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Extrapolation of Functions of Many Variables by Means of Metric Analysis

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Abstract. The paper considers a problem of extrapolating functions of several variables. It is assumed that the values of the function of m variables at a finite number of points in some domain D of the m-dimensional space are given. It is required to restore the value of the function at points outside the domain D. The paper proposes a fundamentally new method for functions of several variables extrapolation. In the presented paper, the method of extrapolating a function of many variables developed by us uses the interpolation scheme of metric analysis. To solve the extrapolation problem, a scheme based on metric analysis methods is proposed. This scheme consists of two stages. In the first stage, using the metric analysis, the function is interpolated to the points of the domain D belonging to the segment of the straight line connecting the center of the domain D with the point M, in which it is necessary to restore the value of the function. In the second stage, based on the auto regression model and metric analysis, the function values are predicted along the above straight-line segment beyond the domain D up to the point M. The presented numerical example demonstrates the efficiency of the method under consideration.

1 Introduction

One of the main problems of data processing in many areas is the problem of functions of several variables extrapolation. In the presented paper, the method of a function of many variables extrapolation, developed by us, uses the interpolation scheme of metric analysis. Below is a brief description of the metric analysis of interpolation of the values of functions of several variables and its application [3].

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2 Extrapolation scheme

At the first and second stages of the scheme of extrapolation of functions of many variables presented in the article, the interpolation of metric analysis is used. The extrapolation scheme uses the interpolation method for the functional dependence:

$$Y = F(X_1, \dots, X_m) = F(\vec{X}), \tag{1}$$

where the function $F(\vec{X})$ is unknown and is subject to recovery, either at one point $\vec{X}^*$ or in a set of given points on the basis of known values of the function $Y_k, k = 1, \dots, n,$ in points $\vec{X}_k = (X_{k1}, \dots, X_{km})^T$ [1].

According to the method of interpolation, based on metric analysis, interpolation values are found as solutions of problems of minimum of the measure of metric uncertainty with respect to the point $\vec{X}^* = (X_1^*, \dots, X_m^*)^T$ [1]–[4]

$$\sigma^2_{ND}(Y^*; \vec{z}^*) = (W\vec{z}^*, \vec{z}^*)\,, \tag{2}$$

where W is a matrix of the metric uncertainty and the interpolation value is determined by a linear combination

$$Y^* = \sum_{i=1}^{n} z_i^* Y_i, \qquad \sum_{i=1}^{n} z_i^* = 1, \tag{3}$$

and is given by

$$Y^* = \frac{(W^{-1}\vec{Y}, \vec{1})}{(W^{-1}\vec{1}, \vec{1})}\,. \tag{4}$$

The matrix of the metric uncertainty is defined by

$$W = \begin{pmatrix} \rho^2(\vec{X}_1, \vec{X}^*)_{\vec{w}} & (\vec{X}_1, \vec{X}_2)_{\vec{w}} & \dots & (\vec{X}_1, \vec{X}_n)_{\vec{w}} \\ (\vec{X}_2, \vec{X}_1)_{\vec{w}} & \rho^2(\vec{X}_2, \vec{X}^*)_{\vec{w}} & \dots & (\vec{X}_2, \vec{X}_n)_{\vec{w}} \\ \dots & \dots & \dots & \dots \\ (\vec{X}_n, \vec{X}_1)_{\vec{w}} & (\vec{X}_n, \vec{X}_2)_{\vec{w}} & \dots & \rho^2(\vec{X}_n, \vec{X}^*)_{\vec{w}} \end{pmatrix}, \tag{5}$$

where $\rho^2(\vec{X}_i, \vec{X}^*)_{\vec{w}} = \sum\limits_{k=1}^{m} w_k (X_{ik} - X_k^*)^2$, $(X_i, X_j)_{\vec{w}} = \sum\limits_{k=1}^{m} w_k (X_{ik} - X_k^*) \cdot (X_{jk} - X_k^*)$, $i \neq j = 1, \dots, n$, where w_k, $k = 1, \dots, m$ are metric weights (see [3]).

