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Lattice QCD with mixed action – Borici-Creutz valence quark on staggered sea

Subhasish Basak[1,⋆], *Jishnu* Goswami[2], and *Dipankar* Chakrabarti[2]

[1] *School of Physical Sciences, NISER Bhubaneswar, Odisha 752050, India*
[2] *Department of Physics, Indian Institute of Technology Kanpur, Kanpur 208016, India*

Abstract. Mixed action lattice QCD with Borici-Creutz valence quarks on staggered sea is investigated. The counter terms in Borici-Creutz action are fixed nonperturbatively to restore the broken symmetries. On symmetry restoration, the usual signatures of partial quenching / unitarity violation like negative scalar correlator are observed. The size of unitarity violation due to different discretization of valence and sea quark is determined by measuring Δ_{mix}.

1 Introduction

Minimally doubled fermions are promising techniques to study light quarks on lattice. They are known to preserve chiral symmetry for a degenerate quark doublet and are local. This can be helpful for $N_f = 2$ lattice simulations, and is relatively simpler and possibly faster than Ginsparg-Wilson fermions. There are two main variants of the minimally doubled fermions – Karsten-Wilczek [1, 2], Borici-Creutz [3, 4]. In this work, we study mixed action lattice QCD with Borici-Cruetz valence quarks in Asqtad improved staggered sea, restricting to light quarks only.

Presently, there is a dearth of numerical studies with BC fermions and very few literature that would suggest its usefulness and performance in lattice QCD simulations. Previously, using BC fermions we have studied discrete chiral symmetry breaking in a two-dimensional Gross-Neveu model [7] and mass spectroscopy in two-dimensional field theories [8] in lattice. In this work, we extend our investigations to four-dimensional lattice. Here, we consider a mixed action approach with BC valence quarks on an Asqtad improved staggered sea. The coefficients of the counterterms of the renormalized action are fixed nonperturbatively to restore the broken hypercubic symmetry of the BC action.

At finite lattice spacing, mixed action QCD violates unitarity but it is believed to have the correct continuum limit. The scalar correlator is known to be sensitive to the unitarity violation, due to different discretizations of valence and sea quarks. After the counterterms are tuned and the breaking of hypercubic symmetries minimized, the scalar correlator shows the effect of partially quenched QCD and becomes negative for valance quark mass smaller than sea quark mass.

In the leading-order mixed action chiral perturbation theory, a lattice spacing-dependent low-energy constant Δ_{mix} appears as a free parameter in the mass formula for a meson made up of one valence and one sea quark. The Δ_{mix} gives a measure of the unitarity violation. The Δ_{mix} obtained in

⋆Speaker, e-mail: sbasak@niser.ac.in

this work is found to be comparable to the mixed action with domain-wall fermions on a staggered sea [9, 10].

2 Borici-Creutz action and Point split method

The free BC action in the discretized four-dimensional space-time lattice is written as

$$\begin{aligned} S_{BC} &= \sum_x \Bigg[\frac{1}{2} \sum_\mu \bar{\psi}(x)\, \gamma_\mu \,(\psi(x+\hat{\mu}) - \psi(x-\hat{\mu})) - \\ & \frac{i}{2} \sum_\mu \bar{\psi}(x)\, (\Gamma - \gamma_\mu)\; (2\psi(x) - \psi(x+\hat{\mu}) - \psi(x-\hat{\mu})) + m\, \bar{\psi}(x)\psi(x) \Bigg] \end{aligned} \tag{1}$$

where $\Gamma = \frac{1}{2}(\gamma_1+\gamma_2+\gamma_3+\gamma_4)$, $\{\Gamma, \gamma_\mu\} = 1$, and we have taken lattice spacing $a = 1$. In the momentum space, the action turns out to be diagonal and is

$$S_{BC} = \int \frac{d^4 p}{(2\pi)^4}\, \bar{\psi}(p) \left[\sum_\mu \Big(\gamma_\mu \sin(p_\mu) + i\,(\Gamma - \gamma^\mu) \cos(p_\mu) \Big) - 2i\,\Gamma + m \right] \psi(p). \tag{2}$$

