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Generation of Quantum Correlations in Bipartite Gaussian Open Quantum Systems

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Abstract.
We describe the generation of quantum correlations (entanglement, discord and steering) in a system composed of two coupled non-resonant bosonic modes immersed in a common thermal reservoir, in the framework of the theory of open systems. We show that for separable initial squeezed thermal states entanglement generation may take place, for definite values of squeezing parameter, average photon numbers, temperature of the thermal bath, dissipation constant and strength of interaction between the two bosonic modes. We also show that for initial uni-modal squeezed states Gaussian discord can be generated for all non-zero values of the strength of interaction between the modes. Likewise, for an initial separable state, a generation of Gaussian steering may take place temporarily, for definite values of the parameters characterizing the initial state and the thermal environment, and the strength of coupling between the two modes.

1 Introduction

Characterizing and quantifying quantum correlations represent a key subject in the theory of quantum information [1–3]. In particular, quantum correlations in states of continuous variable systems play a crucial role in applications of quantum information processing and transmission [4]. In this respect, quantum entanglement is considered to be a strong physical resource for quantum information tasks and protocols [5]. However, not all non-classical properties of quantum correlations are described by entanglement. In this sense, quantum discord was introduced [6, 7] as a measure of quantum correlations which includes entanglement and which can also be present in separable states. Steering is also a type of quantum nonlocality first identified in the Einstein-Podolsky-Rosen paper [8], which is distinct from both nonseparability and Bell nonlocality.

In order to obtain a realistic description of quantum processes it is necessary to take decoherence and dissipation into consideration. Recently we studied, in the framework of the theory of open systems, the dynamics of quantum correlations of two, both uncoupled and respectively coupled, bosonic modes embedded in a common thermal bath, for initial Gaussian states of the system [9–15].

In the present work we analyze the possibility to generate quantum correlations in a system of two coupled bosonic modes interacting with a common thermal reservoir. The initial state of the subsystem is taken of Gaussian form and the evolution under the quantum dynamical semigroup assures the preservation in time of the Gaussian form of the state. We calculate the logarithmic

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negativity and show that for separable initial squeezed thermal states entanglement generation may take place, for definite values of squeezing parameter, average photon numbers, temperature of the thermal bath, dissipation constant and strength of interaction between the two bosonic modes. After its generation one can observe temporary suppressions and revivals of the entanglement. We also show that for initial uni-modal squeezed states the generation of Gaussian quantum discord takes place for all non-zero values of the strength of interaction between the bosonic modes. Finally, we illustrate that, for an initial separable squeezed thermal state, a generation of Gaussian quantum steering may take place temporarily, for definite values characterizing the initial state and the thermal environment, and the strength of interaction between the two modes. The thermal noise and dissipation destroy the steerability between the two parts and, unlike Gaussian quantum discord, which is decreasing asymptotically in time, the Gaussian quantum steering vanishes suddenly in a finite time.

The paper is organized as follows. In Sec. 2 we write the Markovian master equation for the density operator of the open system interacting with a general environment and solve the evolution equation for the covariance matrix of the state of the bimodal bosonic system. Then we describe in Sec. 3 the dynamics of quantum entanglement, Gaussian quantum discord and Gaussian quantum steering for the considered open system. A summary is given in Sec. 4.

2 Master equation for two bosonic modes interacting with the environment

In order to study the dynamics of the subsystem consisting of two coupled bosonic modes (harmonic oscillators) in weak interaction with a thermal reservoir, we use the axiomatic formalism based on completely positive quantum dynamical semigroups. In this framework the Markovian irreversible time evolution of an open system is described by the following Gorini-Kossakowski-Sudarshan-Lindblad master equation for the density operator $\rho(t)$ [16–19]:

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H,\rho(t)] + \frac{1}{2\hbar}\sum_j (2B_j\rho(t)B_j^\dagger - \{\rho(t), B_j^\dagger B_j\}_+), \tag{1}$$

where H denotes the Hamiltonian of the open system and the operators $B_j, B_j^\dagger$ are defined on the Hilbert space of H, and describe the interaction of the subsystem with a general environment.

