$N\pi$ scattering in the Roper channel

M. Padmanath[1,★], *C. B.* Lang[2], *Luka* Leskovec[3] and *Sasa* Prelovsek[1,4,5]

[1] *Institüt für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*

[2] *Institute of Physics, University of Graz, A-8010 Graz, Austria*

[3] *Department of Physics, University of Arizona, Tucson, AZ 85721, USA*

[4] *Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia*

[5] *Jozef Stefan Institute, 1000 Ljubljana, Slovenia*

Abstract. We present results from our recent lattice QCD study of $N\pi$ scattering in the positive-parity nucleon channel, where the puzzling Roper resonance $N^*(1440)$ resides in experiment. Using a variety of hadron operators, that include qqq-like, $N\pi$ in p-wave and $N\sigma$ in s-wave, we systematically extract the excited lattice spectrum in the nucleon channel up to 1.65 GeV. Our lattice results indicate that $N\pi$ scattering in the elastic approximation alone does not describe a low-lying Roper. Coupled channel effects between $N\pi$ and $N\pi\pi$ seem to be crucial to render a low-lying Roper in experiment, reinforcing the notion that this state could be a dynamically generated resonance. After giving a brief motivation for studying the Roper channel and the relevant technical details to this study, we will discuss the results and the conclusions based on our lattice investigation and in comparison with other lattice calculations.

1 Introduction

Understanding the baryon excitations is crucial to enhance our access to strong interactions in the low energy domain. One of the multiple interesting light baryon excitations, is the $N^*(1440)$ resonance - the first excitation of nucleon, also known as Roper resonance with $(I)J^P = (1/2)1/2^+$. It has been elaborately studied and discussed in literature, but continues to be puzzling since its discovery in 1964 [1]. This resonance predominantly decays to $N\pi$, while decay modes such as $N\pi\pi$, $N\rho$ and $\Delta\pi$ are also observed collectively to a significant fraction.

The most striking feature of this resonance is that it appears lower in mass with respect to $N^*(1535)$, the lowest negative parity excitation of the nucleon with $(I)J^P = (1/2)1/2^-$. This hierarchy is in contrast with results from most phenomenological studies that assumed the $N^*(1440)$ resonance to be of three quark nature [2, 3]. A variety of phenomenological suggestions that go beyond the three quark picture followed these puzzling observations [4].

There has been multiple lattice studies of the excited nucleon spectrum as well, most of which indicated an inverted hierarchy with respect to the experiments [5–11]. Unlike other lattice investigations, Ref. [5] observed a low lying Roper resonance using fermion discretization with good chiral properties and different eigenenergy extraction techniques. The question on the importance of the role of chiral symmetry remains to be confirmed from future calculations. All these previous lattice

---

★Speaker e-mail: Padmanath.M@physik.uni-regensburg.de

calculations utilized three-quark interpolating fields (assuming $N^*(1440)$ to be stable) and identified the first excited lattice level with $N^*(1440)$. A recent lattice study that included five quark interpolating fields used strictly local $qqqq\bar{q}$ interpolators [12]. However, either of these kinds of interpolating fields could suffer from lack of coupling with strongly decaying broad resonance, such as $N^*(1440)$.

In this talk, we present the results from our recent lattice investigation of the excited nucleon spectrum within an energy region below 1.65 GeV, where the Roper resonance is observed [13]. In order to determine the complete discrete spectrum we included for the first time $N\pi$ in p-wave as well as $N\sigma$ in s-wave in order to account for their scattering, along with qqq type interpolating fields. We investigate the possible description of the Roper resonance based on our lattice results within elastic approximation of $N\pi$ scattering. In Section 2, we briefly describe the methods and technicalities in our calculation. We discuss the results in Section 3 and then we summarize.

2 Lattice methodology

We investigate the excited nucleon spectrum on an $N_f = 2 + 1$ ensemble generated by the PACS-CS collaboration with lattice extension $V = 32^3 \times 64$, physical volume $L^3 \simeq (2.9\ \text{fm})^3$ and $m_\pi = 156(7)(2)$ MeV [14]. The valence and the sea quarks are realized using non-perturbatively improved Wilson-clover fermions.

