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Degeneracy of vector-channel spatial correlators in high temperature QCD

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Abstract. We study spatial isovector meson correlators in $N_f = 2$ QCD with dynamical domain-wall fermions on $32^3 \times 8$ lattices at temperatures up to 380 MeV with various quark masses. We measure the correlators of spin-one isovector operators including vector, axial-vector, tensor and axial-tensor. At temperatures above T_c we observe an approximate degeneracy of the correlators in these channels, which is unexpected because some of them are not related under $SU(2)_L \times SU(2)_R$ nor $U(1)_A$ symmetries. The observed approximate degeneracy suggests emergent $SU(2)_{CS}$ (chiral-spin) and SU(4) symmetries at high T.

1 Introduction

The behaviour of strongly interacting matter at high temperatures and its description in the phase diagram of QCD are important questions for a better understanding of the theory as well as for any kind of future applications. It is a priori not clear how two main features of QCD, confinement and chiral symmetry breaking, will behave in regions of higher temperature and/or density. For the latter it has been established that above a critial temperature QCD becomes chirally symmetric. In this regime the effects of the anomalous breaking of the $U(1)_A$ symmetry are suppressed. There is strong evidence from lattice calculations using chirally-invariant fermions that the $U(1)_A$ symmetry gets restored along with a gap opening in the eigenmode spectrum of the Dirac operator [1–3].

In this work we calculate spatial correlation functions of quark bilinears to probe their symmetry structure. The operators of interest are all possible local isovectors with spin $J = 0$ and $J = 1$. As fermion discretization we use the chirally invariant domain-wall formalism, and calculate the correlations at four temperatures from $\sim 1.2T_c$ to $\sim 2.2T_c$. The motivation for this is two-fold:

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β	$m_{ud}a$	a [fm]	# configs	L_s	T [MeV]	T/T_c
4.10	0.001	0.113	800	24	~ 220	~ 1.2
4.18	0.001	0.096	230	12	~ 260	~ 1.5
4.30	0.001	0.075	260	12	~ 320	~ 1.8
4.37	0.005	0.065	120	12	~ 380	~ 2.2

Table 1. Ensembles for $32^3 \times 8$ lattices used in this work. L_s is the length of the fifth dimension in the domain wall fermion formulation. The critical temperature for this set of parameters is $T_c = 175 \pm 5$ MeV. The mass range of degenerate up and down quarks m_{ud} is 2–15 MeV.

- In the high temperature, chirally symmetric regime correlation functions connected by $SU(2)_L \times SU(2)_R$ transformations are expected to become degenerate. The effects of $U(1)_A$ breaking are expected to be suppressed. This regime eventually becomes $U(2)_L \times U(2)_R$ symmetric [4].
- In a series of numerical studies at $T = 0$, where near-zero modes of the Dirac operator have been truncated from the quark propagators, extended $SU(2)_{CS}$ (chiral-spin) and SU(4) symmetries [5, 6] have been observed in the hadron spectrum [7–10]. The low eigenmodes of the Dirac operator are strongly suppressed at high temperatures [1, 2], which raises the question whether these symmetries also occur naturally in the high T spectrum.

2 Lattices and operators

The configurations are generated using the Symanzik action for the gauge sector and the Möbius domain wall formalism as fermion discretization [11, 12]. The length of the fifth dimension for the domain wall fermions is chosen separately for each ensemble in a manner which guarantees good chiral properties at moderate cost. The gauge links are stout smeared three times before the computation of the Dirac operator; the boundary conditions for quarks are set antiperiodic in t−direction and periodic in spatial directions. The ensembles and parameters including the lattice spacing a are listed in Table 1, see also [2, 13].

