EPJ Web of Conferences **175**, 06005 (2018) https://doi.org/10.1051/epjconf/201817506005
Lattice 2017

Electromagnetic corrections to the hadronic vacuum polarization of the photon within $\mathrm{QED_L}$ and $\mathrm{QED_M}$

Andrea Bussone[1,⋆], *Michele* Della Morte[1], and *Tadeusz* Janowski[1]

[1] *CP*[3]*-Origins, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark*

Abstract. We compute the leading QED corrections to the hadronic vacuum polarization (HVP) of the photon, relevant for the determination of leptonic anomalous magnetic moments, a_ℓ. We work in the electroquenched approximation and use dynamical QCD configurations generated by the CLS initiative with two degenerate flavors of non-perturbatively $\mathrm{O}(a)$-improved Wilson fermions. We consider $\mathrm{QED_L}$ and $\mathrm{QED_M}$ to deal with the finite-volume zero modes. We compare results for the Wilson loops with exact analytical determinations. In addition we make sure that the volumes and photon masses used in $\mathrm{QED_M}$ are such that the correct dispersion relation is reproduced by the energy levels extracted from the charged pions two-point functions. Finally we compare results for pion masses and the HVP between $\mathrm{QED_L}$ and $\mathrm{QED_M}$. For the vacuum polarization, corrections with respect to the pure QCD case, at fixed pion masses, turn out to be at the percent level.

1 Introduction

The $(g-2)_\mu$ is one of the most precise measurement in particle physics and it serves as a stringent test of the Standard Model (SM). The persistent $3-4\ \sigma$ discrepancy between the experimental value and theoretical calculation [1] has generated a lot of interests in the past years. The dominant contribution to the anomalous magnetic moment is due to QED and at the level we are today in the experiment we need to include all the possible contributions from the SM. In addition, a number of SM extensions have been recently proposed addressing the violations of lepton flavor universality observed in the measurements of R_K and $R_{K^\star}$. For example in Refs. [2, 3] is pointed out that in models with a vectorial coupling to a Z' or in some fundamental composite Higgs model the discrepancy between the $(g-2)_\mu$ measurement and the SM prediction can be alleviated, while explaining at the same time the flavor anomalies. The lattice regularization can provide a non-perturbative determination of the hadronic vacuum polarization contribution to the muon magnetic anomaly, that is the one dominating the error and represents the second most important contribution. Especially in view of the new planned experiments E989 at FNAL and E34 at J-PARC that will improve the determination of a_μ by a factor four on the experimental side. The dispersive approach to calculate the leading hadronic contribution to the muon anomaly is still the most accurate, and it obviously contains all the SM contributions. In order to make contact with it we need to take into account QED effects. As a further motivation, in Ref. [4] an alternative method to measure the hadronic contributions using experimental data employing a space-like kinematics is proposed, which allows for a direct comparison with lattice estimates.

⋆Speaker, e-mail: bussone@cp3-origins.net

Finally we also point out that the Light-by-Light (LbL) contribution to the anomaly, a_μ^{LbL}, is of the same order in the electromagnetic coupling α expansion as the hadronic leading order one with an extra insertion of a photon line, so the two terms must be considered at the same time.

In this work we present preliminary results on the electromagnetic corrections to the HVP. The 3σ discrepancy mentioned above translates in a 4% effect on a_μ^{HLO}, furthermore, the lattice estimates of such a pure QCD term have currently uncertainties around 5%. It is then natural to ask whether one can isolate a correction of order 1%, which is the expected size for a QED contribution. That is the main issue we try to address here, since such a correction can in fact almost completely resolve the theory vs experiment discrepancy. A recent work in the direction of including QED effects, as well as strong isospin breaking corrections, has been presented in Ref. [5].

