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Overlap between Lattice QCD and HRG with in-medium effects and parity doubling*

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Abstract. We investigate the fluctuations and correlations involving baryon number in hot hadronic matter with modified masses of negative-parity baryons, in the context of the hadron resonance gas. Temperature-dependent masses are adopted from the recent lattice QCD results and from a chiral effective model which implements the parity doubling structure with respect to the chiral symmetry. Confronting the baryon number susceptibility, baryon-charge correlation, and baryon-strangeness correlation and their ratios with the lattice QCD data, we show that the strong downward mass shift in hyperons can accidentally reproduce some correlation ratios, however it also tends to overshoot the individual fluctuations and correlations. This indicates, that in order to correctly account for the influence of the chiral symmetry restoration on the fluctuation observables, a consistent framework of in-medium effects beyond hadron mass shifts is required.

1 Introduction

Fluctuations and correlations of conserved charges provide diagnostic tools for the nature of strongly interacting matter described by Quantum Chromodynamics (QCD). The first-principles calculations by lattice QCD (LQCD) have provided not only equation of state at the physical quark masses but also the fluctuations and correlations of the net-baryon, net-electric charge, and net-strangeness [1–6]. The fluctuations and correlations can also be measured in heavy-ion collisions to identify the state of created matter. In particular, non-Gaussian (higher-order) fluctuations have been expected to probe critical properties of the system, such as the QCD critical point in the beam-energy scan program at RHIC and a remnant of the $O(4)$ criticality at small baryon density [7, 8].

For a physical interpretation of the thermodynamic properties, the hadron resonance gas (HRG) model [9, 10] has been used as a reference. The equation of state and fluctuations have been well described by the model below the chiral crossover temperature $T = 154 \pm 9$ MeV [11, 12], and the experimental data of fluctuations have been analyzed in terms of a gas of hadronic states [13–16]. At vanishing baryon density, the critical behavior due to the chiral phase transition appears at the sixth order of net-baryon fluctuations [8]. This is because the singular contribution to the free energy is suppressed in the lower-order fluctuations even in the vicinity of the expected second order transition.

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Therefore, one expects that the lower order fluctuations and correlations may be well described by the hadronic degrees of freedom, and they would provide a reliable baseline for exploring the critical behavior [7].

According to the recent LQCD calculations, however, some correlations between conserved charges cannot be explained by a conventional HRG model. Particularly interesting quantities are those involving net-baryon number, baryon-charge (BQ) and baryon-strangeness (BS) correlations. Since mesons do not contribute to these quantities, one may gain access to a role of baryonic degrees of freedom and interactions which are not visible in the equation of state and other meson-dominated quantities due to their heavy masses [17].

In the HRG model the interaction of hadrons is replaced by resonances, and in the first approximation their widths can be neglected. Then, the partition function of the interacting hadronic system can be written as a mixture of free gases of all stable and resonant hadrons [18]. The validity of the vanishing-width approximation for the fluctuations has been recently examined based on the S-matrix formalism for K–π [19] and π–N–Δ [20] systems. It has been found that an explicit treatment of the width can have a substantial effect on the fluctuations.

On the other hand, at finite temperature and density one expects changes in the spectral property of hadrons. In particular, a search for those modifications because of the partial restoration of the QCD chiral symmetry has been one of the central subjects in heavy-ion collisions [21]. Since the vacuum masses are used in the HRG model, the observed agreement of the equation of state in the HRG and LQCD suggests no need for a strong mass reduction in the dominant degrees of freedom such as light mesons. Although chiral symmetry predicts substantial medium modifications of low-lying mesons [22, 23], their reliable estimates in LQCD have been limited to the screening masses [24].

Recently, masses of the non-strange and strange baryons with positive and negative parity have been extracted from the temporal correlation functions by the FASTSUM collaboration [25, 26]. The negative-parity states clearly show downward mass shifts, whereas the positive-parity baryons stay insensitive to temperature. The obtained spectra follow an expectation from the parity doublet picture of the chiral symmetry, indicating that the masses of a negative- and positive-parity partners tend to degenerate when approaching the chiral crossover. The above medium modification was used to possibly explain a missing contribution in the correlations between the conserved charges [26].