Next, a metric analysis scheme is used to predict (extrapolate) the function of a single variable using an autoregressive model of metric analysis.

Let us consider a function $y = f(x)$ of one variable x with known values $Y_1 = f(x_1), \dots, Y_n = f(x_n)$ for $x_1 < \dots < x_n \in [x_1, x_n]$. It is required to find the extrapolated value Y_{n+1} for x_{n+1}.

The problem of finding the extrapolated value Y_{n+1} is reduced to the problem of interpolation of functions of several variables by means of a nonlinear autoregressive model [3, 4]:

$$\begin{aligned} y(x_{l+1}) &= Y_{l+1} = F(Y_1, \dots, Y_l), \\ y(x_{l+2}) &= Y_{l+2} = F(Y_2, \dots, Y_{l+1}), \\ \dots & \quad \dots \dots \dots \dots \\ y(x_n) &= Y_n = F(Y_{n-l}, \dots, Y_{n-1}). \end{aligned} \tag{6}$$

Then the extrapolation of the function $y = f(x)$ is reduced to the interpolation of the function of l variables $Y = F(y_1, y_2, \dots, y_l)$ with values in $n - l$ points

$$\begin{aligned} \vec{X}_1 &= (Y_1, \dots, Y_l)^T, \\ \vec{X}_2 &= (Y_2, \dots, Y_{l+1})^T, \\ \dots & \dots \\ \vec{X}_{n-l} &= (Y_{n-l}, \dots, Y_{n-1})^T. \end{aligned} \tag{7}$$

The extrapolated value $Y_{\text{ext}} = Y_{n+1}$ is defined as an interpolation value of the function $Y = F(y_1, y_2, \ldots, y_l)$ at the point $\vec{X}^*$:

$$Y_{n+1} = F(\vec{X}^*) = \frac{(W^{-1}\vec{1}, \vec{Y})}{(W^{-1}\vec{1}, \vec{1})}, \tag{8}$$

where $\vec{X}^* = (Y_{n-l+1}, \ldots, Y_n)^T$, W^{-1} is the inverse matrix for the $(n-l) \times (n-l)$ matrix of metric uncertainty, $\vec{Y} = (Y_{l+1}, \ldots, Y_n)^T$ is the $(n-l)$-dimensional vector of the values of the extrapolated function.

The number l determines the dimension of the space of vectors, and its value in the test was found as a solution of extremal problem [1, 2]

$$l^* = \operatorname{argmin} ||\vec{Y}_{\text{re}} - \vec{Y}_{\text{ext}}||, \tag{9}$$

where minimization is done in l, $\vec{Y}_{\text{re}}$ is the vector of realized values and $\vec{Y}_{\text{ext}}$ is the vector of extrapolated values.

The extrapolation scheme for the function of several variables (1) at a given point $\vec{X}^* = (X_1^*, \ldots, X_m^*)^T$ consists of two stages.

At the first stage, the point $\vec{X}_0 = (X_{01}, \ldots, X_{0m})^T$ is selected inside the cluster of the realized values of the function $Y = F(\vec{X})$. Then the points $\vec{X}_0$ and $\vec{X}^*$ are connected by a straight line segment

$$(1-s) \cdot \vec{X}_0 + s \cdot \vec{X}^*, \quad 0 \leq s \leq 1, \tag{10}$$

which is divided into L equal segments with nodes

$$\vec{S}_k = (S_{k1}, \ldots, S_{km})^T, \quad k = 1, \ldots, L+1, \quad \vec{S}_{L+1} = \vec{X}^*. \tag{11}$$

In the points

$$\vec{S}_k = (S_{k1}, \ldots, S_{km})^T, \quad k = 1, \ldots, l, \quad l < L+1, \tag{12}$$

belonging to the cluster and rectilinear segment (10), the interpolation of the values of the function (1) is performed using the scheme (4)–(5) on the set of known values of the function Y_i, $i = 1, \ldots, n$ at the points $\vec{X}_i = (X_{i1}, \ldots, X_{im})^T$ belonging to the cluster.