The zeros of the free massless Dirac operator are at $(0,0,0,0)$ and $(\pi/2, \pi/2, \pi/2, \pi/2)$. The spinor $\psi(p)$ contains two degenerate flavors. We can construct the fields for these two flavors at two different poles by the method of point splitting [11, 12]. We define

$$\begin{aligned} d(p) &\equiv \frac{1}{4}\,\Gamma \sum_\mu (1 - \sin p_\mu)\, \psi(p) \\ u(p) &\equiv \frac{1}{4} \sum_\mu (1 - \cos(p_\mu + \pi/2))\, \psi(p + \pi/2) \end{aligned} \tag{3}$$

$$\tag{4}$$

such that $d(0) = \Gamma\psi(0)$ and $d(\pi/2) = 0$, implying d is defined as an excitation around the pole at $(0,0,0,0)$. Similarly, $u(0) = \psi(\pi/2)$ is defined around the pole at $(\pi/2, \pi/2, \pi/2, \pi/2)$. In our work we have implemented the point split fields as defined in Eq. (3), where the summation over the Lorentz index μ is carried out.

The BC action (1) has a special direction in Euclidean space which is the major hypercube diagonal given by Γ. This breaks reflection symmetry of the hypercube, leading to the breaking of parity and time symmetry [5]. Because of the broken hypercubic symmetry, the counterterms are necessary for a renormalized theory. The allowed counterterms for the BC action are dimension-4 counterterm $c_4(g_0)\, \bar{\psi}\, \Gamma \sum_\mu D_\mu \psi$ and dimension-3 counterterm $ic_3(g_0)\, \bar{\psi}\, \Gamma\, \psi$ [13]. Then the complete renormalized action, with the gauge interaction turned on, reads

$$\begin{aligned} S_{BC} &= \sum_x \Bigg[\frac{1}{2} \sum_\mu \Big(\overline{\psi}(x)\, (\gamma_\mu + c_4(g_0)\Gamma + i\gamma'_\mu)\, U_\mu(x) \psi(x+\hat{\mu}) \\ & - \overline{\psi}(x+\hat{\mu})\, (\gamma_\mu - c_4(g_0)\Gamma - i\gamma'_\mu)\, U^\dagger_\mu(x)\psi(x) \Big) \\ & + \overline{\psi}(x)\, (m + i\tilde{c}_3(g_0)\Gamma)\; \psi(x) \Bigg], \end{aligned} \tag{5}$$

where $\tilde{c}_3 = c_3 - 2$, $\gamma'_\mu = \Gamma - \gamma_\mu = \Gamma\gamma_\mu\Gamma$ and g_0 is the bare coupling parameter. The coefficients of the dimension-3 and -4 counterterms, c_3 and c_4, respectively, have been evaluated in one-loop lattice perturbation theory [13]. Mixed action study has been carried out with the above renormalized BC action for valence quarks.

2.1 Simulation details

Mixed action simulation is carried out with BC fermions on three ensembles of publicly available MILC lattices with 2+1 dynamical flavors of Asqtad improved staggered fermions [14], with a fixed ratio $am_l/am_s = 1/5$. The details of MILC configurations used in this work are listed in Table 1. Since the measurement of $a^2\,\Delta_{\rm mix}$ is one of the important parts of this study, we prefer to use lattices

Table 1. Details of MILC lattices [14] used in this work

Lattice dim.	$\beta = 10/g_0^2$	am_l/am_s	a (fm)	volume (fm)^3	# configs
$16^3 \times 48$	6.572	0.0097 / 0.0484	≈ 0.15	$\sim (2.4\text{ fm})^3$	40
$20^3 \times 64$	6.76	0.01 / 0.05	≈ 0.13	$\sim (2.6\text{ fm})^3$	40
$28^3 \times 96$	7.09	0.0062 / 0.031	≈ 0.09	$\sim (2.5\text{ fm})^3$	30

of three different spacings in lieu of multiple am_l/am_s ratios (for a fixed lattice spacing) as it has been shown in multiple studies that $\Delta_{\rm mix}$ does not depend significantly on the sea masses [15].

The BC valence quark propagators are constructed using a range of bare quark masses [0.0075 – 0.5] which restricts the $m_\pi L$ around 4. The strange mass is tuned by setting the fictitious $s\bar{s}$ pseudoscalar mass to 686 MeV [16]. A similar strange mass is also obtained by using $m_{\rm ps}/m_{\rm vec} = 0.673$ [14] for tuning. Random wall sources are used for the generation of all quark propagators. All the fits are uncorrelated and the errors are from Jack-knife analysis.

