Neutron stars velocities and magnetic fields

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Abstract. We study a model that explain neutron stars velocities due to the anisotropic emission of neutrinos. Strong magnetic fields present in neutron stars are the source of the anisotropy in the system. To compute the velocity of the neutron star we model its core as composed by strange quark matter and analice the properties of a magnetized quark gas at finite temperature and density. Specifically we have obtained the electron polarization and the specific heat of magnetized fermions as a functions of the temperature, chemical potential and magnetic field which allow us to study the velocity of the neutron star as a function of these parameters.

1 Introduction

Neutron stars exhibit an odd large distribution of velocities. In a reported study [1] of the data from the proper motion of 233 pulsars, velocities as high as 1000km s^{-1} are obtained . These velocities are very high compared with typical velocities of other stars, so that the natural question of how to explain it arise. There are several ideas that involve magnetic fields to explain these velocities:

1. Topological currents [2].
2. Electromagnetic rocket effect: Electromagnetic radiation from an off-centered rotating magnetic dipole imparts a kick to the pulsar along its spin axis [3].
3. Neutrino–Magnetic Field Driven Kicks: Asymmetric neutrino emission induced by strong magnetic fields [4].

We have investigated the third option which can be summarized in the following ideas

- Neutrino emissivity from the process $d \to u + e + \bar{\nu}_e,\;\; u + e \to d + \nu_e,$.
- The polarisation of the electron spin will fix the neutrino emission in one direction along the magnetic field.

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- The neutrinos work as a propulsion mechanism for the neutron star.

In order to compute the pulsar velocity we use momentum conservation of the neutron star by the emission of neutrinos

$$dv = \frac{\chi_e}{M_{NS}}\frac{4}{3}\pi R^3 \epsilon dt, \tag{1}$$

where χ_e is the electron spin polarization which is equal to the fraction of neutrinos emitted in the direction of the magnetic fiel, M_{NS} and R are the mass and radius of the neutron star respectively and ϵ is the neutrino emissivity.

If the change in the emissivity is due to the change in temperature, we can use the cooling equation due to neutrino emission

$$-\epsilon = \frac{du}{dt} = \frac{du}{dT}\frac{dT}{dt} = C_v\frac{dT}{dt}, \tag{2}$$

therefor $C_v dT = -\epsilon dt$ and

$$dv = -\frac{\chi_e}{M_{NS}}\frac{4}{3}\pi R^3 C_v dT, \tag{3}$$

$$v = -\frac{1}{M_{NS}}\frac{4}{3}\pi R^3 \int_{T_i}^{T_f} \chi_e\, C_v dT. \tag{4}$$

The work is organized as follows: In Sec. 2 we analyze the properties of a relativistic fermion gas in a magnetic field and we give exact expressions for the polarization and specific heat of charged fermions as a function of temperature, chemical potential and magnetic field. Numerical results are presented in Sec. 3. We finally conclude and discuss our findings in Sec. 4.

2 Thermodynamics of magnetized fermions

The one-loop thermodynamical potential of a magnetized Fermi gas is given by [5]

$$\Omega_f(B,\mu,T) = -\frac{e_f d_f B}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty}(2-\delta_{l0})\left[E_{lf} \ +\frac{1}{\beta}\ln\left(1+e^{-\beta(E_{lf}-\mu)}\right)\left(1+e^{-\beta(E_{lf}+\mu)}\right)\right], \tag{5}$$

where

$$E_{lf} = \sqrt{p_3^2 + 2|e_f|Bl + m_f^2}, \tag{6}$$

is the energy spectrum of charged particles in a constant magnetic field, f stands for fermion, β is the inverse of the temperature, $l = \nu + 1/2 + s/2$ is the Landau level quantum number. $\nu = 0, 1, 2, \ldots$, and $s = \pm 1$. The s values correspond to the projection of the spin operator eigenvalues along the direction of the magnetic field.