In the following we investigate the fluctuations and correlations of the net-baryon with net-charge and net-strangeness on the basis of the HRG model implementing in-medium mass modifications. We employ temperature dependent masses of the negative-parity octet and decouplet baryons from the lattice QCD and from a chiral effective model with parity doubling [27]. We show that the strong downward mass shift in hyperons can accidentally reproduce some correlation ratios, however it also tends to overshoot the individual fluctuation and correlation observables. This indicates that to quantify fluctuation and correlation of conserved charges in the presence of chiral symmetry restoration it is not sufficient to implement in-medium hadron masses in the statistical sum of the hadron resonance gas.

2 Baryon masses in parity doublet picture

2.1 Lattice QCD

The FASTSUM collaboration presented the temperature dependent masses of N, Δ, and Ω states, extracted from imaginary time correlators of the corresponding interpolating operators in $N_f = 2 + 1$ lattice simulations [25]. Their calculations were performed for heavier light-quark mass than the physical one; $m_\pi = 384$ MeV, while the strange quark is set to the physical one. Thus, the mass of baryons except Ω are heavier than the physical ones. This yields the nucleon with positive parity

Table 1. Assignment of negative parity states in in-medium HRG

P^- State	N	Λ	Σ	Ξ	Δ	Σ^*	Ξ^*	Ω
Mass LGT [25, 26]	1779	1899	1823	1917	2138	2131	2164	2193
$M^i_-(T_c)$ [MeV]	1254	1172	1329	1295	1405	1398	1426	1383
b_i	0.338	0.369	0.257	0.275	0.312	0.257	0.246	0.213
Assignment A	1535	1405	1750	1690	1700	1670	1820	2250
Assignment B	1535	1670	1750	1950	1700	1940	1820	2250
Mass [MeV]	1535	1790	1880	2090	1710	1930	2150	2380
Assignment C	1535	1800	1880	2120	1700	1940	2250	2380

N_+ has $m_{N_+} = 1158$ MeV (939 MeV in PDG), while the positive parity Ω_+ has $m_{\Omega_+} = 1661$ MeV (1672 MeV in PDG). The results of other octet and decuplet states have been shown in [26]. Here we use the masses at $T/T_c = 0.24$, 0.76, 0.84, and 0.95. Note that owing to the heavier pion mass, the crossover temperature $T_c = 185$ MeV, which was determined from the renormalized Polyakov loop, is also higher than the physical one, $T_c = 154$ MeV.

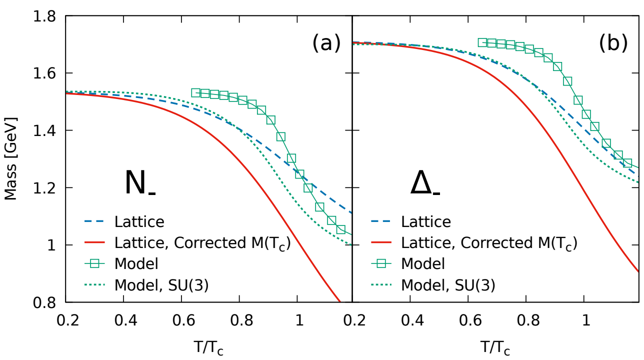

Figure 1. Temperature dependence of the mass of N_- (a) and Δ_- (b) from lattice-motivated parameterization Eq. (1) (lines) and from a chiral effective model (squares).