In Section 2 we discuss our choices of finite-volume zero modes regularization. There we present a test of the quenched QED (qQED) configurations comparing results for Wilson loops with the infinite volume analytical calculations. After that we present preliminary results on the Q(C+qE)D pseudoscalar sector and further details on the $\mathrm{QED_M}$ in order to make sure that photon masses allow to reproduce the correct dispersion relations. In Section 3 we present our results for the EM corrections to the muon anomaly. We present a new emerging strategy that gives us a direct access to the EM correction on the scalar Vacuum Polarization and allows us to determine EM contributions to the anomaly without large systematic effects.

2 QED on the lattice

We do not discuss here all the issues with QED on the lattice, for a review on that the reader is referred to Refs. [6, 7]. Let us restrict to the case of the non-compact formulation, since it will be the one considered in the rest of the work. QED on the lattice is plagued by the well-known zero-mode problem. As a consequence of it in a periodic lattice charged states are forbidden to propagate [8].
In order to correct for that we choose to employ $\mathrm{QED_L}$ [9] and $\mathrm{QED_M}$ [10] as IR-regularizations.
$\mathrm{QED_L}$ has a positive definite Hamiltonian although the conditions to remove the spatial zero-modes result in a non-local constraint[1]. Due to the nature of QED important finite-volume effects are expected to be found, i.e. the corrections are power-like.
In $\mathrm{QED_M}$ the spatial zero-modes are regulated, as in Perturbation Theory (PT), with a Gaussian weight, and the theory has all the advantages of a local Quantum Field Theory. The introduction of a photon mass term solves as well the Gauss's law problem. Finite volume corrections in this case are expected to be exponentially suppressed with the photon mass, but an extra power-like extrapolation to vanishing mass, $m_\gamma \to 0$, has to be performed.

2.1 qQED Wilson loops

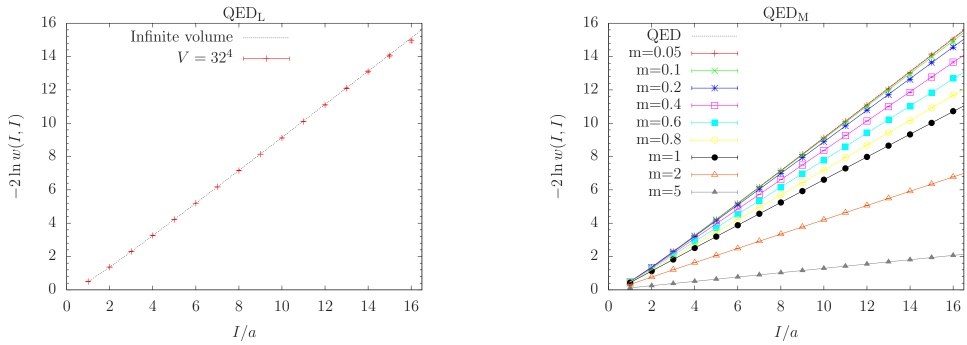
We tested the code for the generation of qQED configurations by comparing the Wilson loops expectation values in an infinite lattice and the one in the finite volume. The square Wilson loops expectation values, $w_{\mu\nu}(I, I)$, with I side length in the (μ, ν) plane, can be exactly calculated in the infinite volume, and key ingredients are given by

$$w_{\mu\nu}(I, I) = \exp\left(2e^2 Q^2 \left[C_\mu(I, 0) - C_\nu(I, I\hat{\nu})\right]\right), \tag{1}$$

$$C_\mu(I, x) = ID(x) + \sum_{\tau=1}^{I-1} (I - \tau) D(x + \tau\hat{\mu}), \tag{2}$$

[1] The potential issues and implications are presented in Ref. [6].

where e is the electric charge and Q the quark charge in units of e. The formulae assume the knowledge of the infinite volume coordinate space quark propagator, i.e. $D(x)$, and they can be applied in both massless and massive case. The first is calculated in QED_L through the Lüscher-Weisz algorithm [11], while in QED_M the Borasoy-Krebs algorithm is employed [12].
The result of the tests is shown in Fig. 1, where the logarithm of the square Wilson loops, $w(I, I)$, averaged over the directions μ, ν, are plotted against the side length for both the infinite volume prediction and a 32^4 volume.



