EPJ Web of Conferences **168**, 06004 (2018) https://doi.org/10.1051/epjconf/201816806004
Joint International Conference of ICGAC-XIII and IK-15 on Gravitation, Astrophysics and Cosmology

On the effective Stefan-Boltzmann law and the thermodynamic origin of the initial radiation density in warm inflation

Yongwan Gim[1,2,⋆] and *Wontae* Kim[1,⋆⋆]

[1] *Department of Physics, Sogang University, 35 Baekbeom-ro, Mapo-gu, Seoul 04107, Korea*
[2] *Research Institute for Basic Science, Sogang University, 35 Baekbeom-ro, Mapo-gu, Seoul 04107, Korea*

Abstract. In this presentation, we are going to explain the thermodynamic origin of warm inflation scenarios by using the effective Stefan-Boltzmann law. In the warm inflation scenarios, radiation always exists to avoid the graceful exit problem, for which the radiation energy density should be assumed to be finite at the starting point of the warm inflation. To find out the origin of the non-vanishing initial radiation energy density, we derive an effective Stefan-Boltzmann law by considering the non-vanishing trace of the total energy-momentum tensors. The effective Stefan-Boltzmann law successfully shows where the initial radiation energy density is thermodynamically originated from. And by using the above effective Stefan-Boltzmann law, we also study the cosmological scalar perturbation, and obtain the sufficient radiation energy density in order for GUT baryogenesis at the end of inflation. This proceeding is based on Ref. [1]

1 Introduction

Inflation is an elegant solution to the interesting problems such as the horizon and flatness problems in the big bang cosmology [2–4], during which the rapid expansion lays the universe in a supercooled phase. And thereafter the reheating process should be assumed for the graceful exit problem. In contrast to the assumption of the supercooled universe after inflation, another way to approach this issue without reheating process has been studied in Ref. [5, 6], which is called as a warm inflation scenario. There are two important ingredients to describe the warm inflation. First one is a damping term during warm inflation. Since the interactions of the inflaton and radiation are inevitable during inflation, the damping term describing the decay rate of the inflaton into other fields should be introduced during warm inflation. Second one is the large-scale initial radiation energy density. It can be naturally assumed to be nonzero, $\rho_{\rm r}(t_{\rm i}) \neq 0$ [7], which is compatible with the Stefan-Boltzmann law of $\rho_r = 3\gamma T^4$ in the hot thermal bath at the initial point of inflation, $t = t_{\rm i}$. Now, one might wonder how to exist the large-scale initial radiation energy density and what is the origin of it in the warm inflation scenario.

2 Effective Stefan-Boltzmann law in warm inflation

The radiation energy density is related to the temperature through the Stefan-Boltzmann law in the standard warm inflation models. As compared to this, if one were to treat the inflaton and radiation

⋆e-mail: yongwan89@sogang.ac.kr
⋆⋆e-mail: wtkim@sogang.ac.kr

on an equal footing in equilibrium, then one would encounter generically non-vanishing trace of the total energy-momentum tensor due to the inflaton part while the radiation part is still traceless. Now, it should be emphasized that the usual Stefan-Boltzmann law commonly rests upon the traceless condition of the energy-momentum tensor, and thus we have to modify the Stefan-Boltzmann law in order to incorporate the non-vanishing trace of the total energy-momentum tensor.

Let us start with the first law of thermodynamics $dE = TdS - PdV$, where E, T, S, P, and V are the energy, temperature, entropy, pressure, and volume of a thermal system, respectively. The relevant energy-momentum tensor is assumed to be perfect fluid written as $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + g_{\mu\nu}p$, where u^μ is the four-velocity of radiation flow satisfying $u^\mu u_\mu = -1$. Assuming that the trace of the energy-momentum tensor is non-vanishing generically, the trace relation is obtained as $-\rho + 3p = T^\mu_\mu$ where $\rho = E/\mathrm{V}$, the differential equation for the energy density is obtained as $T\partial\rho/\partial T - 4\rho = T^\mu_\mu - T\partial T^\mu_\mu/\partial T$, so that the effective Stefan-Boltzmann law to incorporate the non-vanishing trace of the energy-momentum tensor can be obtained as