3 Nonperturbative fixing of the counterterms

3.1 Parity condensate

The BC fermions exhibit parity-flavor breaking, which follows from the absence of hypercubic symmetry of BC action (1). Because the CPT is conserved, the T-symmetry is also broken. The counterterms are necessary for renormalized theory and those that are allowed by the remaining symmetry are added to the action. Two such counterterms of dimension-3 and 4 are introduced. The coefficients of these operators, $\tilde{c}_3(g_0)$ and $c_4(g_0)$, can then be tuned to restore the desired symmetries.

The quantity we measure to determine the size of parity breaking is the parity condensate $|\langle\bar{\psi} i\gamma_5\tau_3\psi\rangle|$, where τ_3 is the third generator of $SU(2)$. First we study the variation of the chiral condensate as we tune $\tilde{c}_3$ keeping c_4 fixed at its perturbative value for the lattice in use. The dependence of parity condensate on $\tilde{c}_3$ for various lattices is shown in Fig.1. The minimum of the condensate for all three ensembles appears around $\tilde{c}_3 = -0.5$. Besides, the actual values and the minimum of parity condensate appear to have much less dependence on the lattice spacing. Next we fix $\tilde{c}_3$ to this nonperturbative value and measure parity condensate for varying c_4. This is plotted in Fig.2. We observe very little dependence of parity condensate on c_4 for all the three lattices. Here too we find that the parity condensate is almost independent of the lattice spacing. Therefore, tuning $\tilde{c}_3$ alone is sufficient for minimizing the breaking (or restoration) of parity.

3.2 Time asymmetry

The BC action has PT-symmetry, and because of breaking of parity, we expect to see the sign of time asymmetry in the theory and lattice calculation. In lattice simulation, this time asymmetry can show up in the spectrum and manifests through nondegeneracy of forward and backward propagating meson states. The pion propagators and the corresponding effective masses for different c_4, while keeping

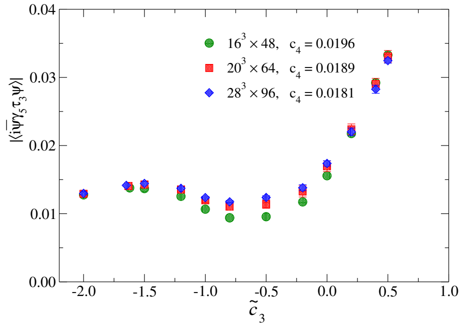

Figure 1. Parity condensate versus $\tilde{c}_3$ for $c_4 = 0.0196, 0.0189, 0.0181$ on $16^3\times48$, $20^3\times64$, $28^3\times96$ lattices

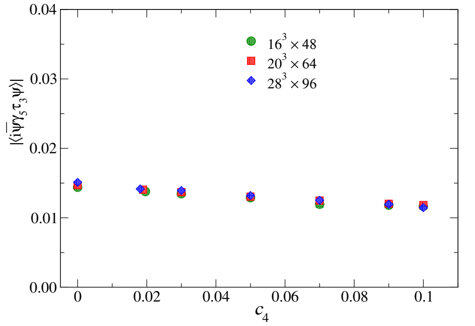

Figure 2. Parity condensate versus c_4 for $\tilde{c}_3 = -0.5$ on $16^3\times48$, $20^3\times64$, $28^3\times96$ lattices

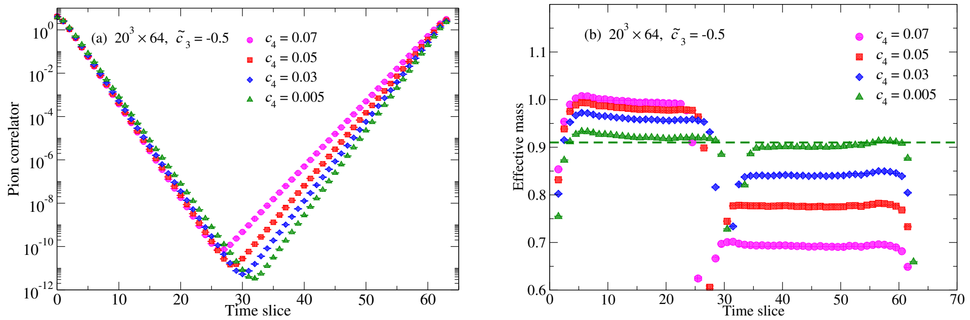

Figure 3. Pion correlator and effective mass for different values of c_4 keeping $\tilde{c}_3 = -0.5$ fixed for $20^3\times64$ lattices