(a) Wilson loop values in the finite lattice (red segments) and the infinite lattice predictions (dashed line).

(b) Wilson loop values in the massless limit (dashed line), massive ones (solid lines) and corresponding values in the finite lattice.

Figure 1: Comparison between the square Wilson loop values in infinite volume and the finite lattice, $V = 32^4$, for the two different IR-regularizations. The photon mass m is given in lattice units.

2.2 Q(C+qE)D pseudoscalar spectrum

In this exploratory study on the electromagnetic corrections of the HVP we work in the electro-quenched approximation and use dynamical QCD configurations generated by the CLS initiative with two degenerate flavors of non-perturbatively O(a) improved Wilson fermions [13]. The relevant parameters for the QCD ensembles are given in Tab. 1 We add to preexistent QCD configurations the

Table 1: QCD ensemble parameters and results taken from Ref. [14].

Run	L/a	β	c_{sw}	κ	κ_c	am_π	$m_\pi L$	a[fm]	m_π[MeV]
A3	32	5.20	2.01715	0.13580	0.1360546	0.1893(6)	6.0	0.079(3)(2)	473
A4	32	5.20	2.01715	0.13590	0.1360546	0.1459(6)	4.7	0.079(3)(2)	364
A5	32	5.20	2.01715	0.13594	0.1360546	0.1265(8)	4.0	0.079(3)(2)	316

qQED ones by forming a **U**(3) gauge theory with un-improved fermions. We implemented QED_L and QED_M at the physical value of electric charge with quark charges $Q_u = 2/3$ and $Q_d = -1/3$, respectively for the up and down quark.
In Tab. 1 preliminary results on the pseudoscalar masses are given. Those are calculated by plateau-fitting cosh-like effective masses. We used point-sources and the errors are calculated through a single-elimination jackknife procedure.

Table 2: Pseudoscalar masses in Q(C+qE$_L$)D (denoted by $m_\gamma = 0$) and Q(C+qE$_M$)D. The resulting pion masses go from about 380 MeV to about 640 MeV.

Run	am_γ	$m_\gamma L$	$am_{\pi^0,u\bar{u}}$	$am_{\pi^0,d\bar{d}}$	$am_{\pi^-} = am_{\pi^+}$	$N_{\rm cnf}$
A3	0	x	.2549(9)	.2071(9)	.2330(9)	312
A3	0.1	3.2	.2556(7)	.2074(8)	.2337(8)	330
A3	0.25	8.0	.2553(7)	.2072(8)	.2331(8)	330
A4	0	x	.2240(8)	.1691(9)	.1994(9)	400
A4	0.1	3.2	.2252(9)	.1699(9)	.2005(9)	380
A4	0.25	8.0	.2246(8)	.1700(10)	.1998(9)	380
A5	0	x	.2105(7)	.1526(9)	.1849(8)	501
A5	0.1	3.2	.2114(7)	.1528(9)	.1856(8)	481
A5	0.25	8.0	.2111(7)	.1531(9)	.1852(8)	481

Notice that for such chiral masses in QCD, i.e. $m_0 \simeq m_c^{\rm QCD}$, a change on the critical mass, m_c, of about 1% may translate into a change of about 100% in the quark masses[2], hence a huge change in pion masses. Most importantly the charged pion masses in the A5 Q(C+qE)D ensemble match the A3 QCD ones. This will be be useful later when computing the EM correction to the HVP. Finally we remark that finite volume and photon mass effects have been checked with PT formulae and are negligible within errors.