$$\rho(T) = 3C_0T^4 - \frac{1}{4}T^\mu_\mu - \frac{3}{4}T^4 \int^T \frac{1}{T^4}\frac{\partial T^\mu_\mu}{\partial T}dT, \tag{1}$$

$$p(T) = C_0T^4 + \frac{1}{4}T^\mu_\mu - \frac{1}{4}T^4 \int^T \frac{1}{T^4}\frac{\partial T^\mu_\mu}{\partial T}dT, \tag{2}$$

where the integration constant C_0 can be fixed from an initial condition. The relations (1) and (2) naturally reduce to the usual Stefan-Boltzmann law for the traceless case, so that $C_0 = \gamma$. However, C_0 will be fixed for the case of the non-vanishing trace for our purpose later by imposing a different boundary condition. In fact, such a modified Stefan-Boltzmann law induced by conformal anomalies had been applied to SU(3) lattice gauge theory in particle physics in the Minkowski spacetime [8] and the recent black hole physics in connection with the information loss problem [9, 10].

From the cosmological point of view, let us assume that the total system of the early universe consists of inflaton and radiation in thermal equilibrium. Then the total energy density $\rho_{\rm tot}$ and pressure $p_{\rm tot}$ are written as [11]

$$\rho_{\rm tot} = \rho_\phi + \rho_{\rm r} = \frac{1}{2}\dot\phi^2 + V_{\rm eff}(\phi, T) + \rho_{\rm r}, \tag{3}$$

$$p_{\rm tot} = p_\phi + p_{\rm r} = \frac{1}{2}\dot\phi^2 - V_{\rm eff}(\phi, T) + p_{\rm r}, \tag{4}$$

where $\rho_{\rm r}$, $p_{\rm r}$ and ρ_ϕ, p_ϕ denote the energy density and pressure of radiation and inflaton, respectively. Specifically, the temperature dependent effective potential $V_{\rm eff}$ for the inflaton is expressed by [12–14]

$$V_{\rm eff}(\phi, T) = -\gamma T^4 + \frac{1}{2}(\delta m_T)^2\phi^2 + V_0(\phi), \tag{5}$$

where $\gamma = \pi^2 g_*/90$ and g_* is an effective particle number. $V_0(\phi)$ is the zero-temperature potential for the scalar field ϕ, and $\delta m_T(\phi, T)$ denotes a thermal correction which will be neglected for simplicity along the lines of Ref. [15].

The trace for the total energy-momentum tensor appears non-vanishing due to the effective potential for the inflaton as

$$T^\mu_\mu = -\rho_{\rm tot} + 3p_{\rm tot} = -4V_{\rm eff}(\phi, T), \tag{6}$$

where the kinetic energy is assumed to be very small as compared to the potential energy from now on. By plugging Eq. (6) into Eqs. (1) and (2), the explicit forms of the pressure and energy density are obtained as

$$\rho_{\rm tot} = V_{\rm eff} + 3\gamma T^4 \ln\left(\frac{T_{\rm GUT}}{T}\right)^4, \quad p_{\rm tot} = -V_{\rm eff} + \gamma T^4 \ln\left(\frac{T_{\rm GUT}}{T}\right)^4. \tag{7}$$

by using the initial condition of $C_0 = 4\gamma \ln T_{\rm GUT}$ from the assumption that there exists only the inflaton field at the initial temperature of our universe $T_{\rm GUT}$, *i.e.*, $\rho_{\rm tot}(T_{\rm GUT}) = \rho_\phi$ and $p_{\rm tot}(T_{\rm GUT}) = p_\phi$. It is worth mentioning that we take T_0 to be the GUT temperature as the maximum temperature of our universe $T_0 = T_{\rm GUT} = 10^{16}$GeV, since all perturbative interactions can be frozen out and ineffective in maintaining or establishing thermal equilibrium for $T > 10^{16}$GeV, and thus the known interactions are not capable of thermalizing the universe at temperature greater than the GUT scale [11]. Comparing Eq. (7) with Eqs. (3) and (4), we can immediately find the effective Stefan-Boltzmann law for the radiation as

$$\rho_{\rm r} = 3\gamma T^4 \ln\left(\frac{T_{\rm GUT}}{T}\right)^4, \qquad p_{\rm r} = \gamma T^4 \ln\left(\frac{T_{\rm GUT}}{T}\right)^4. \tag{8}$$