In Table 1, we list the masses of the octet and decuplet negative-parity states at $T = 0.24T_c$. They are well parameterized, for the given the state i, by [25, 26]

$$M^i_-(T) = M^i_-(T = 0)\omega(T, b_i) + M^i_-(T_c)(1 - \omega(T, b_i)) \quad (1)$$

$$\omega(T, b_i) = \tanh[(1 - T/T_c)/b_i]/\tanh(1/b_i) \quad (2)$$

where $M^i_-(T = 0)$ is fixed to be the value obtained at $T = 0.24T_c$ and $M^i_-(T_c)$ and b_i are the fitting parameters fixed for each channel. The data are given as a function of T/T_c; the value of T_c does not affect the fit but later it is set to the physical value 154 MeV. The parameter b_i corresponds to the width of the chiral crossover. The results of the fits are shown in Table 1.

To correct the unphysical effect from the heavy up and down quarks, $M^i_-(T = 0)$ is set to its PDG mass in the following. The value of $M_-(T_c)$ needs to be corrected as well, and we shall re-scale it by multiplying the factor $M^{\rm PDG}_+(T = 0)/M^{\rm lattice}_+(T = 0)$ with $M_-(T_c)$.

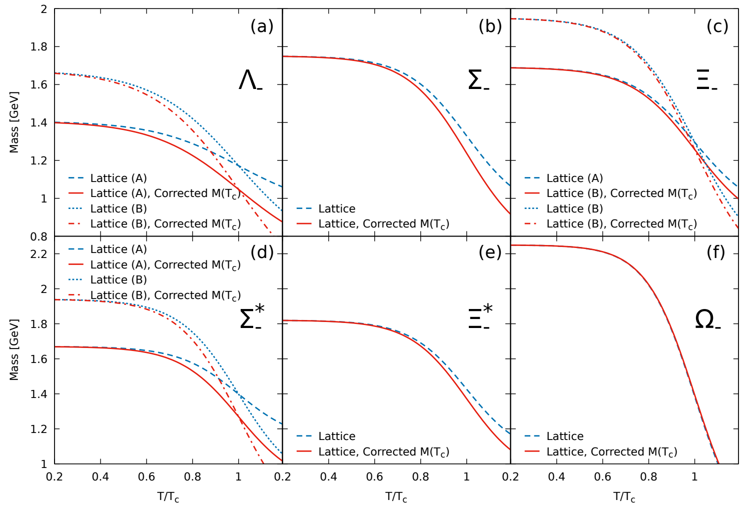

Figure 2. Temperature dependence of octet (upper row) and decuplet (lower row) hyperons from lattice-motivated parameterization Eq. (1).

It is not clear how to assign the negative-parity states to the observed ones because of unknown quantum numbers of some of the candidates. We follow the suggestion in [26] and further adopt two different assignments, A and B. The set A takes the lighter state for Λ_-, Ξ_-, and Σ^*_- while the set B does the heavier ones.

The obtained temperature-dependent masses are displayed in Figs. 1 and 2. The masses of negative-parity states start to drop around $T/T_c \simeq 0.6$, then approach those of their positive-parity partners. In comparing the non-strange baryons with the hyperons, one finds the correction of $M(T_c)$ significantly affects the baryons with more light quarks, as expected.

2.2 Chiral model

The parity-doubled baryons can be modeled in an effective chiral approach [28], where a chiral-invariant mass is naturally introduced. In [27], the mass relations to the light quark σ_q and strange quark σ_s condensates are given for the octet and decuplet states as [1]