The Hamiltonian of two coupled in coordinates non-resonant harmonic oscillators of identical mass m and frequencies ω_1 and ω_2 is given by

$$H = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{m}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) + qxy, \tag{2}$$

where x, y are the coordinates and p_x, p_y are the momenta of the two modes, q is the coupling parameter, and we take the operators B_j as polynomials of first degree in the canonical variables of coordinates and momenta.

The equations of motion for the quantum correlations of the canonical observables are the following (T denotes the transposed matrix) [19]:

$$\frac{d\sigma(t)}{dt} = Z\sigma(t) + \sigma(t)Z^{\mathrm{T}} + 2D, \quad Z = \begin{pmatrix} -\lambda & 1/m & 0 & 0 \\ -m\omega_1^2 & -\lambda & -q & 0 \\ 0 & 0 & -\lambda & 1/m \\ -q & 0 & -m\omega_2^2 & -\lambda \end{pmatrix}, \tag{3}$$

with real diffusion matrix D and dissipation coefficient λ. We introduced the 4×4 bimodal covariance matrix σ, with the elements defined as $\sigma_{ij} = \mathrm{Tr}[(A_iA_j + A_jA_i)\rho]/2, i, j = 1, \ldots, 4$, with

$\mathbf{A} = \{x, p_x, y, p_y\}$, which fully characterize any Gaussian state of a bimodal system (up to local displacements).

The time-dependent solution of Eq. (3) is given by $\sigma(t) = S(t)[\sigma(0) - \sigma(\infty)]S^{\mathrm{T}}(t) + \sigma(\infty)$ [19], where $S(t) \equiv \exp(Zt)$. The values at infinity are obtained from the equation $Z\sigma(\infty) + \sigma(\infty)Z^{\mathrm{T}} = -2D$. The covariance matrix, which is a real, symmetric and positive matrix entirely specifying a two-mode Gaussian state, has the following block structure, where 2×2 Hermitian matrices A and B are the covariance matrices for the single modes, and C contains the cross-correlations between the modes:

$$\sigma(t) = \begin{pmatrix} A & C \\ C^{\mathrm{T}} & B \end{pmatrix}. \tag{4}$$

3 Dynamics of quantum correlations

3.1 Generation of quantum entanglement

We use the logarithmic negativity as a measure to quantify the degree of entanglement of the two-mode Gaussian states: $E = \max\{0, -\log_2 2\tilde{n}_-\}$, where $\tilde{n}_-$ is the smallest of the two symplectic eigenvalues of the partial transpose $\tilde{\sigma}$ of the two-mode covariance matrix σ [20]: $2\tilde{n}_\mp^2 = \tilde{\Delta} \mp \sqrt{\tilde{\Delta}^2 - 4\det\sigma}$ and $\tilde{\Delta} = \det A + \det B - 2\det C$. We obtain [21, 22]:

$$E(t) = \max\{0, -\frac{1}{2}\log_2[4f(t)]\}, \tag{5}$$

where

$$f(t) = \frac{1}{2}(\det A + \det B) - \det C - \left(\left[\frac{1}{2}(\det A + \det B) - \det C\right]^2 - \det\sigma(t)\right)^{1/2}. \tag{6}$$

E determines the strength of entanglement for $E(t) > 0$, and if $E(t) = 0$, then the state is separable. In Refs. [23–28] we described the time evolution of the entanglement for a system of two uncoupled bosonic modes interacting with a common environment.

We take an initial two-mode squeezed thermal state, with the covariance matrix given by [29]

$$\sigma_{st}(0) = \begin{pmatrix} a & 0 & c & 0 \\ 0 & a & 0 & -c \\ c & 0 & b & 0 \\ 0 & -c & 0 & b \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned} a &= (n_1 + \frac{1}{2})\cosh^2 r + (n_2 + \frac{1}{2})\sinh^2 r, \\ b &= (n_1 + \frac{1}{2})\sinh^2 r + (n_2 + \frac{1}{2})\cosh^2 r, \\ c &= (n_1 + n_2 + 1)\sinh r \cosh r. \end{aligned} \tag{8}$$

Here n_1, n_2 denote the mean thermal photon numbers associated with the two modes and r is the squeezing parameter. For $n_1 = 0$ and $n_2 = 0$, (7) becomes the covariance matrix of the two-mode squeezed vacuum state. If the squeezing parameter r satisfies the inequality $r > r_e$ [29], where

$$\cosh^2 r_e = \frac{(n_1 + 1)(n_2 + 1)}{n_1 + n_2 + 1}, \tag{9}$$

then the two-mode squeezed thermal state is an entangled state.