At the second stage, the interpolated values, which were calculated at the first step $\vec{Y} = (Y_1, \ldots, Y_l)^T$ at the points (12), are successively extrapolated to the remaining nodes $\vec{S}_k = (S_{k1}, \ldots, S_{km})^T$, $k = l+1, \ldots, L+1$, where

$$f(s) = F((1-s) \cdot \vec{X}_0 + s \cdot \vec{X}^*), \quad 0 \leq s \leq 1. \tag{13}$$

By using the autoregressive scheme (6)–(9) at the nodes

$$s = s_k, \quad k = l+1, \ldots, L+1, \quad s_k = s_{k-1} + \Delta s, \quad \Delta s = \frac{||\vec{X}^* - \vec{X}_0||}{L}, \tag{14}$$

one can obtain the extrapolated value

$$Y_{\text{ext}} = F(\vec{X}^*) = f(s_{L+1}). \tag{15}$$

3 Numerical result

Example. Function $Y = F(x_1, \ldots, x_{10}) = \sum_{i=1}^{10} x_i^2 + \sum_{i=1}^{10} x_i + 6 \cdot \sin(2(x_1 + x_{10}))$ is considered.

Values of function are set in 8 points:

(0.1;0.6;0.9;0.1;0.1;0.7;0.6;0.1;0;0.9), (0.3;0.7;0.3;0.7;0.2;0.3;0.4;0.8;0.9;0.1), (0.5;0.4;0.1;0.4;0.6;0.1;0;0.3;0.6;0.4), (0.4;0.2;0.3;0.8;0.8;0.2;0.4;0.2;0.3;0.3), (0.2;0.7;0.4;0.3;0.3;0.8;1;0.5;1;0.2), (1;0;0.8;0.6;0.9;0.5;0.3;0.4;0.5;0.3), (0.3;1;0.8;0.5;1;0.9;0.5;0.4;0.3;1), (1;0.8;0.7;0.6;0.8;0.8;1;1;0.9;1)

It is necessary to find the the extrapolated value of Y at the point (1.3;1;0.6;0.8;1;0.8;1.5;1;1.4;1.3).

Extrapolation is made at the points of a straight line connecting the point of the center of domain (0.5;0.5;0.5;0.5;0.5;0.5;0.5;0.5;5;0.5) of the given 8 points and the point (1.3;1;0.6;0.8;1;0.8;1.5;1;1.4;1.3).

For the example in question m=10, n=8, l^*=4, L=20.

Table 1 lists the exact and extrapolated values of the function for k =13,. . . ,21 (see (13)–(14)).

Table 1. Numerical example

k	13	14	15	16	17	18	19	20	21
Exact values	11.58	11.78	12.11	12.61	13.27	14.1	15.1	16.29	17.63
Extrapolated values	11.83	11.97	12.34	12.77	13.32	13.91	14.68	15.37	16.12

4 Conclusion

In this paper a fundamentally new method of the extrapolation for functions of several variables is proposed. The obtained numerical results of extrapolation of functions of several variables, taken from different areas, show that the presented scheme can provide a reliable accuracy of extrapolation results.

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References

[1] A.V. Kryanev, G.V. Lukin, and D.K. Udumyan, *Metric analysis and data processing* (M.: Ed. Science, 2012)

[2] A.V. Kryanev and D.K. Udumyan, Int. Journal of Mathematical Analysis **8** (45), 2221–2228 (2014)

[3] A.V. Kryanev and D. K. Udumyan, Int. Journal of Mathematical Analysis **8** (60), 2971–2978 (2014)

[4] A.V. Kryanev, D.K. Udumyan, G.V. Lukin, and V.V. Ivanov, Applied Mathematical Sciences **8** (22), 1053–1060 (2014)