The radiation energy density (8) starts from zero with the GUT temperature as an initial condition of our universe, and, subsequently, it increases. And eventually it gives the adequate initial radiation energy density for warm inflation and the sufficient temperature after inflation, which will be calculated in Sec. 3.

3 Temperatures at the horizon crossing and at the end of inflation

One of the most important ingredients in warm inflation is that the decreasing radiation energy density during inflation is replenished in such a way that the energy of the inflaton field is transferred to that of radiation in virtue of dissipation. Now, the energy conservation law, $\dot{\rho}_{\rm tot} + 3H(\rho_{\rm tot} + p_{\rm tot}) = 0$, can be separated into the inflaton and radiation parts as [15]

$$\dot{\rho_\phi} + 3H(\rho_\phi + p_\phi) = -\Gamma\dot{\phi}(t)^2, \qquad \dot{\rho}_{\rm r} + 3H(\rho_{\rm r} + p_{\rm r}) = \Gamma\dot{\phi}(t)^2, \tag{9}$$

where $H = \dot{a}/a$ means the Hubble parameter, and $\Gamma\dot{\phi}^2$ is the friction term adopted phenomenologically to describe the decay of the inflaton field and its energy transfers into the radiation bath. And, the Friedmann equation is also given as $H^2 - \rho_{\rm tot}/(3m_{\rm p}^2) = 0$. Based on the slow-roll approximations [15], $\dot{\phi}^2 \ll V_{\rm eff}$, $\ddot{\phi} \ll \Gamma\dot{\phi}$, $\dot{\rho}_{\rm r} \ll 4H\rho_{\rm r}$, $\rho_{\rm r} \ll \rho_\phi$, one can get the following equations,

$$3Hr\dot{\phi} + \partial_\phi V_{\rm eff} = 0, \quad 3H(\rho_{\rm r} + p_{\rm r}) - \Gamma(\phi)\dot{\phi}^2 = 0, \quad H^2 - \frac{1}{3m_{\rm p}^2} V_{\rm eff} = 0, \tag{10}$$

where $\Gamma/(3H) \gg 1$ in the warm inflationary regime.

From now on, we adopt the power-law potential V_0 and damping term Γ [15] as

$$V_0(\phi) = \lambda\phi^n, \qquad \Gamma(\phi) = \Gamma_0 \left(\frac{\phi}{\phi_0}\right)^m \tag{11}$$

in order to perform the specific calculations, where the coefficients Γ_0, ϕ_0 and λ are constants, and the power n and m are fixed as $n = 2$, $m = 2$ for simplicity. In this specific model, the number of e-folds during warm inflation is obtained as

$$N_{\rm inf} = \int_{t_{\rm HC}}^{t_{\rm end}} H(t)dt = \frac{\Gamma_0(\lambda\phi_{\rm HC}^2 - \gamma_{\rm HC}T_{\rm HC}^4)^{\frac{3}{2}}}{6\sqrt{3}m_{\rm p}\lambda^2\phi_0^2} \tag{12}$$

by assuming that $\phi_{\rm end} \ll \phi_{\rm HC}$, where $\phi_{\rm HC}$ and $\phi_{\rm end}$ are the values of the inflaton field corresponding to the horizon-crossing time $t_{\rm HC}$ and the end time of warm inflation $t_{\rm end}$, respectively.