We suppose that the only non-zero quantum diffusion coefficients have the following form (we put $\hbar = 1$) [18]:

$$m^2\omega_1^2 D_{xx} = D_{p_xp_x} = \frac{m\omega_1\lambda}{2}\coth\frac{\omega_1}{2kT}, \quad m^2\omega_2^2 D_{yy} = D_{p_yp_y} = \frac{m\omega_2\lambda}{2}\coth\frac{\omega_2}{2kT}, \tag{10}$$

where k is Boltzmann constant and T the temperature of the thermal reservoir.

In Ref. [30] we described the dynamics of the quantum entanglement of a subsystem composed of two coupled bosonic modes interacting with a common thermal reservoir and have shown that, for a separable initial squeezed thermal state entanglement generation may take place, for definite values of the squeezing parameter, average photon numbers, temperature of the thermal bath, dissipation coefficient and strength of interaction between the two modes.

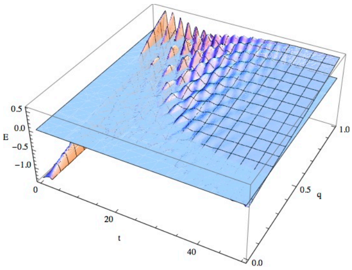

Figure 1. Logarithmic negativity E versus time t and interaction strength q for a separable initial squeezed thermal state with squeezing parameter $r = 0.3$, average photon numbers $n_1 = n_2 = 1$, dissipation constant $\lambda = 0.08$, and $\coth(\omega/2kT) = 1.1$ ($\omega_1 = \omega_2 \equiv \omega = 1, m = \hbar = 1$).

The evolution of separable initial squeezed thermal states with the covariance matrix given by Eq. (7) is illustrated in Fig. 1, where we represent the dependence of the logarithmic negativity $E(t)$ on time t and interaction strength q. One can observe that entanglement generation may take place, for definite values of the strength of interaction between the two modes, of squeezing parameter, average photon numbers, temperature of the thermal bath and dissipation constant. After its generation one can observe temporary suppressions and revivals of the entanglement.

Independent of the initial state, in the limit of large times the system evolves asymptotically to an equilibrium state which may be entangled or separable. The direct interaction between the two modes favours the generation or the preservation in time of the created entanglement, while the temperature of the thermal bath acts towards preventing the generation of entanglement, or suppressing it, once it was created. It is the competition between these two factors which determines the final state of being separable or entangled [30].

3.2 Generation of Gaussian discord

Quantum discord has been defined as the difference between two quantum analogues of classically equivalent expressions of the mutual information, which is a measure of total correlations in a quantum state. It was introduced [6, 7, 31] as a measure of all quantum correlations in a bipartite state, including entanglement. For pure entangled states quantum discord coincides with the entropy of entanglement. Quantum discord can have non-zero values also for some mixed separable states.

For bipartite continuous variable systems, closed formulas of the Gaussian quantum discord have been obtained for bipartite thermal squeezed states [32] and for all two-mode Gaussian states [33].

The Gaussian quantum discord of a general two-mode Gaussian state ρ_{12} can be defined as the quantum discord where the conditional entropy is restricted to generalized Gaussian positive operator valued measurements (POVM) on the mode 2. In terms of symplectic invariants it is given by (the symmetry between the two modes 1 and 2 is broken) [33] $D = f(\sqrt{\beta}) - f(\nu_-) - f(\nu_+) + f(\sqrt{\varepsilon})$, where

$$f(y) = \frac{y+1}{2}\log\frac{y+1}{2} - \frac{y-1}{2}\log\frac{y-1}{2}, \tag{11}$$

$$\varepsilon = \begin{cases} \dfrac{2\gamma^2 + (\beta-1)(\delta-\alpha) + 2|\gamma|\sqrt{\gamma^2 + (\beta-1)(\delta-\alpha)}}{(\beta-1)^2}, \\ \quad \text{if } (\delta-\alpha\beta)^2 \le (\beta+1)\gamma^2(\alpha+\delta) \\ \\ \dfrac{\alpha\beta - \gamma^2 + \delta - \sqrt{\gamma^4 + (\delta-\alpha\beta)^2 - 2\gamma^2(\delta+\alpha\beta)}}{2\beta}, \\ \quad \text{otherwise,} \end{cases} \tag{12}$$