From the thermodynamical potential Eq. (5) we can compute the specific heat as

$$C_{Vf} = T\left(\frac{\partial S_f}{\partial T}\right)_{V,N}, \tag{7}$$

where the entropy is given by $S_f = -(\partial\Omega_f/\partial T)_{V,N}$. So that we obtain

$$C_{Vf} = \frac{e_f d_f B}{2\pi^2}\int_0^\infty dp_3 \sum_{l=0}^{\infty}(2-\delta_{l0})\frac{(E_{lf}-\mu_f)^2}{2T^2[1+\cosh\frac{E_{lf}-\mu_f}{T}]}. \tag{8}$$

We can also compute the electron polarization of charged fermions in a magnetic field which is defined as

$$\chi = \frac{n_- - n_+}{n_- + n_+}, \tag{9}$$

where $n_\pm$ are the number densities of fermions with spin parallel ($s = +1$) and anti–parallel ($s = -1$) to the magnetic field direction.

The general expression for the particle number density can be obtained from Eq. (5) as

$$n = -\left(\frac{\partial\Omega}{\partial\mu}\right)_V = \frac{dm^3}{2\pi^2} b \sum_{l=0}^{\infty}(2-\delta_{l0}) \int_0^\infty dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1+2lb}-x\right)}+1}, \tag{10}$$

where $x_3 = p_3/m$ is the dimensionless momentum and $b = B/B_c^f$ ($B_c^f = m_f^2/e_f$). In such a way, $n_\pm$ are given by

$$n_- = \frac{dm^3}{2\pi^2} b \sum_{\nu=0}^{\infty} \int_0^\infty dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1+2\nu b}-x\right)}+1}, \tag{11}$$

$$n_+ = \frac{dm^3}{2\pi^2} b \sum_{\nu=1}^{\infty} \int_0^\infty dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1+2\nu b}-x\right)}+1}, \tag{12}$$

where we have used the relation between l, ν and s, changing the summation over l to the summation over ν (it is important to notice that the change is $\sum_{l=0}^{\infty}(2-\delta_{l0}) \to \sum_{s=\pm1}\sum_{\nu=0}^{\infty}$).

From Eqs. (9), (10) and (11), we can numerically compute the dependence of χ on the parameters B, T, and μ from the following expression

$$\chi = \left\{ 1 + \frac{2\sum\limits_{\nu=1}^{\infty}\int_0^\infty dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1+2\nu b}-x\right)}+1}}{\int_0^\infty dx_3 \frac{1}{e^{\frac{m}{T}\left(\sqrt{x_3^2+1}-x\right)}+1}} \right\}^{-1}. \tag{13}$$

3 Numerical results

For the numerical analysis of the velocity, we can rewrite Eq. (4) as

$$v = -803.925\,\frac{\text{km}}{\text{s}}\left(\frac{1.4M_\odot}{M_{NS}}\right)\left(\frac{R}{10\,\text{km}}\right)^3\left(\frac{I}{\text{MeV fm}^{-3}}\right), \tag{14}$$

where

$$I = \int_{T_i}^{T_f} \chi_e\, C_v dT. \tag{15}$$

In order to solve the integral in Eq. (15), we need to know the dependence of χ and C_V on the magnetic field, the chemical potential and temperature inside the neutron star. In this way, we impose the conditions that exist inside the core of neutron stars which are determined by the beta decay equilibrium equations between the quark species

$$d \to u + e + \bar{\nu}_e, \quad u + e \to d + \nu_e, \tag{16a}$$

$$s \to u + e + \bar{\nu}_e, \quad u + d \to u + s. \tag{16b}$$

Eqns. (17) along whit charge neutrality and baryon number conservation are the stellar equilibrium conditions that give us how the chemical potentials depends on temperature. To impose these conditions we have to solve the system of equations

$$\mu_u + \mu_e - \mu_d = 0 \ , \ \mu_d - \mu_s = 0, \tag{17a}$$

$$2n_u - n_d - n_s - 3n_e = 0, \tag{17b}$$

$$n_u + n_d + n_s - 3n_B = 0, \tag{17c}$$

where the n_f are given by Eq. (10)

From Eq. (13) we can study how the electron polarization varies as a function of the temperature, chemical potential and magnetic field. In the left panel of Fig. (1) shows the behavior of the polarization as a function of the magnetic field for different values of the temperature and a fixed chemical potential of $x = 10$. Notice that, with the increase of the magnetic field, the polarization grows, while the effect of increasing the temperature is to inhibit the increase of the polarization. Also, notice that for low temperatures, the typical effect of Landau number transitions analogue to the Haas-van Alphen oscillations in the magnetization, can be appreciated. The right panel of Fig. (1) shows the polarization as a function of the temperature for different values of the magnetic field an a fixed chemical potential of $x = 10$. The effect of increasing the temperature is to diminish the polarization, while the increase of the magnetic field increases the polarization.