Quark smearing width and distillation : For the valence quarks sources and propagators we use full distillation [15] with two different smearing widths (narrow [n] and wide [w] using 48 and 24 eigenvectors of the discretized gauge-covariant Laplacian). This technique aids us to build all the required Wick contractions, which are quite cumbersome for this study involving as many as 113 distinct diagrams. Using two smearing widths we hope to get enhanced access to form radial nodes in the wave function, which is favourable for the Roper resonance [11].

Interpolators : We employ the following nucleon ($\mathcal{N}^i$), pion (π) and sigma meson (σ) operators in the dispersion relation studies as well as in constructing the basis of baryon and baryon-meson interpolators used in extracting the excited nucleon spectrum.

$$\mathcal{N}^i_\mu(\mathbf{n}) = \sum_{\mathbf{x}} \epsilon_{abc} [u^{aT}(\mathbf{x}, t)\Gamma^i_2 d^b(\mathbf{x}, t)] [\Gamma^i_1 q^c(\mathbf{x}, t)]_\mu \, e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}, \tag{1}$$

$$\pi^+(\mathbf{n}) = \sum_{\mathbf{x}} \bar{d}(\mathbf{x}, t)\gamma_5 u(\mathbf{x}, t) e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}}, \tag{2}$$

$$\pi^0(\mathbf{n}) = \frac{1}{\sqrt{2}} \sum_{\mathbf{x}} [\bar{d}(\mathbf{x}, t)\gamma_5 d(\mathbf{x}, t) - \bar{u}(\mathbf{x}, t)\gamma_5 u(\mathbf{x}, t)] e^{i\mathbf{x}\cdot\mathbf{n}\frac{2\pi}{L}},$$

$$\sigma(0) = \frac{1}{\sqrt{2}} \sum_{\mathbf{x}} [\bar{u}(\mathbf{x}, t)u(\mathbf{x}, t) + \bar{d}(\mathbf{x}, t)d(\mathbf{x}, t)] . \tag{3}$$

The proton field is given by $\mathcal{N}^i_\mu(\mathbf{n})|_{q=u}$, whereas the neutron is given by $\mathcal{N}^i_\mu(\mathbf{n})|_{q=d}$. With three different choices of $(\Gamma^i_1, \Gamma^i_2) = (\mathbf{1}, C\gamma_5),\ (\gamma_5, C),\ (i\mathbf{1}, C\gamma_t\gamma_4)$, we employ three nucleon fields in this study.

With these single hadron fields, we construct the following 10 baryon and baryon-meson annihilation operators with $(I)J^P = (1/2)1/2^+$ and total momentum zero.

$$O^{N\pi}_{1,2} = -\sqrt{\tfrac{1}{3}} \, [p^{1,2}_{-\frac{1}{2}}(-e_x)\pi^0(e_x) - p^{1,2}_{-\frac{1}{2}}(e_x)\pi^0(-e_x) - ip^{1,2}_{-\frac{1}{2}}(-e_y)\pi^0(e_y) + ip^{1,2}_{-\frac{1}{2}}(e_y)\pi^0(-e_y)$$

$$+ p^{1,2}_{\frac{1}{2}}(-e_z)\pi^0(e_z) - p^{1,2}_{\frac{1}{2}}(e_z)\pi^0(-e_z)] + \sqrt{\tfrac{2}{3}} \, [\{p \to n, \pi^0 \to \pi^+\}] \quad [n],$$