2.3 Further details on QED$_{\rm M}$

In order to find a suitable range of photon masses we explored different values of m_γ. It was emphasized in Ref. [6] that for "small" photon masses there could be a linear t-term in the effective energies for charged states. This term was studied and recognized in Ref. [10] and explicitly subtracted by hand. The presence of this term can be dangerous, because it may signal that the massive formulation is reducing to the so-called TL formulation[3]. In Fig. 2 we show how different choices of photon masses affect the charged pseudoscalar effective mass, for which we could not observe such a linearly rising term.
From Fig. 2 we can draw a number of conclusions:

- QED$_{\rm L}$ is consistent with QED$_{\rm M}$ in the limit of $m_\gamma \to 0$. This suggests that the potential issues present in the L formulations are not affecting spectroscopic quantities.
- For large m_γ the massive photon decouples and the results approach the QCD case.
- As we discussed previously the inclusion of QED increases the quark masses and therefore the light pseudoscalar mesons get heavier (at fixed bare parameters).

Practically for photon masses $m_\gamma \lesssim 0.05$ the charged correlators become too small and it is impossible to reliable extract effective masses. That is a consequence of the reintroduction of the zero mode. We checked that there is no difficulty in extracting the correct dispersion relation for masses $m_\gamma \gtrsim 0.05$, and that it agrees with the continuum one, see Fig. 3. Furthermore the matching between the QCD and Q(C+E)D ensembles survives when we change the particle's momentum. Finally we checked for final volume and photon mass effects and those are found to be negligible within errors.

[2]We recall that we are working with Wilson fermions.

[3]The TL formulation does not satisfy reflection positivity and therefore does not have a positive definite Hamiltonian, furthermore it is a non-local formulation.

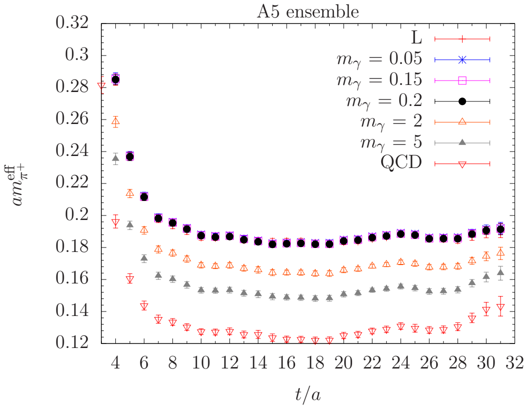
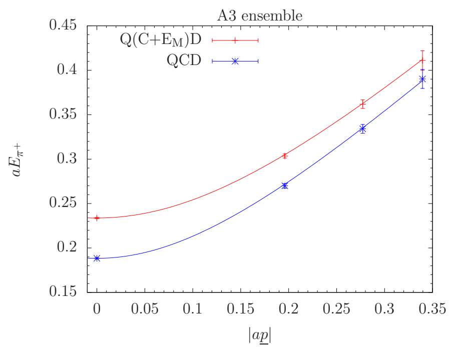
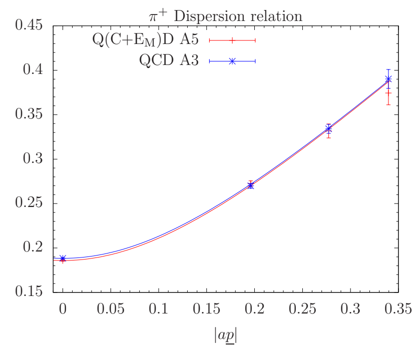


Figure 2: Effective charged pseudoscalar masses for different photon masses and comparison with QCD and Q(C+E$_\mathrm{L}$)D. Notice no t-linear term is present for the values of m_γ explored.



(a) Dispersion relation for the lightest charged pseudoscalar meson in the A3 ensemble.



(b) Dispersion relations for the lightest charged pseudoscalar meson in the two matched simulations.

Figure 3: Dispersion relation for pseudoscalar states with $m_\gamma = 0.1$. The solid lines represent the expectation from the continuum dispersion relation.