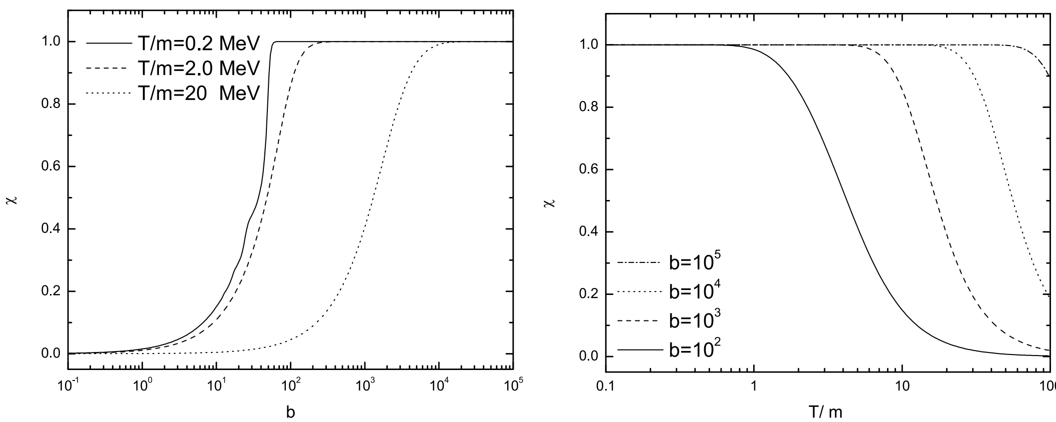

Figure 1. Left panel: polarization χ as a function of the magnetic field and temperature, for a fixed chemical potential of $x = 10$. Right panel: polarization χ as function of the temperature for several values of the magnetic field and fixed chemical potential of $x = 10$.

Fig. (2) shows the polarization as a function of the chemical potential for different values of the magnetic field and two values of temperature. An increase of the chemical potential leads to a decrease in the polarization.

In Fig. (3) we show the specific heat as a function of the temperature for a fixed value of the magnetic field and baryon density. We compare our numerical result whit previous results.

In the left panel of Fig. (3) we show the behaviour of the velocity as a function of the neutron star radius for different values of the magnetic field and a fixed central density, notice that for higher values of the magnetic field, the neutron star can reach higher velocities for small radius values while for low values of the magnetic field the star would requiere larger radius to reach velocities of the order of $v_{\text{kick}} \sim 1000$ km s^{-1}. In the right panel of Fig. (3) we analice how the velocity is affected

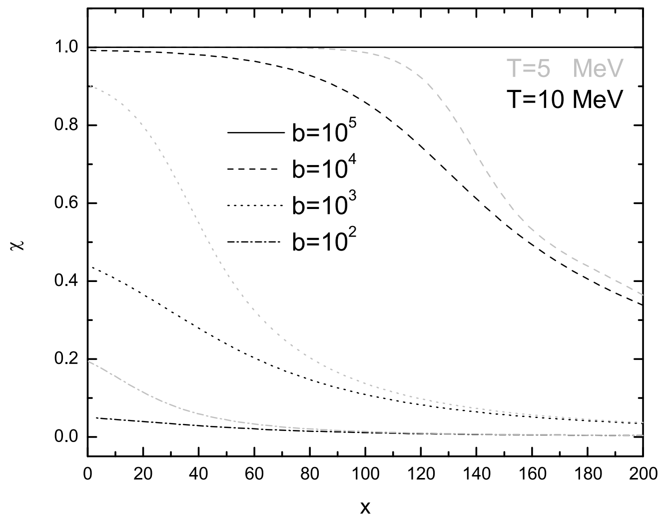

Figure 2. Polarization χ as function of the chemical potential for several values of the magnetic field and two fixed values of the temperature.

