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Dilatonic dyon black hole solutions in the model with two Abelian gauge fields

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Abstract. Dilatonic black hole dyon-like solutions in the gravitational $4d$ model with a scalar field, two 2-forms, two dilatonic coupling constants $\lambda_i \neq 0$, $i = 1, 2$, obeying $\lambda_1 \neq -\lambda_2$ and sign parameter $\varepsilon = \pm 1$ for scalar field kinetic term are considered. Here $\varepsilon = -1$ corresponds to ghost scalar field. These solutions are defined up to solutions of two master equations for two moduli functions, when $\lambda_i^2 \neq 1/2$ for $\varepsilon = -1$. A set of bounds on gravitational mass and scalar charge are presented by using a certain conjecture on parameters of solutions, when $1 + 2\lambda_i^2\varepsilon > 0$, $i = 1, 2$.

1 Introduction

Here we give a brief extension of our previous work [1] devoted to dilatonic dyon black hole solutions. We consider a subclass of dilatonic black hole solutions with electric and magnetic charges Q_1 and Q_2, respectively in $4d$ model with metric g, scalar field φ, two 2-forms $F^{(1)}$ and $F^{(2)}$, corresponding to two dilatonic coupling constants λ_1 and λ_2, respectively. For coinciding dilatonic couplings $\lambda_1 = \lambda_2 = \lambda$ we get a trivial non-composite generalisation of dilatonic dyon black hole solutions in the model with one 2-form which was considered in ref. [1]. The dilatonic scalar field may be either an ordinary one or a phantom (or ghost) one.

Here we present lower bounds on gravitational mass M and scalar charge Q_φ. As in ref. [1] this problem is solved here up to a conjecture, which states one to one (smooth) correspondence between the pair (Q_1^2, Q_2^2), where Q_1 is electric charge and Q_2 is magnetic charge, and the pair of positive parameters (P_1, P_2), which appear in decomposition of moduli functions at large distances. This conjecture is believed to be valid for all $\lambda_i \neq 0$ in the case of ordinary scalar field and for $0 < \lambda_i^2 < 1/2$ for the case of phantom scalar field (in both cases the inequality $\lambda_1 \neq -\lambda_2$ is assumed).

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2 Black hole dyon solutions

Let us consider a model governed by the action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{|g|}\Big\{R[g] - \varepsilon(\partial\varphi)^2 - \sum_{i=1}^{2}\frac{1}{2}e^{2\lambda_i\varphi}(F^{(i)})^2\Big\}, \tag{2.1}$$

where $g = g_{\mu\nu}(x)dx^\mu \otimes dx^\nu$ is metric, φ is the scalar field, $F^{(i)} = dA^{(i)} = \frac{1}{2}F^{(i)}_{\mu\nu}dx^\mu \wedge dx^\nu$ is the 2-form with $A^{(i)} = A^{(i)}_\mu dx^\mu$, $\varepsilon = \pm 1$, G is the gravitational constant, $(\partial\varphi)^2 = g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$, $(F^{(i)})^2 = F^{(i)}_{\mu\nu}F^{(i)\mu\nu}$, $i = 1, 2$. Here $\lambda_1, \lambda_2 \neq 0$ are coupling constants obeying $\lambda_1 \neq -\lambda_2$ and $|g| = |\det(g_{\mu\nu})|$. We put $\lambda_i^2 \neq 1/2$, $i = 1, 2$, for $\varepsilon = -1$.