$\alpha = 4\det A, \beta = 4\det B, \gamma = 4\det C, \delta = 16\det\sigma$, and $\nu_\mp$ are the symplectic eigenvalues of the state, given by $2\nu_\mp^2 = \Delta \mp \sqrt{\Delta^2 - 4\delta}$, where $\Delta = \alpha + \beta + 2\gamma$.

In order to prove the possible generation of the Gaussian quantum discord, we have to consider an initial state of the form of a product state, which is the only kind of states which do not contain quantum discord [3]. The uni-modal squeezed states represent an interesting example of such product states with zero discord, described by the covariance matrix

$$\sigma(0) = \frac{1}{2}\begin{pmatrix} \cosh 2r & \sinh 2r & 0 & 0 \\ \sinh 2r & \cosh 2r & 0 & 0 \\ 0 & 0 & \cosh 2r & \sinh 2r \\ 0 & 0 & \sinh 2r & \cosh 2r \end{pmatrix}, \tag{13}$$

with squeezing parameter r.

In Ref. [34] we have shown that, for such an initial uni-modal squeezed states, the generation of Gaussian quantum discord takes place, immediately after the initial moment of time, for all nonzero values of the strength of interaction between the coupled bosonic modes. This phenomenon of discord generation is illustrated in Fig 2, where it is represented the dependence of the Gaussian quantum discord D on time t and squeezing parameter r for an initial uni-modal squeezed state, for the case of non-resonant modes. We notice that after reaching some maximum value, the generated Gaussian discord non-monotonically decreases in time and tends asymptotically for large times to some definite non-zero value. The non-zero asymptotic value of the Gaussian discord is the result of the fact that the two bosonic modes are coupled. In the absence of their coupling, the discord tends asymptotically for large times to zero, corresponding to an asymptotic product state.

3.3 Generation of Gaussian steering

To infer the steerability between two parties is equivalent with verifying the shared entanglement distribution by an untrusted party, by performing local measurements and classical communications [35]. It has been shown in Refs. [35, 36] that the steerability $A \to B$ is present if and only if the following relation does not hold:

$$\sigma + \frac{i}{2}0_A \oplus \Omega_B \ge 0, \quad \Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{14}$$

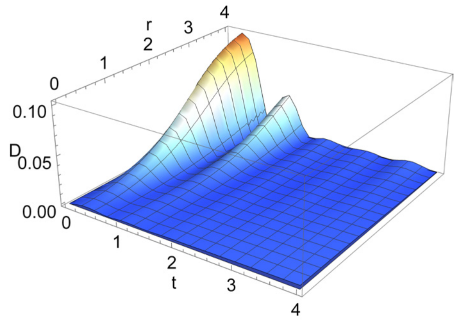

Figure 2. Gaussian quantum discord D versus time t and squeezing parameter r for an initial uni-modal squeezed state, non-resonant modes with $\omega_1 = 1$ and $\omega_2 = 4$, temperature $T = 0$ ($\coth(\omega/2kT) = 1$), strength of interaction between modes $q = 0.5$ and dissipation coefficient $\lambda = 1$.

A measure has been proposed of how much a state with covariance matrix σ is $A \to B$ steerable with Gaussian measurements, by quantifying the amount by which the condition (14) is violated, as follows [37]: $G^{A\to B}(\sigma) = \max\{0, -\ln 2\nu^B\}$, where ν^B is the symplectic eigenvalue of the Schur complement of A in covariance matrix σ. This quantity vanishes if and only if the state σ is not steerable by Gaussian measurements and is invariant under local symplectic transformations. Steering is in general asymmetric with respect to the interchange between steerable parties. The general quantity proposed in Refs. [37, 38] has a particularly simple form when the steered party has one mode:

$$G^{A\to B}(\sigma) = \max\{0, \frac{1}{2}\ln\frac{\det A}{4\det\sigma}\} = \max\{0, S(A) - S(\sigma)\}, \tag{15}$$

where S is the Renyi-2 entropy.

