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Critical endline of the finite temperature phase transition for 2+1 flavor QCD away from the SU(3)-flavor symmetric point

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Abstract. We investigate the critical end line of the finite temperature phase transition of QCD away from the SU(3)-flavor symmetric point at zero chemical potential. We employ the renormalization-group improved Iwasaki gauge action and non-perturbatively $O(a)$-improved Wilson-clover fermion action. The critical end line is determined by using the intersection point of kurtosis, employing the multi-parameter, multi-ensemble reweighting method at the temporal size $N_T = 6$ and lattice spacing as low as $a \approx 0.19$ fm.

1 Introduction

The nature of the finite temperature phase transition of 2+1 flavor QCD at zero chemical potential depends on quark masses. The order of transition and universality class are summarized in the plane of light quark mass, m_l and strange quark mass, m_s, and it is called the Columbia plot [1].

The first order phase transition is expected in the small quark mass region [2]. It is well known that the phase transition is also the first order in the heavy quark mass region and it is crossover in the medium quark mass region by many lattice QCD studies. The boundary between the first order region and crossover region is the second order phase transition of Z_2 universality class.

The nature in the lower-left corner of the Columbia plot has not been fully understood yet. The first lattice QCD calculation was done by using standard Wilson fermions at $N_T = 4$ roughly 20 years ago. It reported the critical mass at the critical endpoint (CEP), m_E, for $N_F = 3$ is heavy , the critical quark mass $m_{q,E} = m_{l,E} = m_{s,E} \gtrsim 140$ MeV or, equivalently, the critical pseudo scalar mass $m_{PS,E} = m_{\pi,E} = m_{\eta_s,E} \gtrsim 1$ GeV [3]. After preliminary study with standard Wilson gauge and staggered fermions which reported the bare critical mass $am_{q,E} \sim 0.035$ [4] at $N_T = 4$, Karsch *et al.* reported preliminary values for the critical mass, $m_{PS,E} \sim 290$ MeV with unimproved gauge and staggered fermion actions and $m_{PS,E} \sim 190$ MeV with improved gauge and staggered fermion actions (p4-action) [5]. These results were obtained by using R-algorithm [6]. Afterward, the results were updated as $m_{PS,E} = 290(20)$ MeV with unimproved gauge and staggered fermion actions and $m_{PS,E} = 67(17)$ MeV with improved gauge and staggered fermion actions (p4-action) [7]. Then, in

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ref. [8], de Forcrand and Philipsen obtained $am_{\mathrm{q,E}} = 0.0260(5)$ by using the RHMC algorithm [9, 10], which is about 25% smaller than the value $am_{\mathrm{q,E}} \approx 0.033$ quoted by works using the R-algorithm. They also performed $N_{\mathrm{F}} = 2+1$ simulations and non zero chemical potential simulations and obtained the critical line and tri-critical point, $am_{\mathrm{s}}^{\mathrm{tri}} \approx 0.7$, where lattice spacing a was approximately 0.3 fm. In ref. [11] with unimproved staggered fermions, is was reported that the ration of $m_{\mathrm{PS,E}}$ and the CEP temperature T_{E} decreased from 1.680(4) to 0.954(12) as increasing N_{T} from 4 to 6. These results are showing very large cut off effect for the critical mass and it is important to push N_{T} and use improved action. Further studies with improved staggered fermions have not found the first order phase transition and quoted only the bound of the critical mass, $m_{\mathrm{PS,E}} \lesssim 50$ MeV, [12–14]. Therefore, the position of the critical endline (CEL), $m_{\mathrm{s}}^{\mathrm{tri}}$ and CEP for $N_{\mathrm{F}} = 3$ is still particularly important problem to be solved at this moment.

Recently we also have investigated the nature of the finite phase transition in small quark mass region by using non-perturbatively $O(a)$-improved Wilson-clover fermion fermions. We have determined CEP at $N_{\mathrm{T}} = 4, 6, 8, 10$ and upper bound of CEP in the continuum limit for $N_{\mathrm{F}} = 3$ [15, 16]. For $N_{\mathrm{F}} = 2 + 1$, we have studied at $N_{\mathrm{T}} = 6$ and determined CEL around the SU(3) flavor symmetric point and confirmed that the slope of CEL at the SU(3) flavor symmetric point is -2 [17]. In this paper, we extend our study for CEL away form the SU(3) flavor symmetric point.