$$O^{N_w}_{3,4,5} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [w], \qquad O^{N_n}_{6,7,8} = p^{1,2,3}_{\frac{1}{2}}(0) \quad [n], \qquad O^{N\sigma}_{9,10} = p^{1,2}_{\frac{1}{2}}(0)\sigma(0) \quad [n]. \tag{4}$$
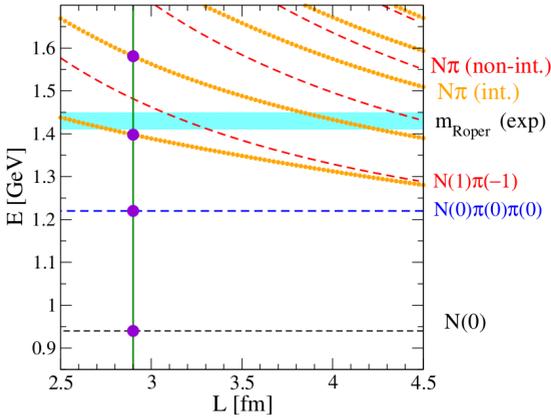
Figure 1. Lattice spectrum as a function of the lattice extension L, predicted by eqn. (7) within elastic $N\pi$ scattering. The experimental mass of the Roper (cyan band) and experimental phase shift information have been used as inputs to determine the energy shifts on the lattice. $N(0)\pi(0)\pi(0)$ (dashed blue line) and $N(0)\sigma(0)$ are assumed to be decoupled with the Roper and hence remain unaffected.

Here e_x, e_y, and e_z refer to the unit lattice momentum vectors along x, y, and z directions, n and w inside square brackets refer to narrow and wide smearing respectively and the subscripts $\pm\frac{1}{2}$ to the proton fields refer to the spin projections. The basis in eqn (4) contains all the baryon-meson interpolators with non-interacting levels below 1.65 GeV.

The σ meson is a broad resonance and can decay into 2π; $O^{N\sigma}$ is expected to couple to the $N\sigma$ as well as $N(0)\pi(0)\pi(0)$ levels. In this work, we implemented only the $O^{N\sigma}$ interpolator with the expectation that it will effectively represent a mixture of $N\sigma$ and $N(0)\pi(0)\pi(0)$.

Extracting the spectrum : Expanding the correlation matrix,

$$C_{ij}(t) = \langle\Omega|O_i(t+t_{src})\bar{O}_j(t_{src})|\Omega\rangle = \sum_n \langle\Omega|O_i|n\rangle e^{-E_n t}\langle n|\bar{O}_j|\Omega\rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t} \tag{5}$$

one can see that the large-time behavior of the eigenvalue $\lambda^{(n)}(t,t_0)$ provides E_n, where as the eigenvectors $u_i^{(n)}$ are related to the operator overlaps Z_i^n. The spectrum is determined by performing non-linear fits to the eigenvalues $\lambda^{(n)}(t)$ extracted from the generalized eigenvalue problem [16, 17] for the correlation matrices $C_{ij}(t)$

$$C_{ij}(t)u_j^{(n)} = \lambda^{(n)}(t,t_0)C_{ij}(t_0)u_j^{(n)} , \quad \lambda^{(n)}(t,t_0) \propto e^{-E_n(t-t_0)} \tag{6}$$

Extracting the resonance information : In a scenario, when there is no meson-meson and baryon-meson interactions, we expect to observe levels corresponding to $N(0)$, $N(0)\pi(0)\pi(0)$ and $N(1)\pi(-1)$ on the ensemble we use. With non-trivial attractive interaction the extracted levels get affected and thus the observed levels and their respective energies will have implications in the presence of resonances. Within the approximation of elastic $N\pi$ scattering, the interacting $N\pi$ levels (dotted yellow lines) gets shifted with respect to the respective non-interacting positions (dashed orange lines) as shown in Fig. 1. The figure predicts the lattice spectrum as a function of the lattice extension L given the experimental mass of the Roper (cyan band) and experimental phase shift information as inputs to Lüscher's relation [18, 19]

$$\delta(p) = \mathrm{atan}\left[\frac{\sqrt{\pi}pL}{2\,Z_{00}(1;(\frac{pL}{2\pi})^2)}\right], \; E_{n.i.} = E_{N(p)} + E_{\pi(p)} \tag{7}$$

where $E_{H(p)}$ is the energy of the hadron. The numerically extracted lattice spectrum from the correlation matrices we compute is compared with these analytic predictions and explored in search of a possible description of $N^*(1440)$ feature as a conventional resonance within the elastic approximation of $N\pi$ scattering.