We consider a family of dyonic-like black hole solutions to the field equations corresponding to the action (2.1) which are defined on the manifold

$$\mathcal{M} = (2\mu, +\infty) \times S^2 \times \mathbb{R}, \tag{2.2}$$

and have the following form

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = H_1^{h_1}H_2^{h_2}\Big\{-H_1^{-2h_1}H_2^{-2h_2}\left(1 - \frac{2\mu}{R}\right)dt^2 + \frac{dR^2}{1 - \frac{2\mu}{R}} + R^2 d\Omega_2^2\Big\}, \tag{2.3}$$

$$\exp(\varphi) = H_1^{h_1\lambda_1\varepsilon}H_2^{-h_2\lambda_2\varepsilon}, \tag{2.4}$$

$$F^{(1)} = \frac{Q_1}{R^2}H_1^{-2}H_2^{-A_{12}}dt \wedge dR, \tag{2.5}$$

$$F^{(2)} = Q_2\tau. \tag{2.6}$$

Here Q_1 and Q_2 are (colored) charges - electric and magnetic, respectively, $\mu > 0$ is the extremality parameter, $d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the canonical metric on the unit sphere S^2 ($0 < \theta < \pi$, $0 < \phi < 2\pi$), $\tau = \sin\theta d\theta \wedge d\phi$ is the standard volume form on S^2,

$$h_i = K_i^{-1}, \qquad K_i = \frac{1}{2} + \varepsilon\lambda_i^2, \tag{2.7}$$

$i = 1, 2$, and

$$A_{12} = (1 - 2\lambda_1\lambda_2\varepsilon)h_2. \tag{2.8}$$

Functions $H_s > 0$ obey the equations

$$R^2\frac{d}{dR}\left(R^2\frac{\left(1 - \frac{2\mu}{R}\right)}{H_s}\frac{dH_s}{dR}\right) = -K_s Q_s^2\prod_{l=1,2}H_l^{-A_{sl}}, \tag{2.9}$$

with the following boundary conditions imposed

$$H_s \to H_{s0} > 0 \tag{2.10}$$

for $R \to 2\mu$, and

$$H_s \to 1 \tag{2.11}$$

for $R \to +\infty$, $s = 1, 2$.

In (2.9) we denote

$$(A_{ss'}) = \begin{pmatrix} 2 & A_{12} \\ A_{21} & 2 \end{pmatrix}, \tag{2.12}$$

where A_{12} is defined in (2.8) and

$$A_{21} = (1 - 2\lambda_1\lambda_2\varepsilon)h_1. \tag{2.13}$$

These solutions may be obtained just by using general formulae for non-extremal (intersecting) black brane solutions from [2]. The composite analogs of the solutions with one 2-form and $\lambda_1 = \lambda_2$ were presented in ref. [1].

The first boundary condition (2.10) guarantees (up to a possible additional demand on analicity of $H_s(R)$ in the vicinity of $R = 2\mu$) the existence of (regular) horizon at $R = 2\mu$ for the metric (2.3). The second condition (2.11) ensures an asymptotical (for $R \to +\infty$) flatness of the metric.

3 Bounds on mass and scalar charge

For ADM gravitational mass we get from (2.3)

$$GM = \mu + \frac{1}{2}(h_1 P_1 + h_2 P_2), \tag{3.1}$$

where the parameters P_s appear in asymptotical relations $H_s = 1 + P_s/R + o(1/R)$, as $R \to +\infty$. The scalar charge just follows from (2.4)

$$Q_\varphi = \varepsilon(\lambda_1 h_1 P_1 - \lambda_2 h_2 P_2). \tag{3.2}$$

Here we outline the following hypothesis, which is supported by certain numerical calculations [1, 3]. For $h_1 = h_2$ this conjecture was proposed in ref. [1].

Conjecture. *For any $h_1 > 0$, $h_2 > 0$, $\varepsilon = \pm 1$, $Q_1 \neq 0$, $Q_2 \neq 0$ and $\mu > 0$: (A) the moduli functions $H_s(R)$, which obey (2.9), (2.10) and (2.11), are uniquely defined and hence the parameters P_1, P_2, the gravitational mass M and the scalar charge Q_φ are uniquely defined too; (B) the parameters P_1, P_2 are positive and the functions $P_1 = P_1(Q_1^2, Q_2^2)$, $P_2 = P_2(Q_1^2, Q_2^2)$ define a diffeomorphism of $\mathbb{R}^2_+$ ($\mathbb{R}_+ = \{x | x > 0\}$); (C) in the limiting case we have: (i) for $Q_2^2 \to +0$: $P_1 \to -\mu + \sqrt{\mu^2 + K_1 Q_1^2}$, $P_2 \to +0$ and (ii) for $Q_1^2 \to +0$: $P_1 \to +0$, $P_2 \to -\mu + \sqrt{\mu^2 + K_2 Q_2^2}$.*