As observables we use correlators of local isovector operators $O_\Gamma(x) = \bar{q}(x)\Gamma\frac{\vec{\tau}}{2}q(x)$, where Γ might be any element of the Clifford algebra; τ_a are the isospin Pauli matrices. We study spatial correlators in z-direction, as suggested in ref. [14]. Therefore we use a zero-momentum projection by summing over all lattice points in slices orthogonal to the measurement direction:

$$C_\Gamma(n_z) = \sum_{n_x,n_y,n_t} \langle O_\Gamma(n_x, n_y, n_z, n_t) O_\Gamma(\mathbf{0}, 0)^\dagger \rangle \,. \tag{1}$$

Thus we can identify the propagating components of the Clifford algebra and organize them to spin $J = 0$ and $J = 1$ operators: The Pseudoscalar (PS) is given by $\Gamma = \gamma_5$, and the Scalar (S) operator by $\Gamma = \mathbb{1}$. For the Vector and Axial-vector operators Γ has the following components:

$$\mathbf{V} = \begin{pmatrix} \gamma_1 = V_x \\ \gamma_2 = V_y \\ \gamma_4 = V_t \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} \gamma_1\gamma_5 = A_x \\ \gamma_2\gamma_5 = A_y \\ \gamma_4\gamma_5 = A_t \end{pmatrix}. \tag{2}$$

Conservation of the vector current requires that V_z does not propagate in z-direction. As the axial vector current j_5^μ is not conserved at zero temperature, the relevant component $\gamma_3\gamma_5$ of the Axial-vector does propagate at zero temperature and eventually couples to the Pseudoscalar. Above the

Name	Dirac structure	Abbreviation	
Pseudoscalar	γ_5	PS	$\}$ U(1)$_A$
Scalar	$\mathbb{1}$	S	
Axial-vector	$\gamma_k\gamma_5$	**A**	$\}$ SU(2)$_A$
Vector	γ_k	**V**	
Tensor-vector	$\gamma_k\gamma_3$	**T**	$\}$ U(1)$_A$
Axial-tensor-vector	$\gamma_k\gamma_3\gamma_5$	**X**	

Table 2. Bilinear operators considered in this work and their transformation properties (last column). This classification assumes propagation in z-direction. The open vector index k denotes the components $1, 2, 4$, *i.e.* x, y, t.

critical temperature — after U(1)$_A$ and SU(2)$_L\times$SU(2)$_R$ restoration — A_z behaves as its parity partner V_z and does not propagate in z-direction. For propagation in z-direction the tensor elements $\sigma_{\mu\nu}$ of the Clifford algebra are organized in the following way in components of Tensor- and Axial-tensor operators:

$$\mathbf{T} = \begin{pmatrix} \gamma_1\gamma_3 = T_x \\ \gamma_2\gamma_3 = T_y \\ \gamma_4\gamma_3 = T_t \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} \gamma_1\gamma_3\gamma_5 = X_x \\ \gamma_2\gamma_3\gamma_5 = X_y \\ \gamma_4\gamma_3\gamma_5 = X_t \end{pmatrix}. \tag{3}$$

Table 2 summarizes our operators and gives the U(1)$_A$ and SU(2)$_L\times$ SU(2)$_R$ relations of these operators. Given restoration of the U(1)$_A$ and SU(2)$_L\times$SU(2)$_R$ symmetries at high T we expect degeneracies of correlators calculated with the corresponding operators.

For measurements at zero temperature the three components of the vectors give the same expectation value due to the SO(3) symmetry in continuum. On the finite temperature lattices the corresponding symmetry group is D_{4h} where the Vector has one two-dimensional (V_x,V_y) and one one-dimensional (V_t) irreducible representation, and similar for **A**, **T**, **X**. Thus only x- and t-components are shown subsequently for the relevant operators.

3 Results

Figure 1 shows the spatial correlation functions normalized to 1 at $n_z = 1$ for the operators given in Table 2. As argument we show n_z which is proportional to the dimensionless product zT for fixed N_t, the temporal extent of the lattice.

As we describe in more detail later, we find that all correlators connected by SU(2)$_L\times$ SU(2)$_R$ and U(1)$_A$ transformations coincide within small deviations at $T > 220$ MeV, which means that at these temperatures both chiral symmetries get restored. More interestingly, there are additional degeneracies of correlators. In total we observe three different multiplets:

$$E_1: \quad PS \leftrightarrow S \tag{4}$$
$$E_2: \quad V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x \tag{5}$$
$$E_3: \quad V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t. \tag{6}$$

E_1 describes the Pseudoscalar-Scalar multiplet connected by the U(1)$_A$ symmetry, that is realized in the absence of chiral zero-modes [15]. Note that we only consider the isospin triplet channels so S corresponds to the a_0- rather than the σ-particle. The E_2 multiplet on the other hand contains some