$$\begin{aligned}
m_{N_\pm} &= (a_N \mp b_N)\, 3\sigma_q + m_{N0}\,, & m_{\Delta_\pm} &= (a_\Delta \mp b_\Delta)\, 3\sigma_q + m_{\Delta 0} \\
m_{\Sigma_\pm} &= (a_N \mp b_N)\left(2\sigma_q + \sqrt{2}\sigma_s\right) + m_{N0} + m_1\,, & m_{\Sigma^*_\pm} &= (a_\Delta \mp b_\Delta)\left(2\sigma_q + \sqrt{2}\sigma_s\right) + m_{\Delta 0} + m_s \\
m_{\Lambda_\pm} &= (a_N \mp b_N)\left(2\sigma_q + \sqrt{2}\sigma_s\right) + m_{N0} + m_3\,, & m_{\Xi^*_\pm} &= (a_\Delta \mp b_\Delta)\left(\sigma_q + 2\sqrt{2}\sigma_s\right) + m_{\Delta 0} + 2m_s \\
m_{\Xi_\pm} &= (a_N \mp b_N)\left(\sigma_q + 2\sqrt{2}\sigma_s\right) + m_{N0} + m_2\,, & m_{\Omega_\pm} &= (a_\Delta \mp b_\Delta)\, 3\sqrt{2}\sigma_s + m_{\Delta 0} + 3m_s\,.
\end{aligned} \tag{3}$$

The parameters $m_{1,2,3}$ are related via the Gell-Mann–Okubo relation as

$$m_1 = 2m_2 - 3m_3\,. \tag{4}$$

The parameters in the above expressions are determined at zero temperature as in Table 2.

[1]In [27] the same m_0 was assumed common to the octet and decuplet baryons. Here we lift this constraint and introduce two independent masses. Their splitting can be deduced from the spin-spin interaction.

a_N	b_N	m_1 [GeV]	m_2 [GeV]	a_Δ	b_Δ	m_s [GeV]
1.22	1.08	0.248	0.367	1.16	0.862	0.139

Table 2. Set of parameters in the baryon-mass relations. We set $m_{N0} = 0.9$ GeV and $m_{\Delta 0} = 1.15$ GeV.

The masses at $T = 0$ are also summarized in Table 1. Thermal modifications of the baryon masses are driven by the quark condensates. Following [27], we shall use the in-medium condensates measured in LQCD by the HotQCD collaboration [12]. One readily finds that some of the hyperon states have larger masses than those expected from the lattice baryon-spectrum in [25, 26]. Thus, we assign these hyperons to the PDG states as in the bottom line of Table 1 and refer to set C. For a comparison with LQCD, we also apply the set C to the lattice-motivated parametrization (1).

The non-strange baryons exhibit the thermal behavior as in Fig. 1. To some extent, their trends are similar to the results by the FASTSUM collaboration, whereas the parametrization (2) leads to a sizable difference in the mass dropping near T_c. This is particularly evident when the re-scaling of $M_-(T_c)$ is made.

Figure 3 displays the mass of the hyperons in the assignment C. The most distinct difference from the non-strange baryons lies in the mass of the hyperons near T_c; the model calculation shows a rather moderate downward-shift, while the lattice QCD results lead to a strong mass drop. Comparing Λ (Σ), Ξ, and Ω states between the model and the lattice-inspired scaling, one clearly finds that the difference becomes more significant as the baryon contains more strangeness. This may be attributed to the difference in the masses between the light and strange quarks ; The FASTSUM setup with a large pion mass $m_\pi = 384$ MeV and the physical kaon mass is rather close to the flavor SU$_f$(3) limit, while the model calculation done with the physical pion mass is dominated by SU$_f$(2). Therefore, it will be intriguing to see whether the strong mass reduction of Ω and Ξ would still persist in the lattice simulations with a lighter pion.

3 Fluctuations and correlations from in-medium HRG

We employ the hadron resonance gas model to explore the fluctuations and correlations in the hot hadronic matter. In this work we entirely rely on the zero-width approximation; all the resonances are treated as point-like particles and their spectral functions are given by the delta function. Consequently, the thermodynamic pressure at temperature T and chemical potentials μ_j ($j = B, Q, S$) is given by the summation of ideal gas pressure of each particle species as

$$p^{\rm HRG}(T, \mu_B, \mu_Q, \mu_S) = \sum_{i=\rm hadrons} p^{\rm ideal}(T, \mu_i; m_i) \tag{5}$$

where $\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S$ is the chemical potential of particle i with baryon number B_i, electric charge Q_i, and strangeness S_i and the ideal gas pressure with degeneracy d reads

$$p^{\rm ideal}(T, \mu; m) = \mp \frac{dT}{2\pi^2}\int_0^\infty dp p^2 \ln[1 \mp e^{-(\sqrt{p^2+m^2}-\mu)/T}]. \tag{6}$$