$\tilde{c}_3$ fixed at -0.5 as obtained above, are shown in Fig. 3. The pion operator is constructed from the d-quark field as defined in Eq. (3). The asymmetry in the forward and backward propagating parts and consequently difference in effective masses is rather evident. The asymmetry in pion propagators and the mass difference vanishes when c_4 is lowered to about 0.005. This behavior does not change with the changing values of $\tilde{c}_3$ implying that the size of time symmetry breaking is almost entirely driven by c_4. Additionally, the value of c_4 at which the time symmetry is restored remain fairly constant for all the three lattices that we used. In Fig.4, we plot the mass difference of the forward and backward

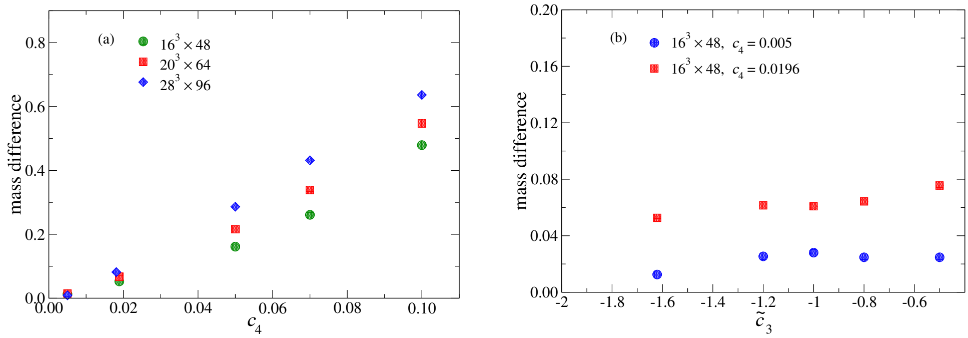

Figure 4. Variations of forward-backward mass difference with the coefficients of the counter terms (a) c_4 when $\tilde{c}_3 = -0.5$ and (b) $\tilde{c}_3$ when $c_4 = 0.0196,\ 0.005$

propagating pion masses against the variation of c_4 while keeping $\tilde{c}_3 = -0.5$ in (a) while the variation against $\tilde{c}_3$ is shown keeping c_4 fixed at 0.005 in (b). Fig. 4(b) clearly indicates the absence of any role of $\tilde{c}_3$ in restoring time symmetry. Also, the plot shows that, although the mass differences are different for different lattices in absence of time symmetry, the mass differences converge to zero at about $c_4 = 0.005$ upon restoration of symmetry. The smallness of c_4 suggests that T symmetry is only weakly broken. We have used $\tilde{c}_3 = -0.5$ and $c_4 = 0.005$ in all our subsequent simulations irrespective of lattice ensembles.

4 Mixed action pion mass

The pion spectrum is obtained for valence quark mass in the range of [0.0075 – 0.5]. In Figs. 5(a) and (b), we plotted the pion mass m_π and pion mass squared m_π^2 respectively as function of bare valence quark mass m. At small quark masses, the plot deviates from straight line due to the logarithmic

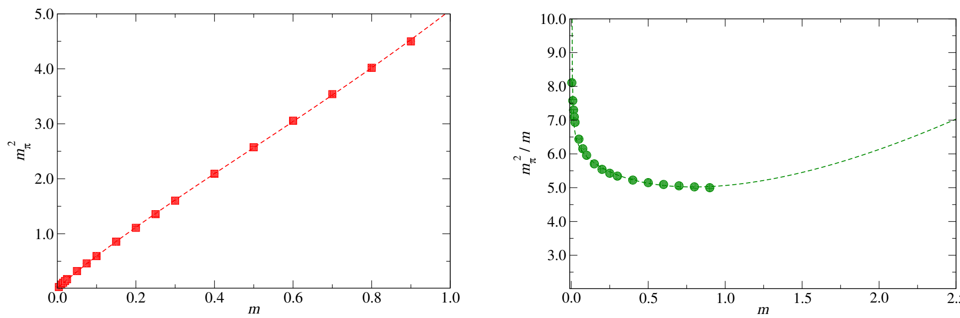

Figure 5. Pion mass as a function of valence quark mass m on $20^3 \times 64$ lattice. In (b), the dotted curve is a fit to the mass formula in Eq. (7)