Table 1. Simulation parameters at $\kappa_{\mathrm{s}} = 0.128000$ (very large m_s runs).

		# of conf.		
β	κ_{l}	$N_{\mathrm{S}} = 10$	$N_{\mathrm{S}} = 12$	$N_{\mathrm{S}} = 16$
1.720	0.143788	5060		
1.730	0.143365	5150		
1.730	0.143380			11982
1.730	0.143390	5007		9457
1.730	0.143410		14202	
1.740	0.142955			14432
1.740	0.142970	6590	23367	
1.740	0.142990	7000		
1.740	0.143012	1670		
1.740	0.143042	1580		
1.745	0.142733	3480		
1.745	0.142755			9574
1.745	0.142760	9950	14341	
1.745	0.142767			9924
1.745	0.142770	5900		
1.745	0.142780		5050	
1.745	0.142790		2940	

2 Simulations

We employ the renormalization-group improved Iwasaki gauge action [18] and non-perturbatively $O(a)$-improved Wilson-clover fermion action [19]. CEP is determined by using the intersection

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Table 2. Simulation parameters at $\kappa_s = 0.132800$ (large m_s runs).

		# of conf.		
β	κ_l	$N_S = 10$	$N_S = 12$	$N_S = 16$
1.720	0.143160		6553	5464
1.730	0.142702	9290		
1.730	0.142750		12011	6290

point of kurtosis of chiral condensate. This method is expounded in Ref [15] and used in our recent studies [15–17]. Expectation value, susceptibility and skewness of chiral condensate are used for confirming phase transition and determination of the transition point. Chiral condensate and its higher moments are computed from traces of inverse Wilson clover Dirac operator up to power of −4, $\mathrm{Tr}D^{-1,-2.-3,-4}$, by using 10 noise vectors. We have checked that 10 noises are good enough for some parameter sets. We employ the multi-parameter, multi-ensemble reweighting method [20] to determine CEP very small statistical error. We reweight both κ_l and κ_s, so that we can determine many CEP without doing simulations at many parameter sets. Our simulations are performed at the temporal size $N_T = 6$ and lattice spacing $a \approx 0.19$ fm. The spatial size N_S is $10, 12, 16, 24$. We have confirmed $m_{PS}L > 4$ at almost all transition points, where m_{PS} is pseudo scalar mass and L is physical spatial extent. Simulation parameters are shown in Table 1 and Table 2. Configurations around symmetric point ($\kappa_l = \kappa_s$) were generated in previous study [17].

We also performed O(100) zero temperature runs at $\beta = 1.72, 1.73, 1.74$ for physical scale setting which are covering almost all transition points of finite temperature simulations.