The signs are negative for mesons and positive for baryons. The fluctuations and the correlations at vanishing chemical potentials can be expressed as the generalized susceptibilities

$$\chi_{ijk}^{BQS} \equiv \frac{\partial^{i+j+k} p^{\rm HRG}/T^4}{\partial^i(\mu_B/T)\partial^j(\mu_Q/T)\partial^k(\mu_S/T)} \tag{7}$$

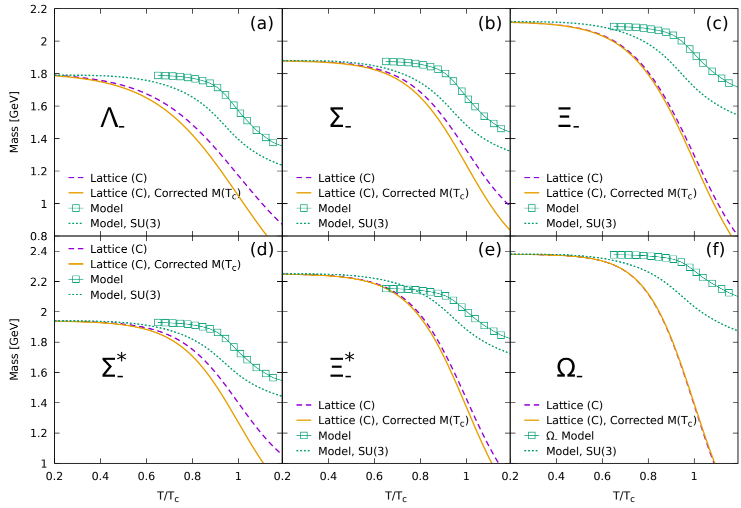

Figure 3. Comparison of hyperon masses between the lattice-motivated parametrization (lines) and the chiral effective model (squares) in the assignment C. As in Fig. 2, the octet and decuplet hyperons are shown in upper and lower panels, respectively.

Although heavy particles ($m \gg T$) are thermally suppressed, the existence of missing states are expected likely in the strange sectors [29, 30]. In fact, the assignment C in Table 1 contains several 1-star states whose spins are undetermined in the latest PDG [31]. For those state we shall assume the same spin with its partner. For consistency we also include other unconfirmed state; we have in total 28 nucleons, 22 Δ baryons, 18 Λ baryons, 21 Σ baryons, 8 Ξ baryons, 3 Ω baryons, and their isospin multiplets. We also include a deuteron, triton, ^{3}He and ^{4}He as they are considered to be thermal ingredients in QCD thermodynamics and heavy ion collisions [10].

In this work we focus on χ_2^B, χ_{11}^{BQ}, and χ_{11}^{BS} at $\mu_B = \mu_Q = \mu_S = 0$. Since $\chi_{ijk}^{BQS} \propto (B/T)^i(Q/T)^j(S/T)^k$, mesons do not contribute to these susceptibilities. Thus they are good measures of in-medium effects in the baryon sector on the QCD thermodynamics. In general, particle masses in a medium can depend not only on temperature but also on chemical potentials. We neglect such an intrinsic chemical-potential dependence in the baryon masses in the present calculations. This is well justified at small chemical potential since the quark condensates are not much affected.