3 Results and discussion

In Fig. 2, we present our results for the excited nucleon spectrum. These are based on correlation matrices for subsets of the operators basis ($O_1^{N\pi}$, $O_3^{N_n}$, $O_{6,8}^{N_w}$, $O_9^{N\sigma}$) as shown in the Fig. 2 legend. Basis 1 (within the red rectangle) represents the set with all five operators in the above basis, which we call the complete set containing all types of interpolators. Including the rest of the operators in (4) makes the spectrum noisier. The horizontal dashed blue and red lines denote the multi-hadron levels $m_N + 2m_\pi$ and $E_{N(1)} + E_{\pi(-1)}$ in the non-interacting limit.
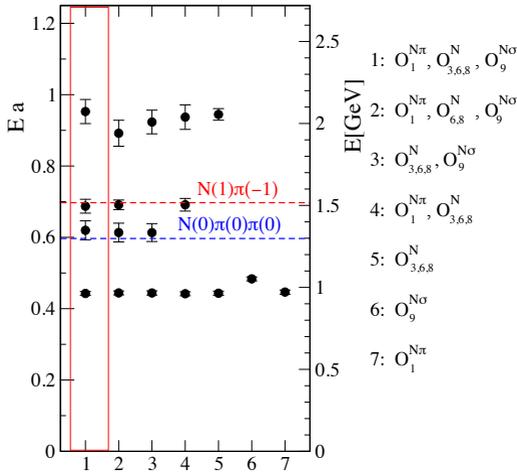

Figure 2. Lattice excitation spectrum of the nucleon for various choices of the interpolator basis. Basis 1 is referred to as the complete set, as it contains all types of interpolators and is highlighted with a red rectangle in the figure. The non-interacting scattering levels are shown as dashed horizontal lines (red for $N(1)\pi(-1)$ and blue for $N(0)\pi(0)\pi(0)$).

The main observation is that in basis 1 there are only three levels below 1.65 GeV. The ground state represents the nucleon and the remaining two levels lie close to the non-interacting positions of $m_N + 2m_\pi$ and $E_{N(1)} + E_{\pi(-1)}$. As expected the ground state representing the nucleon can be seen to be quite stable with respect to different bases used, except for basis 6. In basis 6 only $O_9^{N\sigma}$ is used and hence the extracted level could be a mixture of N, $N(0)\pi(0)\pi(0)$ and $N(0)\sigma(0)$, since the σ meson can also mix with the vacuum. The first excited state in basis 5, where only interpolators of type *qqq* are used, appears significantly above 1.65 GeV. This observation agrees with the previous lattice results, where the quark fields are realized with clover fermions, using only *qqq* type interpolators. It is also to be noted that no interpolator basis renders more than three eigenstates below 1.65 GeV. This indicates not only that the *qqq* type interpolators have weak coupling with the scattering levels, but also the first excited state with a strong *qqq* Fock component appears significantly above 1.65 GeV. Considering the closeness of the first and second excitations to the non-interacting energies of the scattering levels, we relate them with the scattering channels $N(0)\pi(0)\pi(0)$ and the $N(1)\pi(-1)$ respectively. This identification is supported by features of the spectrum for different choices of the operator basis as demonstrated in Fig. 2. Exclusion of $O^{N(0)\sigma(0)}$ from basis 1 leads to the disappearance of first excited state as can be seen in basis 4. Similarly the $N(1)\pi(-1)$ Fock component is observed to be crucial to determine the second excitation in basis 1, as can be inferred from basis 3.