In Ref. [39] we considered the system of two uncoupled bosonic modes interacting with a common thermal reservoir and described the behaviour of the Gaussian quantum steering, when the initial state of the system is an entangled squeezed thermal state. We have shown that the suppression of the Gaussian steering takes place in a finite time, for all temperatures of the thermal reservoir and all values of the squeezing parameter, this behaviour being similar to the well-known phenomenon of entanglement sudden death. This kind of evolution of Gaussian steering and entanglement is in contrast with the dynamics of quantum discord, which decreases to zero asymptotically in time for uncoupled modes.

However, if we take a separable initial squeezed thermal state, with the covariance matrix of the form (7), then it is possible to generate Gaussian steering, for definite values of the squeezing parameter r and strength of interaction q between the coupled modes. In Fig. 3 we illustrate the generation of the Gaussian $A \to B$ quantum steering G for an initial separable squeezed thermal state, for a zero temperature of the thermal bath, as a function of time t and squeezing parameter r. We notice that after its generation, the Gaussian steering is suppressed in a finite time. There are also possible temporary revivals and suppressions of the Gaussian steering. Compared to the Gaussian quantum discord, which is decreasing asymptotically in time, the Gaussian quantum steering suffers a sudden death behaviour like quantum entanglement. The thermal noise and dissipation destroy Gaussian steering in a finite time. For large times the asymptotic value of Eq. (15) has only negative values for non-zero temperatures and is 0 for $T = 0$. Therefore for large times the Gaussian quantum steering is completely missing, independent of the fact that the two bosonic modes are coupled or not.

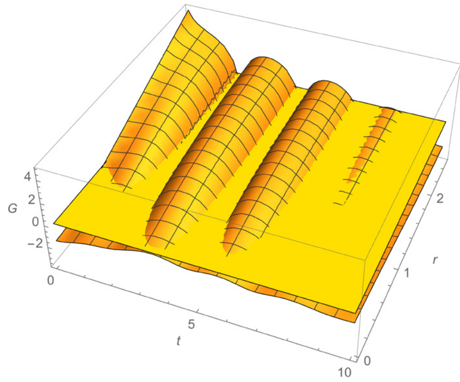

Figure 3. Gaussian quantum steering G versus time t and squeezing parameter r for a separable initial squeezed thermal state with $n_1 = 0.5, n_2 = 1$, in a thermal environment with temperature $T = 0$, dissipation coefficient $\lambda = 0.03$, strength of interaction between modes $q = 0.9$ and $\omega_1 = \omega_2 = 1$.

4 Summary

We investigated the generation of quantum correlations in a system composed of two coupled bosonic modes, embedded in a common thermal bath, in terms of the covariance matrix for Gaussian input states. The main conclusion is that the generation of quantum correlations strongly depends on the initial states of the subsystem, frequencies of the modes, parameters characterizing the thermal reservoir and the intensity of the coupling between the two modes. The competition between the direct interaction between the two bosonic modes and the influence of the thermal environment is decisively determining the possible generation of the quantum correlations and their time evolution.

For an initial separable state, for definite values of the mentioned parameters and for sufficiently large values of the interaction strength between the modes, this interaction makes possible the generation of the entanglement. After its generation, temporary suppressions and revivals of entanglement may also happen, and in the limit of large times the system evolves asymptotically to an equilibrium state which, depending on the values of the strength of interaction between the two bosonic modes, on temperature and dissipation coefficient, may be separable or entangled. Generation of Gaussian quantum discord in the case of an initial uni-modal squeezed state always happens, for all values of the strength of interaction between the two modes. Compared to the entanglement, Gaussian discord keeps for all finite times a non-zero value, including for asymptotic large times. Likewise, for an initial separable state, a generation of Gaussian quantum steering may take place temporarily, for definite values characterizing the initial state and the thermal environment, and the strength of interaction between the modes. In the limit of large times, the Gaussian steering is always zero.

In the context of the debate relative to the physical interpretation of quantum correlations, the present results, in particular the possibility of generating and preserving them in thermal environments for enough long times, might be useful in controlling quantum correlations in open systems and also for applications in the protocols of quantum information processing and communication.

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