We display χ_2^B, χ_{11}^{BQ}, and $-\chi_{11}^{BS}$ in Fig. 4. In the baryon number susceptibility χ_2^B, the HRG model agrees with the LQCD results below T_c. [2] With the in-medium mass shift, a decrease in the negative-parity baryon masses lead to an enhancement of the susceptibility. One sees a moderate enhancement in all the three scenarios, Lattice (A) (B) and (C), and it becomes stronger when $M(T_c)$ is corrected, since the correction further reduces the masses just below T_c. On the other hand, the different assignments of the parity-partners lead to a minor change. The results with the masses from the chiral effective model follows the same trend, but the amount of the enhancement is much smaller than the tanh-parameterization, owing to the weaker downward mass shifts in the hyperon sectors (see Fig. 3.) In any case, all the results with mass reduction overshoot the lattice data of χ_2^B.

[2]The present HRG leads to a slightly larger χ_2^B than the conventional one because of including the 1-star states and the multi-baryons.

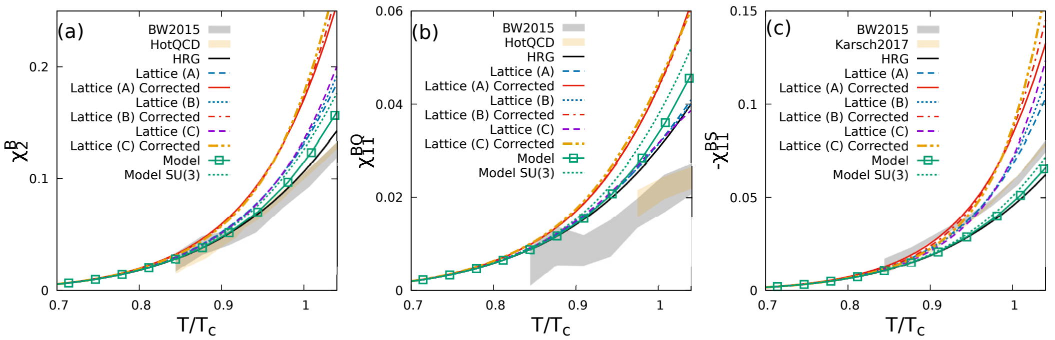

Figure 4. Fluctuations and correlations from the HRG model with and without the mass shifts. From left to right, (a) Baryon number susceptibility, (b) Baryon-charge correlation, and (c) Baryon-strangeness correlation are shown together with the LQCD results from HotQCD [6, 32] and Budapest-Wuppertal Collaboration [33]

The χ_{11}^{BQ} shows a different tendency in Fig. 4(b); the results of Lattice (A)-(C) without the correction of $M(T_c)$ do not exhibit any enhancement from the HRG, even below the "Model" result. In the χ_{11}^{BQ}, $I = 1$ states do not contribute because neutral particles do not contribute and the positively-charged states cancel with the negatively-charged states. Charged N states and doubly charged Δ^{++} states contribute positively, while negatively-charged Ξ_-^-, Ξ_-^{*-} and Ω_-^- states contribute oppositely. Without a strong mass-shift of these multi-strange states, the χ_{11}^{BQ} becomes larger due to the smallness of the hyperon contribution suppressed by the Boltzmann factor, as in the "Model" case. With the strong mass shift of the hyperons, they tend to cancel a part of the positive contribution, and suppress the χ_{11}^{BQ}. The strong enhancement with the corrected $M(T_c)$ clearly signals that the effect of the correction is smaller for Ξ states and larger for N and Δ states, as seen in Figs. 1-3.

The behavior of $-\chi_{11}^{BS}$ in Fig. 4(c) can be understood in a similar way to the χ_2^B; the "Model" result does not differ much from the HRG, because of a weaker mass reduction. Owing to the different trend in the hyperon sectors, the curves deviate more sensitively depending on the fate of the masses.

The observed differences can be more pronounced by taking the ratio between the susceptibilities. In Fig. 5(a), we display the χ_{11}^{BQ}/χ_2^B together with the LQCD data. The results from Lattice (A) and (B) reproduce the one shown in [26]; they follows the trend seen in the HotQCD data. As explained above, however, the individual susceptibilities cannot be reproduced. The coincidence comes from the reduction of χ_{11}^{BQ} owing to the strong mass reductions in the charge asymmetric states. The "Model" case shows the opposite trend that the ratio is above the HRG, due to the much milder mass-reduction in the hyperon sectors. Therefore, the data cannot be explained solely by the mass reduction.