Next, we are going to determine the temperature bounds at the end of inflation via cosmological perturbation. The thermal fluctuations produce the power spectrum P_ζ for the comoving curvature ζ, $P_\zeta = \pi^{1/2} H^{5/2} \Gamma^{1/2} T/(2\dot{\phi}^2)$, and the power spectral index $n_{\rm s}$ for the scalar perturbation is defined as $n_{\rm s} - 1 = d \ln |P_\zeta| / d \ln k$. After some tedious calculations with the slow-roll equations (10) , the spectral index is finally obtained as

$$n_{\rm s} - 1 = \frac{1}{N_{\rm inf} + N_{\rm inf}^2 \ln\left(\frac{T_{\rm GUT}}{T_{\rm HC}}\right)^4} - \frac{1}{12 N_{\rm inf}\left(1 - \ln\left(\frac{T_{\rm GUT}}{T_{\rm HC}}\right)^4\right)} - \frac{7}{4N_{\rm inf}}, \tag{13}$$

where the number of e-folds is assumed to be $N_{\rm inf} = 60$ in order for solving the horizon problem. The spectral index can respect the data of Planck 2015 when the temperature at the horizon crossing $T_{\rm HC}$ lies in the interval of

$$8.026 \times 10^{15} {\rm GeV} \leq T_{\rm HC} \leq 9.985 \times 10^{15} {\rm GeV}. \tag{14}$$

In order to evaluate the temperature at the end of warm inflation, we will use the procedure presented in Ref. [16]. By using Eq. (10), the total number of e-folds $N_{\rm tot}$ from the scale at the horizon crossing $a_{\rm HC}$ to the scale at the present time a_0 is written as $N_{\rm tot} = \ln(a_0/a_{\rm HC}) = \ln\left((\sqrt{3} k_0 m_{\rm p})^{-1} \sqrt{\lambda \phi_{\rm HC}^2 - \gamma_{\rm HC} T_{\rm HC}^4}\right)$, where the scale of the present time is fixed as $a_0 = 1$ and the scale at the horizon crossing is given as $a_{\rm HC} = k_0 / H(t_{\rm HC})$. Next, the relation of $T_{\rm rec} = (1 + z_{\rm rec}) T_{\rm CMB}$, where $z_{\rm rec}$ is the red-shift factor given as $1 + z_{\rm rec} = a_0/a_{\rm rec}$, indicates that the temperature diminishes from the recombination era to present universe due to the expansion of the universe as $N_0 = \ln(a_0/a_{\rm rec}) = \ln(T_{\rm rec}/T_{\rm CMB})$. For the radiation-dominated era, the adiabatic expansion of the universe is assumed as $dS = 0$ [11], then the number of e-folds can be rewritten in the radiation-dominated era $N_{\rm rad}$ as

$$N_{\rm rad} = \ln\left(\frac{a_{\rm rec}}{a_{\rm end}}\right) = \frac{1}{3}\ln\left(\frac{s_{\rm end}}{s_{\rm rec}}\right) = \frac{1}{3}\ln\left(\frac{4\gamma_{\rm end} T_{\rm end}^3 \ln\left(\frac{T_{\rm GUT}}{T_{\rm end}}\right)^4}{4\gamma_{\rm rec} T_{\rm rec}^3}\right), \tag{15}$$

where the entropy at the end of inflation is $S_{\rm end} = 4\gamma_{\rm end} a_{\rm end}^3 T_{\rm end}^3 \ln(T_{\rm GUT}/T_{\rm end})^4$. By the way, $S_{\rm rec} = 4\gamma_{\rm rec} a_{\rm rec} T_{\rm rec}^3$ since the radiation only consists of photons without the inflaton, so that the usual Stefan-Boltzmann law is used.

To perform the specific calculations, we choose the effective particle number at the electroweak energy scale as $g_{\rm HC} = g_{\rm end} = 106.75$ and at the recombination era as $g_{\rm rec} = 2$ [11]. The temperature of CMB is known as $T_{\rm CMB} = 2.725K$, and the spectral index for $k_0 = 0.05{\rm Mpc}^{-1}$ is $n_{\rm s} = 0.9655 \pm 0.0062$ from Planck 2015 [17, 18]. By solving the equation of $N_{\rm tot} = N_0 + N_{\rm rad} + N_{\rm inf}$, the range of $T_{\rm end}$ is, finally, obtained as

$$2.409 \times 10^{13}\ {\rm GeV} \leq T_{\rm end} \leq 2.216 \times 10^{14}\ {\rm GeV}, \tag{16}$$

where this range lies below the well-known upper bound of the temperature of the universe to avoid monopole proliferation [11] and above the lower bounds in Refs. [19–21]. In addition, the corresponding energy density for radiation is consequently $2.852 \times 10^{56} {\rm GeV}^4 \leq \rho_{\rm end} \leq 1.291 \times 10^{60} {\rm GeV}^4$, which is a sufficient radiation energy density to accommodate the GUT baryogenesis at the end of inflation [22].

