$. Due to the dual-null decomposition, the causal region of a grid is clear, and there are in-built accuracy checks: the integrability conditions or consistency conditions $\partial_-\partial_+ u = \partial_+\partial_- u$.

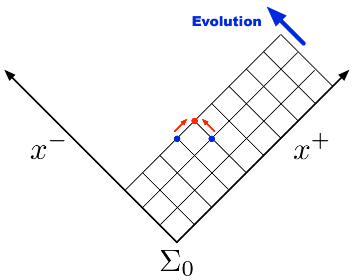

Figure 1. Numerical grid structure. Initial data are given on null hypersurfaces $\Sigma_\pm$ ($x^\mp = 0$, $x^\pm > 0$) and their intersection Σ_0. Generally, the initial data have to be given as $(\Omega, f, \vartheta_\pm, \psi)$ on Σ_0, and $(\nu_\pm, \pi_\pm)$ on $\Sigma_\pm$. We then evolve the data $u = (\Omega, \vartheta_\pm, f, \nu_\pm, \psi, \pi_\pm)$ on a constant-x^- slice to the next.

3 Numerical evolutions of the collision of scalar pulses

There are several exact solutions of the colliding plane waves, which produce curvature singularity after their collisions. We prepare a similar situation in our code and examine such a strong curvature effect in higher-dimensional GR and in Einstein-GB gravity. We first note that in the construction of exact solutions, the wave fronts are assumed to be a step function, while in our simulations the wave fronts are a continuous function. We have found that we can compare the behaviors more easily when we place colliding matter rather than colliding gravitational waves.

We put a perturbed scalar field (ψ) in the flat background on the initial surfaces and evolve it. The initial scalar field is set as $\psi = 0$ and has momentum

$$\begin{cases} \pi_+ = a\exp(-b(x^+/\sqrt{2}-c)^2) \\ \pi_- = 0 \end{cases} \text{on } \Sigma_+, \qquad \begin{cases} \pi_+ = 0 \\ \pi_- = a\exp(-b(x^-/\sqrt{2}-c)^2) \end{cases} \text{on } \Sigma_-. \tag{8}$$

3.1 Evolutions in GR

The two typical evolutions are shown in Fig. 2. We plot the behaviors of the scalar field and the Kretschmann scalar, $\mathcal{I}^{(5)}$, for five-dimensional GR. We set $a = 0.2$ and 0.4, and $b = 10$, $c = 2$ for these plots. For small pulses [Fig. 2(a)], we see that two pulses just pass through each other and the curvature $\mathcal{I}^{(5)}$ turns back to the flat again. On the contrary, for large pulses [Fig. 2(b)], the nonlinear curvature evolution appears after the collision of pulses. The latter behavior is similar to the exact solutions of the plane-wave collision (see, e.g., figures in Ref. [8]). We actually find that in all blow-up regions, both expansions are $\vartheta_\pm < 0$. In four-dimensional plane-symmetric space-time, if the curvature blows up, then it means the appearance of a naked singularity, since there is no chance to form a horizon. However, in higher dimensions, we expect such a blow-up will be hidden in a horizon as the expansions suggest.

3.2 Evolutions in Einstein-GB

We also evolved the same initial data by the set of evolution equations with nonzero $\alpha_{\rm GB}$.

Figure 3(a) displays the Kretschmann scalar, $\mathcal{I}^{(5)}(= R^{ijkl}R_{ijkl})$, for both $\alpha_{\rm GB} = +1$ and $\alpha_{\rm GB} = -1$ cases for the same initial data with the large-amplitude case ($a = 0.4$) in Fig. 2(b). We see that the local peak of $\mathcal{I}^{(5)}$ at the collision of two pulses (at $x^+ = x^- = 2\sqrt{2}$) is smaller (larger) when $\alpha_{\rm GB} > 0$ ($\alpha_{\rm GB} < 0$) than that in GR.

In Fig. 3(b), we plot the "evolution" behavior of the Kretschmann scalar, $\mathcal{I}^{(5)}$, at the origin ($x^+ = x^-$) where two pulses collide. At later times, we see that the curvature will diverge for all the cases (GR and Einstein-GB) due to the large amplitude of the initial pulses, but these growing behaviors are

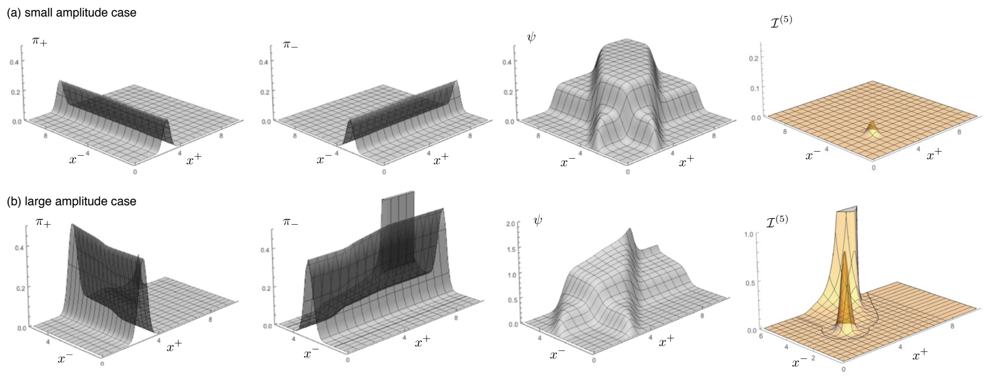

Figure 2. Evolutions of colliding two scalar pulses in five-dimensional GR: (a) the small-amplitude case [$a = 0.2$ in Eqs. (8)], and (b) the large-amplitude case ($a = 0.4$). The scalar momentum $\pi_\pm$, scalar field ψ, and the Kretschmann scalar $\mathcal{I}^{(5)}$ are plotted in the (x^+, x^-) coordinates. Initial data were set at both Σ_- ($x^+ = 0$, $x^- > 0$) and Σ_+ ($x^+ > 0$, $x^- = 0$) and evolved. For small pulses, we see that they just cross, and space-time turns back toward flat again, while for large pulses, we see that nonlinear curvature evolution appears after the collision of pulses. The latter behavior is similar to the exact solutions of the plane-wave collision.