Identification of the dominant nature of the levels from above procedures is further verified by their signatures in the overlap factors, Z_i^n. We present the normalized overlap factors from the basis 1 in Fig. 3. The overlap factors Z_i^n for any operator i are normalized with the largest among $|Z_i^{m=1,...5}|$. The ground state is observed to couple strongly to all interpolators. The operator $O^{N\pi}$ can be seen to overlap strongest with the second excitation, implying that the state is related to $N(1)\pi(-1)$. $O^{N\sigma}$ couples strongest to the ground state, which could be a result of the σ meson mixing with the vacuum.
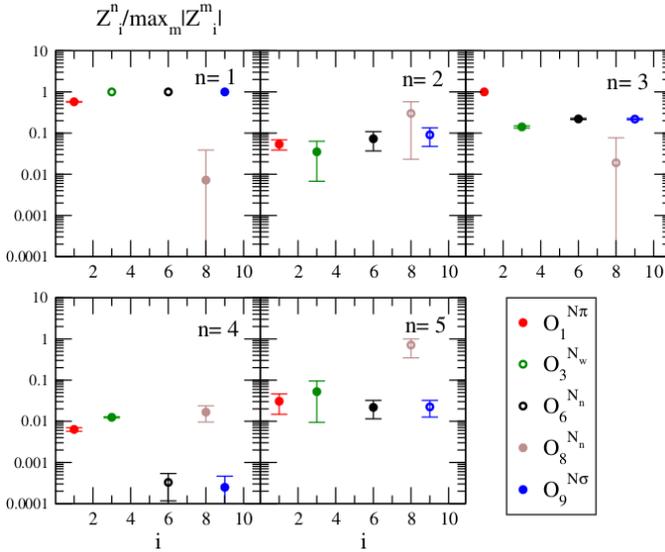
Figure 3. The normalized overlaps $\tilde{Z}_i^n = \langle\Omega|O_i|n\rangle$, extracted from the correlation matrix (5) based on the complete set (basis 1). The ratios of overlaps Z_i^n are with respect to the largest among $|Z_i^{m=1,\ldots5}|$ for a given operator i.

However, the most interesting observation in this figure is that the $O^{N\sigma}$ is found to have similar overlap with first and second excited states. Furthermore $|\langle\Omega|O^{N\sigma}|n=2\rangle| \lesssim |\langle\Omega|O^{N\sigma}|n=3\rangle|$ suggests that the scattering channels could be significantly coupled.

Extracting the phase shift information is not straightforward as the Roper channel is significantly inelastic above the $N\pi\pi$ threshold. Numerical study of three-body channels has not been performed in lattice QCD up to now, though analytic treatments exist [20, 21]. Hence we restrict ourselves to the elastic approximation of $N\pi$ scattering. Within this approximation, we determine the infinite volume phase shifts from the lattice energy levels using Lüscher's relation as in eqn. (7). The energy level related to $N(1)\pi(-1)$ in basis 1 lies on top of the non-interacting energy $E_{N(1)} + E_{\pi(1)}$, as can be seen from basis 1 in Figs. 2. Hence the energy shift of this level with respect to the non-interacting level and the corresponding infinite volume phase shift are consistent with zero within the sizable errors. This observation remains stable with respect to various choices of dispersion relation to determine the non-interacting levels, correlated or uncorrelated fits, etc. This indicates that $N\pi$ scattering in the elastic approximation alone does not render a low lying Roper resonance.

We quantify this inference further by comparing the actual lattice data we compute with analytical predictions for lattice data in Fig. 4. These analytic predictions are based on Lüscher's formulae (assuming elasticity) with experimental inputs on the masses and the phase shifts related to the Roper resonance. One can clearly see that below $E < 1.65$ GeV numerical results indicate only three eigenlevels, whereas analytic predictions indicate additional levels if the Roper can be described as a conventional resonance within the elastic $N\pi$ scattering approximation.

Absence of signatures for the Roper resonance could be due to various reasons. The Roper could be a dynamically coupled channel phenomenon involving multiple scattering channels like $\Delta\pi$, $N\rho$ and $N\pi\pi$ in the interpolator basis. Or it could be a hybrid phenomenon, where the gluonic degrees of freedom are excited [22, 23]. It could also be that our basis lack some genuine interpolators like a pentaquark operator or non-local qqq type interpolators, which could actively scan radial and orbital excitations within the qqq structure. Lastly this work, like most other lattice studies, involved quark fields that are not chiral at finite lattice spacing a. This is probably reflected in the contrasting results from Ref. [5] with respect to other lattice investigations, including ours.
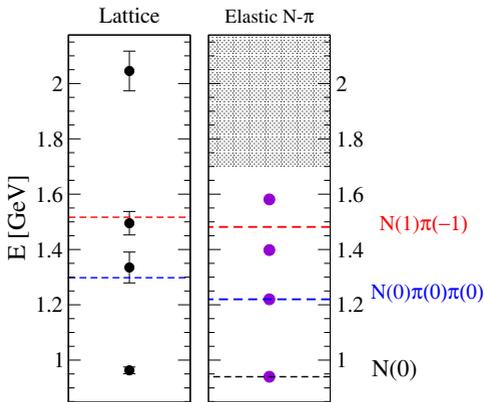
Figure 4. Energy values from our simulation (on the left pane) being compared with analytic predictions based on Lüscher's formulae (on the right pane). The relevant non-interacting scattering levels are shown as dashed horizontal lines.