correction coming from one-loop chiral perturbation theory,

$$m_\pi^2 = 2Bm + (2B)^2 \frac{m^2}{(32\pi^2 f^2)} \log(m)\,, \tag{6}$$

where B is the LEC and f is the pion decay constant. The chiral log has been studied previously for different lattice fermions in the context of mixed action in [17–19]. A more complete mass formula in one-loop partially quenched chiral perturbation theory is given by [18, 20],

$$m_\pi^2 = C_1 m + C_{1L} m \log(m) + C_2 m^2 + C_{2L} m^2 \log(m), \tag{7}$$

where C_1, C_{1L}, C_2, C_{2L} are independent low energy constants. The existence of chiral log in our mixed action simulation is more prominently visible when the ratio of m_π^2/m is plotted against m.

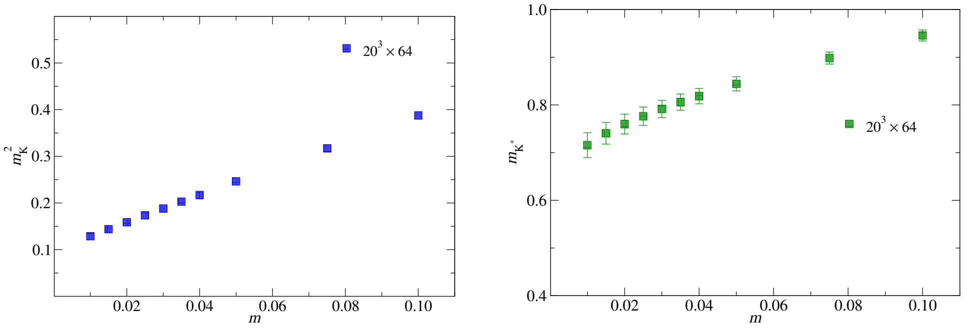

Figure 6. m_K^2 and $m_{K^\star}$ vs light-quark mass on $20^3 \times 64$ lattice

Generally, the strange quark mass is determined by tuning the unphysical pseudoscalar $s\bar{s}$ meson mass to 687 MeV. The strange mass $am_s = 0.030$ gives the fictitious pseudoscalar $s\bar{s}$ mass $m_{s\bar{s}} = 682$ MeV (with $a = 0.13$ fm) which corresponds to bare strange mass $m_s = 45.46$ MeV on $20^3 \times 64$ lattice. With that fitted strange mass, the vector meson $\phi = s\bar{s}$ is found to have $m_\phi \approx 1120$ MeV [PDG [22] value 1020 MeV]. In Fig.6, m_K^2 and m_{K^*} for the above mentioned strange mass are plotted against the lighter quark masses. The lightest pion mass in the plot is $m_\pi \approx 400$ MeV (corresponding to the light-quark mass $am = 0.01$). For this pion mass, the ρ mass is found to be 912 MeV, and kaon masses on this coarse lattice are obtained as $m_K \approx 550$ MeV [PDG value 496 Mev] and $m_{K^*} \approx 1060$ MeV [PDG value 892 MeV].

5 Scalar correlator

Lattice QCD with mixed action is inevitably partially quenched and violates unitarity. The full QCD can be recovered in the continuum limit only. The scalar meson $(\bar{\psi}\psi)$ is known to be sensitive to this unitarity violation, it gives the scalar correlator a negative value when valence quark mass is less than sea quark [23]. This effect of partial quenching is shown in Fig. 7, which is observed only after the parity is restored by nonperturbative tuning of $\tilde{c}_3$. The errors become large for partially quenched scalar correlators, but its negative value when $m_{\text{val}}(= 0.005) < m_{\text{sea}}(= 0.01)$ is observable. From the plot for $\tilde{c}_3 = -0.5$, it is evident that as one increases valence quark mass, for a fixed sea quark mass, the negative contribution reduces.