again ordered by $\alpha_{\rm GB}$. Supposing that the curvature singularity will be formed at the final phase of this evolution (analogues to the plane-wave collision), then we can say that introducing the GB terms cannot stop the formation of the singularity, but it will shift its appearance later if $\alpha_{\rm GB} > 0$.

Figure 3(right) shows the magnitude of the Kretschmann scalar, $\mathcal{I}^{(n)}$, at the moment of the collision of scalar pulses (at the first peak of $\mathcal{I}^{(n)}$). We plot the cases $\alpha_{\rm GB} = 0, \pm 0.1, \pm 0.5, \pm 1.0$ and the dimensions $n = 4, 5, 6$, and 7. We see that for $n = 4$, all three cases have the same magnitude, which is consistent with the fact that the GB correction does not appear at $n = 4$. For larger dimensions, the magnitude becomes lower. We also find that introducing positive $\alpha_{\rm GB}$ (i.e. the normal higher-curvature correction) reduces its magnitude.

4 Summary and Discussions

We investigated fully nonlinear dynamics in n-dimensional Gauss-Bonnet gravity using a model of colliding scalar pulses. We observed that curvature (Kretschmann scalar) evolves more mildly in the presence of the normal GB terms ($\alpha_{\rm GB} > 0$) and in higher-dimensional space-time. The appearance of the singularity is inevitable in our model, but the basic feature is matched with the expected effect of the cosmologists; i.e., the avoidance (or lower possibility) of the appearance of the singularity.

As is shown in other models (e.g., Refs. [1, 9]), in higher-dimensional GR, the chance of appearances of naked singularities is suppressed compared to the four-dimensional GR cases. This is suggested by the existence of many freedoms in gravity which suppresses the growth of curvature and makes the formation of horizons less eccentric. The introduction of the GB terms seems to work for this direction. We hope that these results will be used as a guiding principle for understanding the fundamental dynamical features of the Einstein-GB gravity.

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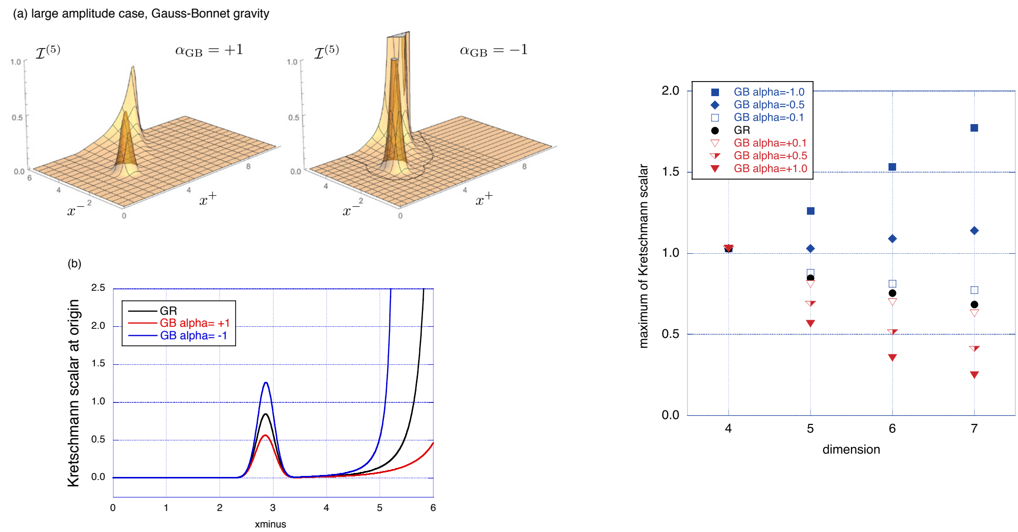

Figure 3. (a) Kretschmann scalar, $\mathcal{I}^{(5)}$, of the evolutions of colliding two scalar pulses in five-dimensional Einstein-GB gravity with $\alpha_{\rm GB} = \pm 1$. The initial data are the same with the large-amplitude case in Fig. 2(b). We see that the local peak of $\mathcal{I}^{(5)}$ at the collision of two pulses (at $x^+ = x^- = 2\sqrt{2}$) is smaller (larger) when $\alpha_{\rm GB} > 1$ ($\alpha_{\rm GB} < 1$). (b) Kretschmann scalar, $\mathcal{I}^{(5)}$, at the origin ($x^+ = x^-$), of these evolutions together with one with $\alpha_{\rm GB} = 0$ (i.e. GR). (Right) The Kretschmann scalar, $\mathcal{I}^{(n)}$, at the moment of the collision of scalar pulses (at $x^+ = x^- = 2\sqrt{2}$). We plot for the models with $\alpha_{\rm GB} = 0, \pm 0.1, \pm 0.5, \pm 1.0$ and for the dimensions $n = 4, 5, 6, 7$. For larger dimensions, the magnitude becomes lower in GR. We also find that introducing positive $\alpha_{\rm GB}$ (i.e. the normal higher-curvature correction) reduces its magnitude.

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