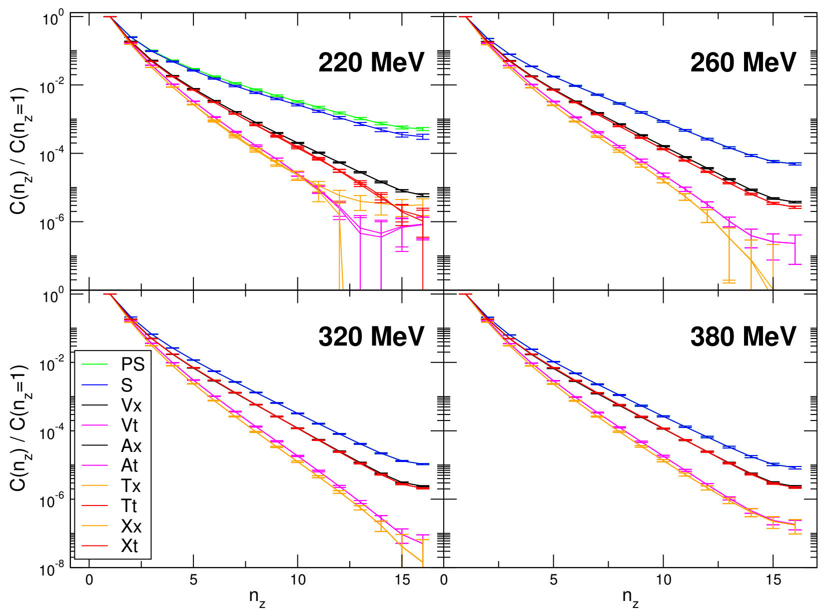

Figure 1. Normalized spatial correlators. The temperatures correspond to the ensembles listed in Table 1.

operators that are not connected by either $SU(2)_L \times SU(2)_R$ or $U(1)_A$ transformations; this holds also for the E_3 multiplet.

The left side of Figure 2 shows the correlators of the E_1 and E_2 multiplets in detail at the highest available temperature $T = 380$ MeV. Here we also show correlators calculated with non-interacting quarks. The non-interacting (*free*) data have been generated on the same lattice sizes using a unit gauge configuration and verified by analytic calculation. Due to the small quark mass the differences between chiral partners is negligible for the free case, therefore they are omitted.

We observe a precise degeneracy between S and PS correlators, which is consistent with the $U(1)_A$ restoration on these lattice ensembles [1]. The logarithmic slope of the interacting (*dressed*) S and PS correlators is substantially smaller than that for free quarks. In the latter case the slope is given by $2\pi/N_t$. A system of two free quarks cannot have 'energy' smaller than twice the lowest Matsubara frequency [14]. For the E_2 multiplet we observe asymptotic slopes that are quite close to $2\pi/N_t$ in agreement with previous studies [16].

Figures 3 and 4 show ratios of correlators connected by various symmetries at different temperatures. We use these ratios as quantities to measure the level of symmetry breaking. Figure 3 shows ratios of PS and S correlators on the left, and ratios of V_x and A_x correlators on the right. Figure 4 shows ratios of X_t and T_t correlators on the left, as well as of V_x and T_t correlators on the right side. The chiral symmetries $U(1)_A$ and $SU(2)_L \times SU(2)_R$ are restored at $T > 220$ MeV, as is evident from both Figures (see also, *e.g.* [17, 18]). Interesting is the level of residual breaking at $T = 220$ MeV: The corresponding deviation from unity for $U(1)_A$ connected correlators is at least one order of magnitude higher than for $SU(2)_L \times SU(2)_R$ connected correlators. [1]

[1]This effect might be spoiled by chirality violating eigenmodes of the Dirac operator, which are non-neglible for this ensemble and temperature [2]. They are absent for higher temperatures.