6 Δ_{mix}

In mixed action calculations, we can have three different types of mesons: mesons composed of (*i*) two valence quarks, (*ii*) two sea quarks and (*iii*) a mix of one valence and one sea quark. Each of

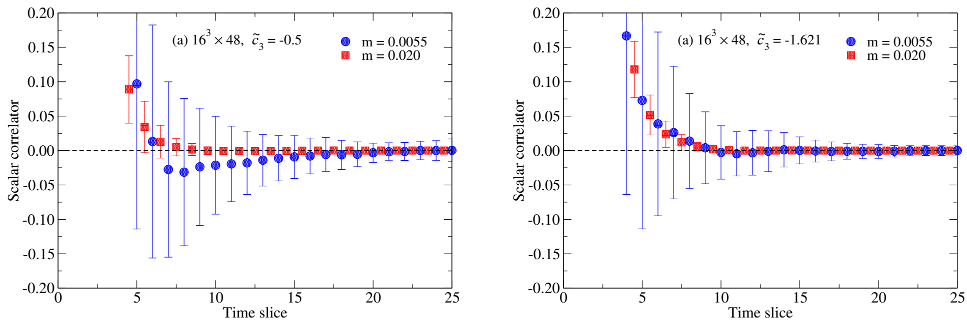

Figure 7. Scalar correlator for different values $\tilde{c}_3$ for $m_{\rm val}(= 0.005) < m_{\rm sea}(= 0.01)$. For $\tilde{c}_3$= -0.5 (nonperturbatively tuned) and -1.6211 (perturbatively calculated), we compare the correlators for $m_{\rm val} > m_{\rm sea}$.

these undergo lattice spacing dependent mass renormalization. In the leading order, the pseudoscalar meson masses for BC valence and Asqtad sea are given by

$$m^2_{v_1 v_2} = B_v(m_{v_1} + m_{v_2}) \tag{8}$$
$$m^2_{s_1 s_2,t} = B_0(m_{s_1} + m_{s_2}) + a^2\Delta_t \tag{9}$$
$$m^2_{vs} = B_v m_v + B_0 m_s + a^2\tilde{\Delta}_{\rm mix}, \tag{10}$$

where $m_{v_1 v_2}$ ($m_{s_1 s_2}$) is the pseudoscalar meson mass made up of valance (sea) quark and antiquark while m_{vs} is the mass of the mixed meson composed of valence and sea quarks. The degree of unitarity violation at finite lattice spacing depends on the size of $\Delta_{\rm mix}$. The $a^2\Delta_t$ are the taste splittings of the *staggered* pions, where $t = A, T, V, I$ and $a^2\Delta_5 = 0$ [24] and $a^2\tilde{\Delta}_{\rm mix} = a^2\Delta_{\rm mix} + a^2\Delta'_{\rm mix}$, where the primed term is given in Ref. [25]. The $\tilde{\Delta}_{\rm mix}$ can be extracted from the meson spectrum

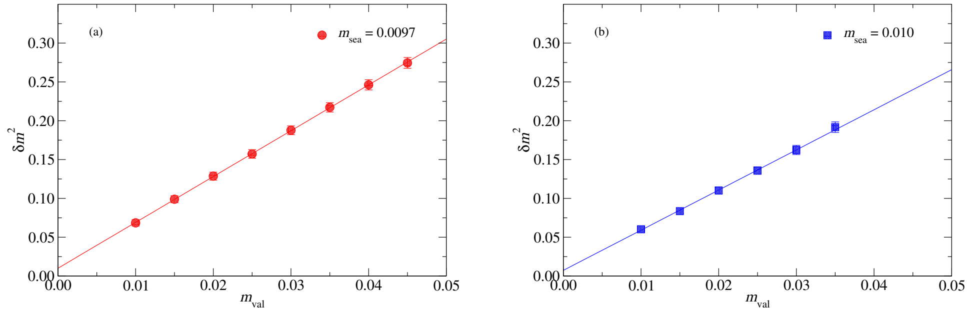

Figure 8. δm^2 plotted as a function of m_v (a) $16^3 \times 48$ lattice (b) $20^3 \times 64$ lattice

$$\delta m^2 \equiv m^2_{vs} - \frac{1}{2} m^2_{ss,5} = B_v m_v + a^2\tilde{\Delta}_{\rm mix}. \tag{11}$$

The Fig. 8 shows the result of variation of δm^2 with m_v and the linear extrapolation of δm^2 in the bare valence mass m_v gives the $a^2\tilde{\Delta}_{\rm mix}$ as the y intercept. In this work, the values of $a^2\tilde{\Delta}_{\rm mix}$ obtained are 0.03(1) GeV^4 for 0.15 fm $16^3 \times 48$ lattice and 0.03(8) GeV^4 for 0.12 fm $20^3 \times 64$ lattice.

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