3 Electromagnetic corrections to the muon anomaly

The HVP tensor is given by

$$\Pi_{\mu\nu}(q) = \int \mathrm{d}^4x\, \mathrm{e}^{iq\cdot x}\langle V_\mu(x)V_\nu(0)\rangle, \tag{3}$$

where $V_\mu(x)$ is the quark electromagnetic current. Such a current is conserved in two-flavor Q(C+E)D as it is obtained from a combination of $\mathbf{1}$ and τ^3 matrices in flavor space, i.e.

$$V_\mu(x) = \overline{\Psi}(x)\gamma_\mu\left[\frac{Q_u}{2}\left(\mathbf{1}+\tau^3\right)+\frac{Q_d}{2}\left(\mathbf{1}-\tau^3\right)\right]\Psi(x), \tag{4}$$

where Ψ represents the flavor doublet $(u\,,d)^\mathrm{T}$. The corresponding vector transformations are indeed preserved by electromagnetic interactions. On the lattice we use the one-point-split current, that is exactly conserved, therefore the normalization constant Z_V is one.

3.1 Hadronic Vacuum Polarization

For the calculation of the HVP we neglect quark-disconnected diagrams, which are Zweig suppressed, and we recall that we are working in the electroquenched approximation. Point sources are used throughout this preliminary study. We extract the scalar HVP from both the diagonal components ($\mu = \nu$), after we have taken into account the contact term, and the non-diagonal ones ($\mu \neq \nu$). We also employ the Zero Mode Subtraction (ZMS) modification of the tensor [15]. Results are shown in Fig. 4 where we plot the unsubtracted and subtracted scalar HVP as a function of $r_0^2 \hat{q}^2$, with r_0, from Ref. [13], being the Sommer parameter and $\hat{q}_\mu = 2\sin(q_\mu/2)$. Notice that r_0/a as any other gluonic scale does not receive QED corrections in the quenched approximation.
In Fig. 4a we see good agreement between the Q(C+E$_\mathrm{M}$)D and Q(C+E$_\mathrm{L}$)D results. Again, we interpret this as an indication that the L formulation, despite a number of theoretical issues, seems to provide a valid IR-regularization for the HVP.

3.2 A new strategy

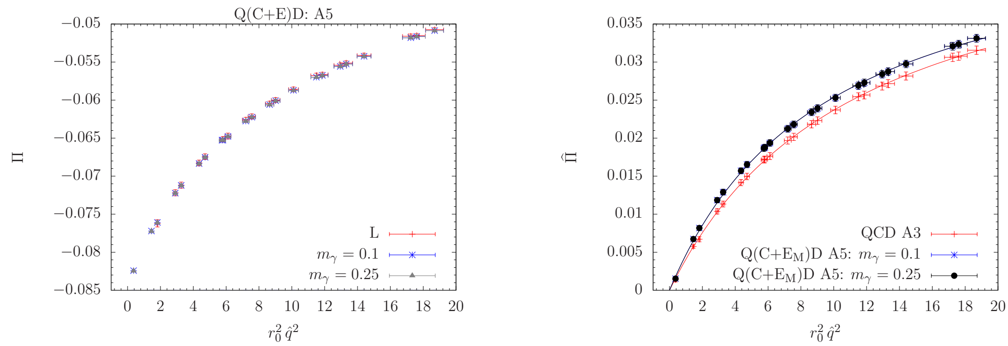
Ideally we would like to compare the HVP with and without electromagnetic effects. The two HVPs will be different functions of the renormalized parameters, and to have a meaningful comparison we need to consider them at the same renormalized parameters values. One way to go could be to rescale the bare values for the change in the quark mass and the strong coupling (reflected in the change of the lattice spacing), when considering the electromagnetic corrections to the HVP. We expect all those changes to be at the percent level. In this preliminary study we neglect the shift in the absolute scale. In Ref. [16] an estimate is provided, implying that a change of 1% on the lattice spacing is reflected in a $\simeq 1.5\%$ change in $a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 f(q^2, m_\mu^2)\hat{\Pi}(q^2)$, (see e.g., Ref. [14] for the definitions of $f(q^2, m_\mu^2)$ and $\hat{\Pi}(q^2)$). A naïve strategy to compute the electromagnetic effects on the muon anomaly would be:

- fit the scalar HVP in QCD and Q(C+E)D,
- compute separately the muon anomaly a_μ in the two different theories,
- take the resulting difference after the extrapolation to infinite volume, physical point and continuum.