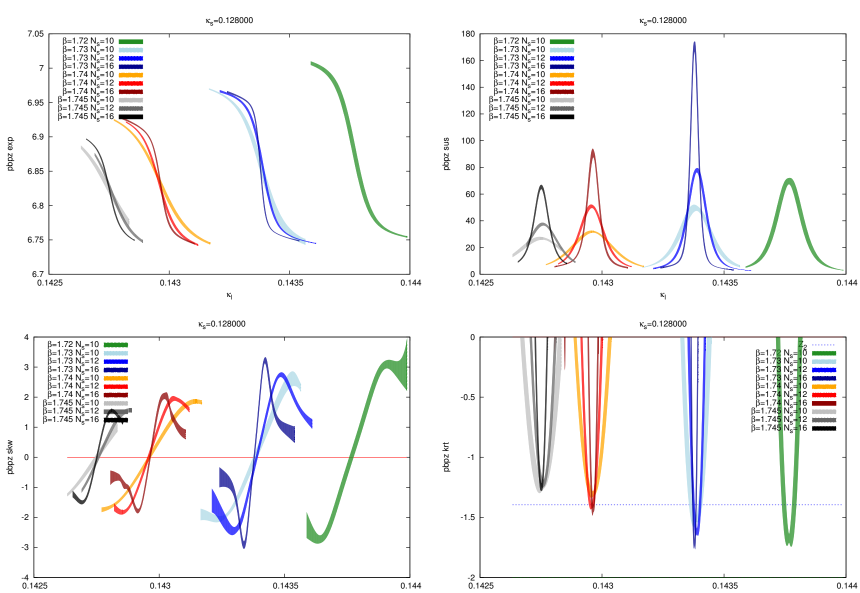

Figure 1. Expectation, susceptibility, skewness and kurtosis as a function of κ_l at $\kappa_s = 0.128000$.

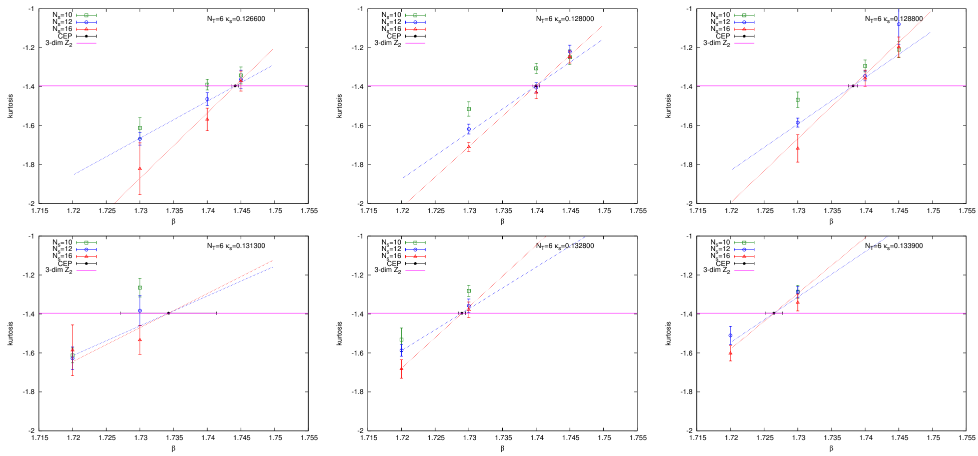

Figure 2. Kurtosis intersection as a function of β.

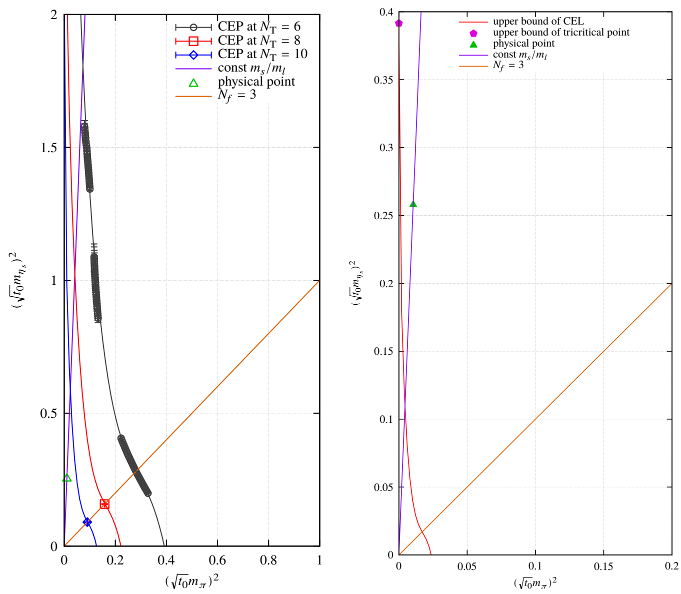

Figure 3. Preliminary results for CEL at $N_{\mathrm{T}} = 6, 8, 10$ (left), and in the continuum limit (right).

3 Simulation results

We show expectation value, susceptibility, skewness and kurtosis of chiral condensate at $\kappa_{\mathrm{s}} = 0.128000$ as example in Fig. 1. It shows that the re-weighting method works well and we can find the phase transition precisely. Kurtosis intersection plots are shown in Fig. 2. We determine CEP by assuming Z_2 universality class.