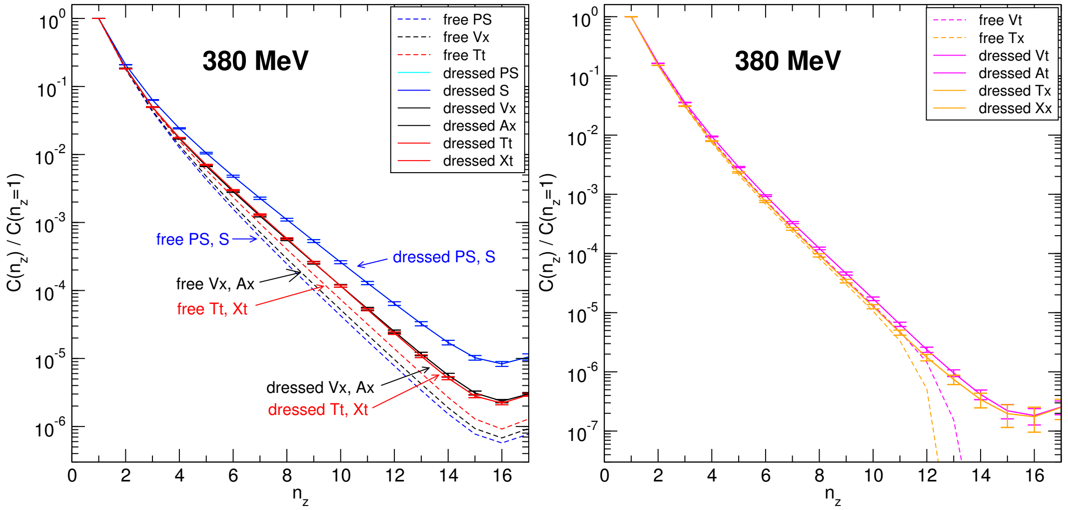

Figure 2. *Left side:* E_1 and E_2 multiplets (4-5) for interacting (*dressed*) and non-interacting (*free*) calculations at $T = 380$ MeV. *Right side:* E_3 multiplet (6) for interacting (*dressed*) and non-interacting (*free*) calculations.

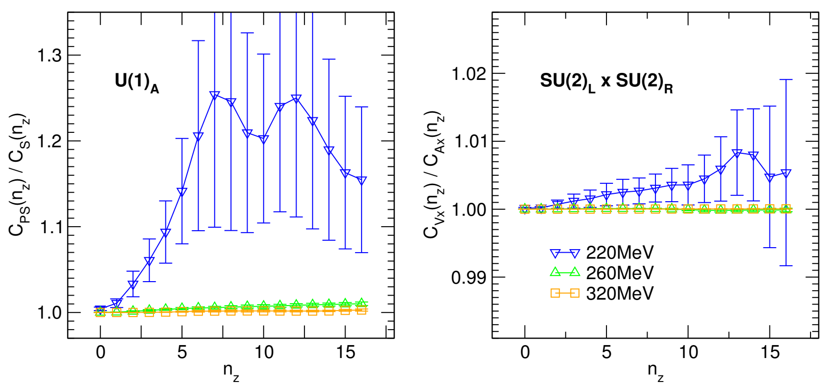

Figure 3. Ratios of PS and S as well as of V_x and A_x correlators from Figure 1, that are related by chiral $\mathrm{U}(1)_A$ and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ transformations.

Figures 2 and 4 suggest a possible higher symmetry that connects V_x and T_t channels. The right panel of Figure 4 shows the corresponding ratio, which demonstrates an approximate degeneracy at the level of 5% above $T \simeq 320$ MeV. We notice that this degeneracy is not expected in the free quark limit which is plotted by a dashed curve. This unexpected symmetry requires that the cross-correlator calculated with the V_x and T_t operators (both create the 1^{--} states) should vanish. We have carefully checked that it indeed vanishes to high accuracy.

The right side of Figure 2 shows the E_3 multiplet. Here again we observe a precise degeneracy in all $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and $\mathrm{U}(1)_A$ connected correlators, as well as the approximate degeneracy in all four correlators. We also see qualitatively different data between free and dressed correlators at $n_z \geq 11$, as also seen in [19]. For this set of operators the spatial correlations of free quarks become