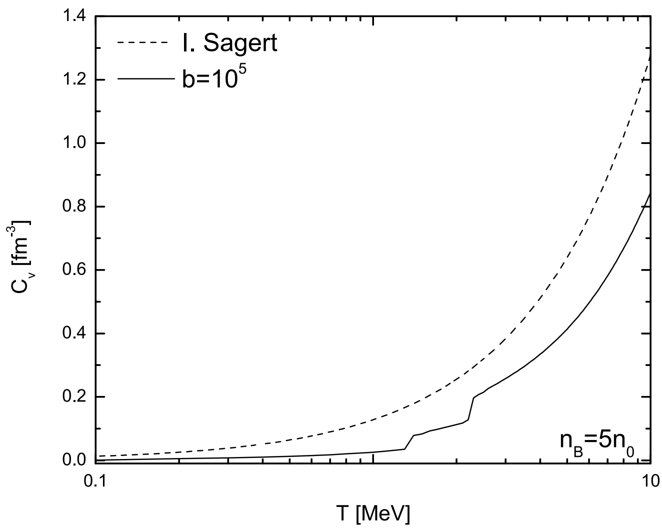

Figure 3. Heat capacity as function of the temperature for a fixed magnetic field and baryon density.

when the baryon density changes, an increase in the baryon density implies that the stars can reach higher velocities for the same value of magnetic fields an radius.

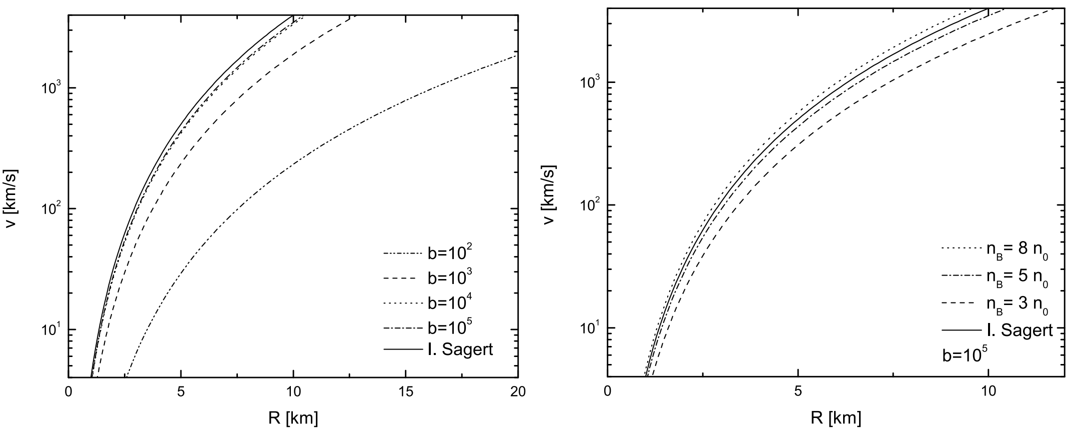

Figure 4. Left panel: velocity as a function of the neutron star radius for different values of the magnetic field and a fixed central density of $n_B = 5\, n_0$. Right panel: velocity as a function of the neutron star radius for different values of the central density and a fixed magnetic field.

4 Conclusions

We have studied a model that can explain the observations of large velocities found on neutron stars. Our model is based on the magnetic induced anisotropic emission of neutrinos as a mechanism of propulsion. We have computed the exact numerical expressions for the electron polarization and the specific heat of a magnetized quark gas as a function of the temperature, chemical potential and magnetic field.

The velocity of NS was computed for SQM in presence of a magnetic field in stellar equilibrium. We have obtained kick velocities $v_{\rm kick} \sim 1000$ km s^{-1} for different values of magnetic fields and star radius. We also have studied the dependence of the kick velocity with the central densities of the star obtaining that when the central density increases the stars can reach higher velocities for the same value of magnetic fields.

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References

[1] G. Hobbs, D.R. Lorimer, A.G. Lyne, M. Kramer, MNRAS **360**, 974 (2005), astro-ph/0504584
[2] J. Charbonneau, A. Zhitnitsky, JCAP **8**, 010 (2010), 0903.4450
[3] D. Lai, *Physics of neutron star kicks*, in *Astrophysics and Space Science Library*, edited by K.S. Cheng, H.F. Chau, K.L. Chan, K.C. Leung (2000), Vol. 254 of *Astrophysics and Space Science Library*, p. 127, astro-ph/9912522

[4] I. Sagert, J. Schaffner-Bielich, Astron. Astrophys. (2007), [Astron. Astrophys.489,281(2008)], 0708.2352

[5] E.J. Ferrer, V. de La Incera, J.P. Keith, I. Portillo, P.L. Springsteen, Physical Review C **82**, 065802 (2010), 1009.3521