In Ref. [12] a local pentaquark operator with color structure $\epsilon_{abc}\bar{q}_a[qq]_b[qq]_c$ ($[qq]_c = \epsilon_{cde}q_cq_dq_e$) was used to explore the Roper region, but a low-lying Roper state was not found. The local pentaquark operator are related to baryon-meson interpolators via Fierz relations, with the important ones being the $N(1)\pi(-1)$ and $N(0)\sigma(0)$ in the Roper region. Since we include such baryon-meson interpolators with good partial wave projection, we expect that our simulation does incorporate good localized pentaquark operators as well. However, it remains to be a challenge to include pentaquark interpolators with more complicated structures.

Interpolators of type qqq with particularly designed non-local internal structure might also be important and be better suited to scan the radial and orbital excitations rather than interpolators with varying smearing widths. Most of the studies in the past with such non-local qqq operators have also not yielded low-lying state with mass below 1.6 GeV [9]. However no work has been reported studying the Roper channel with a large number of specially-designed non-local qqq operators as well as relevant baryon-meson interpolators in the basis.

Dynamically coupled channel phenomena to describe the Roper resonance have been explored recently in the [24] using Hamiltonian Effective Field Theory (HEFT). Various scenarios involving (I) $N\pi - N\sigma - \Delta\pi$ coupled to a low-lying bare qqq type interpolator representing the Roper, (II) coupling only to $N\pi - N\sigma - \Delta\pi$ channels and (III) $N\pi - N\sigma - \Delta\pi$ coupled to a low-lying bare qqq type interpolator representing nucleon were explored using HEFT to explain the experimentally observed phase shifts. Figure 5 depicts the comparison of our lattice spectrum with the lattice spectrum predicted from the above HEFT calculation. Excluding the stars in the spectrum, which are related to the scattering levels that are not included in basis used in our work, we compare the number of expected levels (squares and circles) between the lattice and the predictions based on HEFT. Our results are compatible with the cases (II) and (III), where one can see that there are only two states in the region between 1.2 to 1.7 GeV. However, scenario (I) is disfavored by our results as it predicts three levels in this energy range. A recent follow up HEFT calculation, based on constraints from our lattice QCD data, has claimed that the Roper resonance is best described as a dynamical phenomenon through strongly coupled baryon-meson channels [25].

4 Summary

In this talk we present the results from our recent lattice QCD study of $N\pi$ scattering in the positive-parity nucleon channel. This channel is interesting for the presence of a low lying first excitation of the nucleon, called Roper resonance $N^*(1440)$. Employing a set of interpolators that includes qqq-like, $N\pi$ in *p*-wave and $N\sigma$ in *s*-wave, we systematically extract the excited nucleon spectrum up
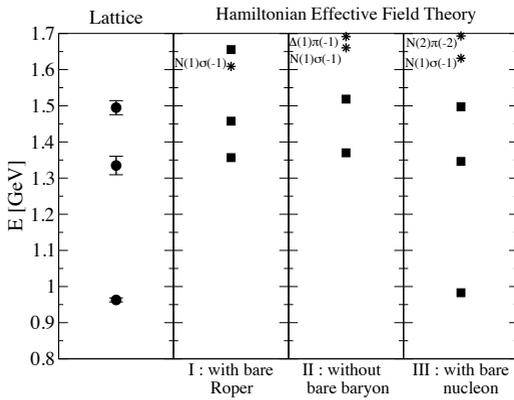
Figure 5. Comparison of our results to predictions based on HEFT [24] with different Hamiltonians. Our results are consistent with the scenarios (II) and (III), whereas (I) is disfavored, as it predicts additional levels in the energy region being investigated.

to an energy of 1.65 GeV. The extracted lattice spectrum disfavors the description of the low lying Roper resonance as a conventional resonance in the Nπ scattering within the elastic approximation. The overlap factors from our study indicate a possibility of Roper being a dynamical coupled channel phenomenon. Other possible reasons for absence of signature of Roper resonance in our calculations were also discussed.