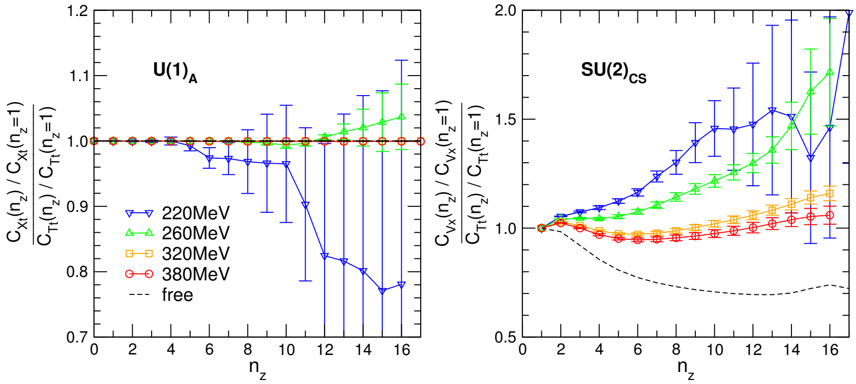

Figure 4. Ratios of normalized X_t and T_t as well as of V_x and T_t correlators from Figure 1, that are related by $\mathrm{U}(1)_A$ and $\mathrm{SU}(2)_{CS}$ transformations.

negative near the symmetry point, which is a finite size effect. The absence of this behaviour in the dressed correlators implies that we do not observe free non-interacting quarks but instead systems with some interquark correlation, which is in accordance with the known results for energy density and pressure at high temperatures [20].

4 $\mathrm{SU}(2)_{CS}$ and $\mathrm{SU}(4)$ symmetries

In this section we introduce the $\mathrm{SU}(2)_{CS}$ and SU(4) transformations, which connect operators from multiplet E_2 (5) as well as from multiplet E_3 (6) and contain chiral transformations as a subgroup. The basic ideas of $\mathrm{SU}(2)_{CS}$ and SU(4) symmetries at zero temperature are given in [6]. Here we adapt the group structure to our setup.

We use the γ-matrices given by

$$\gamma_i\gamma_j + \gamma_j\gamma_i = 2\delta_{ij} \tag{7}$$

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4. \tag{8}$$

The generators of the $\mathrm{SU}(2)_{CS}$ chiral-spin group, defined in the Dirac spinor space and diagonal in flavour space, are given by

$$\vec{\Sigma} = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\}. \tag{9}$$

$\mathrm{SU}(2)_{CS}$ contains $\mathrm{U}(1)_A$ as a subgroup. The $\mathfrak{su}(2)$ algebra $[\Sigma_\alpha, \Sigma_\beta] = 2i\epsilon^{\alpha\beta\gamma}\Sigma_\gamma$ is satisfied with any $k = 1, 2, 3, 4$. The $\mathrm{SU}(2)_{CS}$ transformations mix the left- and right-handed components of the quark field. It is not a symmetry of the free massless quark Lagrangian. For z-direction correlators the following representations of $\mathrm{SU}(2)_{CS}$ are relevant:

$$R_1 : \ \{\gamma_1, -i\gamma_5\gamma_1, \gamma_5\} \tag{10}$$

$$R_2 : \ \{\gamma_2, -i\gamma_5\gamma_2, \gamma_5\} \tag{11}$$

These R_1 and R_2 $\mathrm{SU}(2)_{CS}$ transformations connect the following operators from the E_2 multiplet:

$$R_1 : \quad V_y \leftrightarrow T_t \leftrightarrow X_t, \tag{12}$$

$$R_2 : \quad V_x \leftrightarrow T_t \leftrightarrow X_t, \tag{13}$$

as well as the operators from the E_3 multiplet:

$$R_1: \quad V_t \leftrightarrow T_y \leftrightarrow X_y, \tag{14}$$

$$R_2: \quad V_t \leftrightarrow T_x \leftrightarrow X_x. \tag{15}$$

Our symmetry group D_{4h} includes both the permutation operator $\hat{P}_{xy}$ and $\mathbb{1}$ transformations, which form a group S_2. $\hat{P}_{xy}$ permutes γ_1 and γ_2, and transforms γ_5 to $-\gamma_5$. Then $P_{xy}R_1$ is isomorphic to R_2. This means that $S_2 \times \mathrm{SU}(2)_{CS}$ contains multiplets

$$(V_x, V_y, T_t, X_t), \tag{16}$$

$$(V_t, T_x, T_y, X_x, X_y). \tag{17}$$

The degeneracy between **V** and **A** means $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry. A minimal group that includes $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ and $\mathrm{SU}(2)_{CS}$ is SU(4). The 15 generators of SU(4) are the following matrices:

$$\{(\tau_a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma_i), (\tau_a \otimes \Sigma_i)\} \tag{18}$$

with flavour index $a = 1, 2, 3$ and $\mathrm{SU}(2)_{CS}$ index $i = 1, 2, 3$. Predictions of $S_2 \times \mathrm{SU}(4)$ symmetry for isovector operators are the following multiplets:

$$(V_x, V_y, T_t, X_t, A_x, A_y), \tag{19}$$

$$(V_t, T_x, T_y, X_x, X_y, A_t). \tag{20}$$

$S_2 \times \mathrm{SU}(4)$ multiplets include in addition the isoscalar partners of V_x, V_y, T_t and X_t operators for the first multiplet in (20) as well as of V_t, T_x, T_y, X_x,X_y for the second multiplet in (20).

5 Conclusions

In this work we studied the high temperature behaviour of spatial correlation functions for connected spin $J = 0$ and $J = 1$ quark bilinears. Above the critial temperature we see restoration of both chiral symmetries, $\mathrm{U}(1)_A$ and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. Additionally, some unexpected observations are made: the approximate symmetries $\mathrm{SU}(2)_{CS}$ and SU(4) emerge as the temperature is increased to $\sim 2T_c$. At the same time there are no indications that the data would converge to the naive free quark limit.

For a discussion of implications of these observations see ref [21].

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References

[1] G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, H. Matsufuru, J.I. Noaki, Phys. Rev. **D87**, 114514 (2013), [Erratum: Phys. Rev.D88,no.1,019901(2013)], 1304.6145
[2] A. Tomiya, G. Cossu, S. Aoki, H. Fukaya, S. Hashimoto, T. Kaneko, J. Noaki, Phys. Rev. **D96**, 034509 (2017), 1612.01908
[3] A. Bazavov et al. (HotQCD), Phys. Rev. **D86**, 094503 (2012), 1205.3535
[4] T.D. Cohen, Phys. Rev. **D54**, R1867 (1996), hep-ph/9601216
[5] L.Ya. Glozman, Eur. Phys. J. **A51**, 27 (2015), 1407.2798
[6] L.Ya. Glozman, M. Pak, Phys. Rev. **D92**, 016001 (2015), 1504.02323
[7] M. Denissenya, L.Ya. Glozman, C.B. Lang, Phys. Rev. **D89**, 077502 (2014), 1402.1887
[8] M. Denissenya, L.Ya. Glozman, C.B. Lang, Phys. Rev. **D91**, 034505 (2015), 1410.8751
[9] M. Denissenya, L.Ya. Glozman, M. Pak, Phys. Rev. **D91**, 114512 (2015), 1505.03285
[10] M. Denissenya, L.Ya. Glozman, M. Pak, Phys. Rev. **D92**, 074508 (2015), [Erratum: Phys. Rev.D92,no.9,099902(2015)], 1508.01413
[11] R.C. Brower, H. Neff, K. Orginos, Nucl. Phys. Proc. Suppl. **153**, 191 (2006), hep-lat/0511031
[12] R.C. Brower, H. Neff, K. Orginos (2012), 1206.5214
[13] G. Cossu, H. Fukaya, S. Hashimoto, A. Tomiya (JLQCD), Phys. Rev. **D93**, 034507 (2016), 1510.07395
[14] C.E. Detar, J.B. Kogut, Phys. Rev. **D36**, 2828 (1987)
[15] J.B. Kogut, J.F. Lagae, D.K. Sinclair, Phys. Rev. **D58**, 054504 (1998), hep-lat/9801020
[16] F. Karsch, E. Laermann (2003), hep-lat/0305025
[17] M. Cheng et al., Eur. Phys. J. **C71**, 1564 (2011), 1010.1216
[18] K.I. Ishikawa, Y. Iwasaki, Y. Nakayama, T. Yoshie (2017), 1706.08872
[19] R.V. Gavai, S. Gupta, R. Lacaze, Phys. Rev. **D78**, 014502 (2008), 0803.1368
[20] F. Karsch, E. Laermann, A. Peikert, Phys. Lett. **B478**, 447 (2000), hep-lat/0002003
[21] C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L.Ya. Glozman, S. Hashimoto, C.B. Lang, S. Prelovsek (2017), 1707.01881