The conjecture could be readily verified for the case $\varepsilon = 1$, $\lambda_1\lambda_2 = 1/2$. Another integrable case $\varepsilon = 1$, $\lambda_1 = \lambda_2 = \lambda$, $\lambda^2 = 3/2$ is more involved [3].

The conjecture implies the following proposition.

Proposition. *For $h_s > 0$, $Q_s \neq 0$, $\lambda_s \neq 0$ $(s = 1, 2)$ and $\lambda_1 + \lambda_2 \neq 0$ we have the following bounds on the gravitational mass M and the scalar charge Q_φ which are valid for all $\mu > 0$:*

$$\frac{1}{2}\sqrt{h_{min}(Q_1^2 + Q_2^2)} < GM, \tag{3.3}$$

$$|Q_\varphi| < |\lambda|_{max}\sqrt{h_{min}(Q_1^2 + Q_2^2)}, \tag{3.4}$$

for $\varepsilon = +1$ $(0 < h_s < 2)$, and

$$\sqrt{\frac{1}{2}(Q_1^2 + Q_2^2)} < GM, \tag{3.5}$$

$$|Q_\varphi| < |\lambda|_{max}\sqrt{h_{max}(Q_1^2 + Q_2^2)}, \tag{3.6}$$

for $\varepsilon = -1$ $(h_s > 2)$. *Here* $h_{min} = \min(h_1, h_2)$, $h_{max} = \max(h_1, h_2)$, *and* $|\lambda|_{max} = \max(|\lambda|_1, |\lambda|_2)$; $h_{min} = (\frac{1}{2} + |\lambda|^2_{max})^{-1}$ *for* $\varepsilon = +1$ *and* $h_{max} = (\frac{1}{2} - |\lambda|^2_{max})^{-1}$ *for* $\varepsilon = -1$.

In ref. [1] Proposition was proved for the case $\lambda_1 = \lambda_2$. In this case the bound (3.3) is coinciding (up to notations) with that from ref. [4], which was proved there by using certain spinor techniques.

4 Conclusions

Here a family of non-extremal black hole dyon-like solutions in a 4d gravitational model with a scalar field and two Abelian vector fields is considered. The scalar field is either ordinary ($\varepsilon = +1$) or phantom one ($\varepsilon = -1$). The model contain two dilatonic coupling constants $\lambda_s \neq 0$, $s = 1, 2$, obeying $\lambda_1 \neq -\lambda_2$.

The solutions are defined up to two moduli functions $H_1(R)$ and $H_2(R)$, which obey two differential equations of second order with boundary conditions imposed. For $\varepsilon = +1$ these equations are integrable for four cases, corresponding to Lie algebras $A_1 + A_1$, A_2, $B_2 = C_2$ and G_2. In the first case $(A_1 + A_1)$ we have $\lambda_1\lambda_2 = 1/2$, while in the second one (A_2) we get $\lambda_1 = \lambda_2 = \lambda$ and $\lambda^2 = 3/2$.

Here we have presented lower bounds on the gravitational mass and upper bounds on the scalar charge for $1 + 2\lambda_s^2\varepsilon > 0$, which are based on the conjectur on the parameters of solutions $P_1 = P_1(Q_1^2, Q_2^2)$, $P_2 = P_2(Q_1^2, Q_2^2)$. For $\varepsilon = +1$ the lower bound on the gravitational mass is in agreement for $\lambda_1 = \lambda_2$ with that obtained earlier by Gibbons et al. by using certain spinor techniques.

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