5 Acknowledgements

References

[1] L.D. Roper, Phys. Rev. Lett. **12**, 340 (1964)

[2] N. Isgur, G. Karl, Phys. Rev. **D18**, 4187 (1978)

[3] S. Capstick, N. Isgur, Phys. Rev. **D34**, 2809 (1986), [AIP Conf. Proc.132,267(1985)]

[4] V. Crede, W. Roberts, Rept. Prog. Phys. **76**, 076301 (2013), `1302.7299` *references within*

[5] K.F. Liu, Y. Chen, M. Gong, R. Sufian, M. Sun, A. Li, PoS **LATTICE2013**, 507 (2014), `1403.6847`

[6] C. Alexandrou, T. Korzec, G. Koutsou, T. Leontiou, Phys. Rev. **D89**, 034502 (2014), `1302.4410`

[7] C. Alexandrou, T. Leontiou, C.N. Papanicolas, E. Stiliaris, Phys. Rev. **D91**, 014506 (2015), `1411.6765`

[8] G.P. Engel, C.B. Lang, D. Mohler, A. Schäfer (BGR), Phys. Rev. **D87**, 074504 (2013), `1301.4318`

[9] R.G. Edwards, J.J. Dudek, D.G. Richards, S.J. Wallace, Phys. Rev. **D84**, 074508 (2011), `1104.5152`

[10] M.S. Mahbub, W. Kamleh, D.B. Leinweber, P.J. Moran, A.G. Williams, Phys. Rev. **D87**, 094506 (2013), `1302.2987`

[11] D.S. Roberts, W. Kamleh, D.B. Leinweber, Phys. Lett. **B725**, 164 (2013), `1304.0325`

[12] A.L. Kiratidis, W. Kamleh, D.B. Leinweber, Z.W. Liu, F.M. Stokes, A.W. Thomas, Phys. Rev. **D95**, 074507 (2017), `1608.03051`

[13] C.B. Lang, L. Leskovec, M. Padmanath, S. Prelovsek, Phys. Rev. **D95**, 014510 (2017), `1610.01422`

[14] S. Aoki et al. (PACS-CS), Phys. Rev. **D79**, 034503 (2009), `0807.1661`

[15] M. Peardon, J. Bulava, J. Foley, C. Morningstar, J. Dudek, R.G. Edwards, B. Joo, H.W. Lin, D.G. Richards, K.J. Juge (Hadron Spectrum), Phys. Rev. **D80**, 054506 (2009), `0905.2160`

[16] C. Michael, Nucl. Phys. **B259**, 58 (1985)

[17] M. Lüscher, Commun. Math. Phys. **104**, 177 (1986)

[18] M. Lüscher, Nucl. Phys. **B354**, 531 (1991)

[19] M. Lüscher, Nucl. Phys. **B364**, 237 (1991)

[20] M.T. Hansen, S.R. Sharpe, Phys. Rev. **D92**, 114509 (2015), `1504.04248`

[21] M.T. Hansen, S.R. Sharpe, Phys. Rev. **D95**, 034501 (2017), `1609.04317`

[22] E. Golowich, E. Haqq, G. Karl, Phys. Rev. **D28**, 160 (1983), [Erratum: Phys. Rev.D33,859(1986)]

[23] L.S. Kisslinger, Z.P. Li, Phys. Rev. **D51**, R5986 (1995)

[24] Z.W. Liu, W. Kamleh, D.B. Leinweber, F.M. Stokes, A.W. Thomas, J.J. Wu, Phys. Rev. **D95**, 034034 (2017), `1607.04536`

[25] J.j. Wu, D.B. Leinweber, Z.w. Liu, A.W. Thomas (2017), `1703.10715`