This procedure is quite inefficient, since the EM effects can be easily washed out by the various systematics, e.g. Padé fit.
Our strategy makes use of the matching of the charged pion masses (traded for the quark masses) in the ensembles with QCD and Q(C+E)D. In Fig. 4b we present the comparison between the QCD result and Q(C+E$_\mathrm{M}$)D one, that gives direct access to the electromagnetic effects.
Our strategy consists in:

- take the difference of the subtracted scalar HVPs, i.e. $\delta\widehat{\Pi} \equiv \widehat{\Pi}^{\mathrm{Q(C+E)D}} - \widehat{\Pi}^{\mathrm{QCD}}$, at fixed pion mass,
- fit $\delta\widehat{\Pi}$ and plug it in the formula for the anomaly $a_\mu^\delta \propto \int dq^2 f(q^2, m_\mu^2)\ \delta\hat{\Pi}(q^2)$. Notice that the change in the lattice spacing between QCD and Q(C+E)D would be relevant here, and once that is considered the expression above is strictly speaking no-longer valid, as one should consider two different f-kernels when converting the muon mass to lattice units.
- finally extrapolate to infinite volume, physical point and continuum limit.

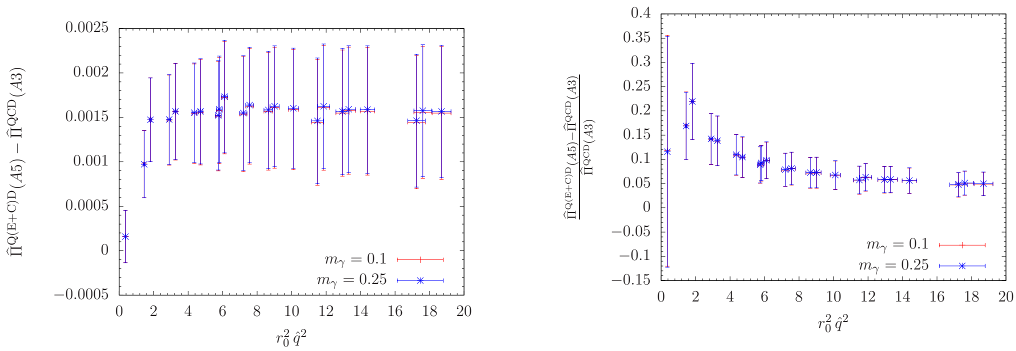
This consists of a one-fewer-fit procedure compared to the first strategy discussed and the systematics are reduced, as one can see in Fig. 5. The signal is quite clear, even though compatible with zero within two σs. The slowly varying relative EM effect on the HVP could in fact be fitted by a constant around 7%. In view of that, in order to get an idea about the size of the EM corrections to a_μ^{HLO},



(a) Unsubtracted HVP in Q(C+qE)D in A5 ensemble.

(b) Subtracted HVP with and without $\mathrm{QED_M}$.

Figure 4: HVP as a function of $r_0^2\hat{q}^2$.



(a) Difference of the HVP with and without $\mathrm{QED_M}$.

(b) Relative difference of the QCD and Q(C+E$_\mathrm{M}$)D HVP.

Figure 5: The EM contribution to the scalar VP $\delta\widehat{\Pi}$ as a function of $r_0^2\hat{q}^2$.

we decided not to perform any fit and simply integrate numerically (using the trapezoidal rule) the function $f(q^2, m_\mu^2)\ \delta\hat{\Pi}(q^2)$ in dq^2 up to $r_0\hat{q}^2 \simeq 20$. We obtain the result

$$a_\mu^\delta \times 10^{10} = 21 \pm 9_{\mathrm{stat}}\,, \tag{5}$$

where the error is statistical only. The effect seems to be of the same size as the theo-exp discrepancy. There are still important sytematic effects to be quantified though, as the change in the lattice spacing and in the pion masses.