Fig. 3 shows CEP at $N_\mathrm{T} = 6$ together with preliminary results for CEL at $N_\mathrm{T} = 6, 8, 10$ and its continuum extrapolation in dimensionless physical scale plane. Here $\sqrt{t_0}$ is Wilson flow lattice scale [21]. This plan corresponds light quark and strange quark mass plane. We see that CEL increase rapidly as decreasing light quark mass. We estimate CEL by fitting data with

$$\begin{aligned} &\text{Fitting 1} : y = a_0 + a_1 x^{2/5} + a_2 x^2 + a_3 x^3 \,, \\ &\text{Fitting 2} : y = b_0 + b_1 x^{2/5} \,, \end{aligned} \tag{1}$$

where $x = (\sqrt{t_0} m_{\pi,\mathrm{E}})^2$ and $y = (\sqrt{t_0} m_{\eta_s,\mathrm{E}})^2$. These fitting functions are inspired by Rajagopal prediction [22]

$$m_\mathrm{s} - m_\mathrm{s}^\mathrm{tri} \sim m_\mathrm{l}^{2/5} \,. \tag{2}$$

We obtain good fitting results, $\chi^2/\mathrm{dof} = 2.32$ for fitting 1 and $\chi^2/\mathrm{dof} = 2.02$ for fitting 2 in range $x < 0.133$. Results of fitting 1 are shown in Fig. 3. Fitting 2 would give a scaling region of $m_\mathrm{l}^{2/5}$. By changing fitting range we could discuss scaling region qualitatively. Since χ^2/dof is very reasonable, here we just quote tentative scaling region at $N_\mathrm{T} = 6$, $m^2_{\eta_s,\mathrm{E}} > 6.45 m^2_{\pi,\mathrm{E}}$. This value is roughly four times smaller than the ratio of the physical strange and light quark mass.

Further estimation is possible by using our resent results for $N_\mathrm{F} = 3$ and assuming that there is no N_T dependence in the shape of CEL. The normalized pseudo scalar mass at CEP and SU(3) flavor symmetric point for each N_T, $\sqrt{t_0} m^\mathrm{sym}_{\mathrm{PS,E},N_\mathrm{T}}$, are

$$\begin{aligned} &\sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,6} = 0.5282(12) \,, \quad \sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,8} = 0.3977(19) \,, \\ &\sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,10} = 0.3006(19) \,, \quad \sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,\infty} < 0.1281(61) \,, \end{aligned} \tag{3}$$

where result in the continuum limit (at $N_\mathrm{T} = \infty$) is upper bound. For example, by scaling $\sqrt{t_0} m_{\pi,\mathrm{E}}$ and $\sqrt{t_0} m_{\eta_s,\mathrm{E}}$ by the ratio of $\sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,8}$ and $\sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E,6}$, we obtain CEL at $N_\mathrm{T} = 8$. Note that, we are ignoring lattice artifact in the estimation of CEL except for $N_\mathrm{T} = 6$ and CEL in the continuum limit of right plot of Fig. 3 is also upper bound since we have only determined upper bound of $\sqrt{t_0} m^\mathrm{sym}_\mathrm{PS,E}$ in the continuum limit. With these caveat, we find $m_\mathrm{s}^\mathrm{tri} < 1.52\, m_\mathrm{s}^\mathrm{phy}$.

4 Summary

We have determined CEL away from the SU(3)-flavor symmetric point at $N_\mathrm{T} = 6$ with non-perturbatively $O(a)$-improved Wilson fermions and presented preliminary results for CEL at $N_\mathrm{T} = 8, 10$ and in the continuum limit We find that 3 series of multi-ensemble, multi-parameter re-weighting well determines CEL, and CEL at $N_\mathrm{T} = 6$ is nice agreement with $m_\mathrm{s} - m_\mathrm{s}^\mathrm{tri} \sim m_\mathrm{l}^{2/5}$ in small m_l region, and $m_\mathrm{s}^\mathrm{tri} < 1.52\, m_\mathrm{s}^\mathrm{phy}$ as very preliminary result. We plan larger N_T simulations for the continuum limit.

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