4 Conclusion

Motivated by the non-zero initial radiation energy density in warm inflation scenario, we performed thermodynamic analysis for the warm inflation model by using the definitions for the inflaton and radiation energy density presented in Ref. [11]. And then we obtained the effective Stefan-Boltzmann law to show that the zero radiation energy density (8) at the Grand Unification epoch just prior to starting inflation became finite when inflation starts, which gives the adequate radiation energy density for warm inflation. By using the effective Stefan-Boltzmann law for the radiation energy density, we studied the number of e-folds and the spectral index of the scalar perturbation under the slow-roll approximations in the power-law potential and damping terms, so that the temperature (16) at the end of warm inflation was successfully calculated, and it satisfies the upper bound lower than the GUT scale [11], and lower bound of the big bang nucleosynthesis [19, 20] by the CMB data [21]. Additionally, we confirmed that a sufficient radiation energy density could be produced for GUT baryogenesis at the end of inflation [22].

As a matter of fact, we have assumed the simplest setting described by the perfect fluid and the power law potential and the damping term as a toy model. The effective Stefan-Boltzmann law can be applied to other models which have the non-vanishing trace, and it might give interesting results. We hope that this issue will be elaborated in the near future.

References

[1] Y. Gim, W. Kim, JCAP **1611**, 022 (2016), 1608.07466
[2] A.A. Starobinsky, Phys. Lett. **B91**, 99 (1980)
[3] K. Sato, Mon. Not. Roy. Astron. Soc. **195**, 467 (1981)
[4] A.H. Guth, Phys. Rev. **D23**, 347 (1981)
[5] A. Berera, Phys. Rev. Lett. **75**, 3218 (1995), astro-ph/9509049
[6] A. Berera, Phys. Rev. **D54**, 2519 (1996), hep-th/9601134
[7] A. Berera, I.G. Moss, R.O. Ramos, Rept. Prog. Phys. **72**, 026901 (2009), 0808.1855
[8] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legeland, M. Lutgemeier, B. Petersson, Nucl. Phys. **B469**, 419 (1996), hep-lat/9602007
[9] Y. Gim, W. Kim, Eur. Phys. J. **C75**, 549 (2015), 1508.00312
[10] M. Eune, Y. Gim, W. Kim, Eur. Phys. J. **C77**, 244 (2017), 1511.09135
[11] E.W. Kolb, M. Turner, Westview press (1990)
[12] L. Dolan, R. Jackiw, Phys. Rev. **D9**, 3320 (1974)
[13] S. Weinberg, Phys. Rev. **D9**, 3357 (1974)
[14] A.D. Linde, Rept. Prog. Phys. **42**, 389 (1979)
[15] L.M.H. Hall, I.G. Moss, A. Berera, Phys. Rev. **D69**, 083525 (2004), astro-ph/0305015
[16] J. Mielczarek, Phys. Rev. **D83**, 023502 (2011), 1009.2359
[17] P.A.R. Ade et al. (Planck) (2015), 1502.02114
[18] P.A.R. Ade et al. (Planck) (2015), 1502.01589
[19] M. Kawasaki, K. Kohri, N. Sugiyama, Phys. Rev. **D62**, 023506 (2000), astro-ph/0002127
[20] S. Hannestad, Phys. Rev. **D70**, 043506 (2004), astro-ph/0403291
[21] J. Martin, C. Ringeval, Phys. Rev. **D82**, 023511 (2010), 1004.5525
[22] M. Bellini, Phys. Rev. **D63**, 123510 (2001), gr-qc/0101062