Also shown in Fig. 5(b) is the $-\chi_{11}^{BQ}/\chi_{11}^{BS}$. Again, while the sets (A) and (B) follow the trend of the HotQCD data, the "Model" shows the opposite. This supports the observation that naive insertion of the in-medium masses of parity doublers into the HRG results in fluctuation observables which are inconsistent with LGT data. Nevertheless, our results do not rule out the manifestation of parity doublers. Several studies [19, 20] have revealed that the conventional treatment of the particles in the HRG is not sufficient and a proper inclusion of the width improves the thermodynamics. Although those analysis have been limited to the S−matrix approaches where interactions are incorporated via the two-body scattering phase shift in the vacuum, a consistent treatment of the resonance widths at finite temperature and densities should clarify the consequences of the partial restoration of the chiral symmetry in the baryonic sector of correlations and fluctuations.

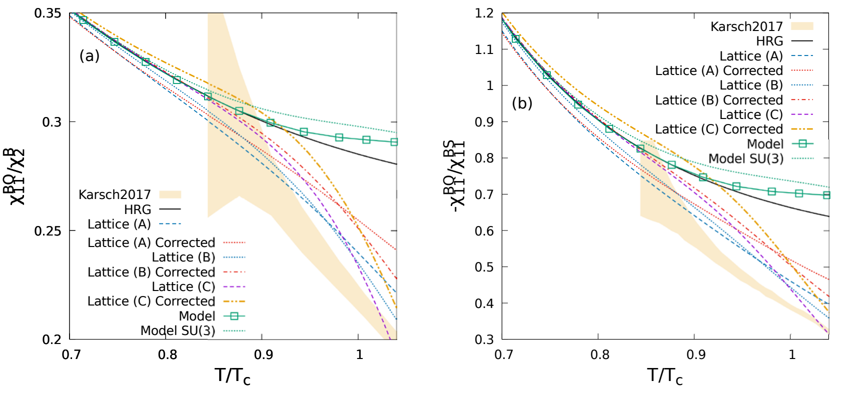

Figure 5. Ratio of the baryon-charge correlation to the baryon number fluctuation (a) and the baryon-strangeness correlation (b). The legends are the same as Fig. 4.

4 Concluding remarks

We have studied the fluctuations and correlations of conserved charges by making use of the hadron resonance gas model with in-medium masses of the low-lying parity-doubled baryons. Motivated by the recent LQCD result on the baryon masses at finite temperature, we have examined the consequences of several scenarios with in-medium masses for the negative-parity octet and decouplet baryons, adopting a lattice-motivated parametrization and that from a chiral effective theory. We have computed the baryon number fluctuation χ_2^B, baryon-charge correlation χ_{11}^{BQ}, and baryon-strangeness correlation χ_{11}^{BS}. The results have been understood on the basis of the presence or absence of the strong mass reduction in the hyperon sectors. We have pointed out that the reproduction of the LQCD results of the ratios χ_{11}^{BQ}/χ_2^B and $-\chi_{11}^{BQ}/\chi_{11}^{BS}$ is just accidental when the in-medium masses are naively introduced in the conventional HRG approach. Since the strong mass shifts of the hyperons observed in LQCD can be regarded as a consequence of the approximate $SU_f(3)$ nature due to a heavy pion mass, it is a critical issue to be examined whether the hyperon mass-shift persists for the physical pion mass.

Although our results clarify the interplay of the different particle species in the behavior of the baryon-charge and baryon-strangeness correlations, we emphasize that the treatment of the resonances in the conventional HRG model is insufficient to quantify correlation and fluctuations of conserved charges. This indicates, that in order to correctly account for the influence of the chiral symmetry restoration on the fluctuation observables, a consistent framework of in-medium effects beyond hadron mass shifts is required.

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