4 Conclusions

We presented a new way to isolate electromagnetic effects for the hadronic contribution to $(g-2)_\mu$. We added quenched QED configurations to preexistent QCD ones. We considered two different regularizations of the finite volume zero modes. We were able to see a clear effect, even for physical quark charges and electromagnetic coupling. The crucial question to be answered in the future is whether the effect we have seen is going to be larger for smaller pion masses or not. Within the limitations of the computation (single lattice spacing, pion mass around 400 MeV), we saw an effect of the same

size as the discrepancy in a_μ between theory and experiments.
In order to reduce the systematics, we are planning to analyze different volumes, pion masses and lattice spacings, already available within the CLS initiative. Other improvements can be achieved by considering the addition of a clover term for the electromagnetic part of the action, as well as improving the vector current, in order to have better control on the continuum extrapolation. We may also have to consider reweighting in the bare gauge coupling in order to match lattice spacings (between QCD and Q(C+E)D) and isolate electromagnetic corrections following the approach we described.

The main extension in order to properly assess isospin breaking corrections remains however the inclusion of the up-down quarks mass splitting (strong isospin breaking). Work in that direction is progressing.

Acknowledgements. We wish to thank Agostino Patella for useful discussions and Georg von Hippel for help in accessing the CLS configurations. This work was supported by the Danish National Research Foundation DNRF:90 grant and by a Lundbeck Foundation Fellowship grant. The computing facilities were provided by the Danish Centre for Scientific Computing and the DeIC national HPC center at SDU.

References

[1] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016) and 2017 update.

[2] S. Di Chiara, A. Fowlie, S. Fraser, C. Marzo, L. Marzola, M. Raidal, C. Spethmann, Nucl. Phys. **B923**, 245 (2017), arXiv:1704.06200.

[3] G. D'Amico, M. Nardecchia, P. Panci, F. Sannino, A. Strumia, R. Torre, A. Urbano, JHEP **09**, 010 (2017), 1704.05438

[4] C.M. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni, Phys. Lett. **B746**, 325 (2015), arXiv:1504.02228.

[5] P. Boyle, V. Gülpers, J. Harrison, A. Jüttner, C. Lehner, A. Portelli, C.T. Sachrajda (2017), arXiv:1706.05293.

[6] A. Patella, PoS **LATTICE2016**, 020 (2017), arXiv:1702.03857.

[7] A. Portelli, PoS **KAON13**, 023 (2013), arXiv:1307.6056.

[8] M. Hayakawa, S. Uno, Prog. Theor. Phys. **120**, 413 (2008), arXiv:0804.2044.

[9] S. Borsanyi et al., Science **347**, 1452 (2015), arXiv:1406.4088.

[10] M.G. Endres, A. Shindler, B.C. Tiburzi, A. Walker-Loud, Phys. Rev. Lett. **117**, 072002 (2016), arXiv:1507.08916.

[11] M. Luscher, P. Weisz, Nucl. Phys. **B445**, 429 (1995), hep-lat/9502017.

[12] B. Borasoy, H. Krebs, Phys. Rev. **D72**, 056003 (2005), hep-lat/0508005.

[13] P. Fritzsch, F. Knechtli, B. Leder, M. Marinkovic, S. Schaefer, R. Sommer, F. Virotta, Nucl. Phys. **B865**, 397 (2012), arXiv:1205.5380

[14] M. Della Morte, B. Jager, A. Juttner and H. Wittig, JHEP **1203** (2012) 055 arXiv:1112.2894.

[15] D. Bernecker, H.B. Meyer, Eur. Phys. J. **A47**, 148 (2011), arXiv:1107.4388.

[16] M. Della Morte et al., JHEP **1710** (2017) 020, arXiv:1705.01775.