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One-loop perturbative coupling of A and $A_\star$ through the chiral overlap operator

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Abstract. Recently, Grabowska and Kaplan constructed a four-dimensional lattice formulation of chiral gauge theories on the basis of the chiral overlap operator. At least in the tree-level approximation, the left-handed fermion is coupled only to the original gauge field A, while the right-handed one is coupled only to the gauge field $A_\star$, a deformation of A by the gradient flow with infinite flow time. In this paper, we study the fermion one-loop effective action in their formulation. We show that the continuum limit of this effective action contains local interaction terms between A and $A_\star$, even if the anomaly cancellation condition is met. These non-vanishing terms would lead an undesired perturbative spectrum in the formulation.

1 Introduction and discussion

Recently, Grabowska and Kaplan proposed a four-dimensional lattice formulation of chiral gauge theories [1]. This formulation is based on the so-called overlap operator, which can be obtained from their five-dimensional domain-wall formulation [2][1] by the traditional way [4–6]. In this formulation, along the fifth dimension, the original gauge field A is deformed by the gradient flow [7–10] for infinite flow time. Since the gradient flow preserves the gauge covariance, this formulation is manifestly gauge invariant, *even if the anomaly cancellation condition is not met*. Although there is a subtlety associated with the topological charge [1, 2, 11–13], the smeared gauge field after the infinite-flow time, $A_\star$, only to which the right-handed (invisible) fermion would be coupled, can be basically considered as pure gauge (see Appendix A). Then one would regard their setup as the system of the left-handed fermion interacting with the gauge field A;[2] this picture was however confirmed only in the tree-level approximation [1]. It is thus a crucial problem whether radiative corrections induce the physical coupling of the right-handed fermion or not.

First, let us see the tree-level decoupling between the physical and invisible sectors. So far, only when the transition of the flowed gauge field along the fifth dimension is abrupt, the four-dimensional lattice Dirac operator has been obtained as an explicit form; this is referred to as the chiral overlap

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[1] As a closely related six-dimensional domain-wall formulation, see Ref. [3].
[2] Grabowska and Kaplan's formulation is a modification of that of Álvarez-Gaumé and Ginsparg [14]. The latter takes $A_\star = 0$ identically without the gradient flow and it breaks the gauge invariance.

operator $\hat{\mathcal{D}}_\chi$. The operator $\hat{\mathcal{D}}_\chi$ is given by [1]

$$a\hat{\mathcal{D}}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon \epsilon_\star + 1} (1 - \epsilon) \right], \tag{1}$$

where a is the lattice spacing, and ϵ ($\epsilon_\star$) is the sign function [15, 16]

$$\epsilon \equiv \frac{H_w(A)}{\sqrt{H_w(A)^2}} \qquad \left(\epsilon_\star \equiv \frac{H_w(A_\star)}{\sqrt{H_w(A_\star)^2}} \right), \tag{2}$$

of the Hermitian Wilson Dirac operator

$$H_w = \gamma_5 \left[\frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^*) - \frac{1}{2} a \nabla_\mu \nabla_\mu^* - m \right], \tag{3}$$

where m is the parameter of the domain-wall height, and γ_μ is the Dirac matrix. In this expression, ∇_μ is the forward gauge covariant lattice derivative and ∇_μ^* is the backward one. With the assumption of abruptness, this Dirac operator depends on the two gauge fields, A and $A_\star$. In the classical continuum limit [1],

$$am\hat{\mathcal{D}}_\chi \overset{a \to 0}{\to} \gamma_\mu D_\mu(A) P_- + \gamma_\mu D_\mu(A_\star) P_+, \tag{4}$$

where $D_\mu(A)$ ($D_\mu(A_\star)$) is the covariant derivative defined with respect to A ($A_\star$), and $P_\pm = (1 \pm \gamma_5)/2$ are the chirality projection operators. Therefore, the coupling between the gauge fields, A and $A_\star$, is not produced in the tree-level approximation.

Let us study how the decoupling between A and $A_\star$ is modified under radiative corrections. The fermion one-loop effective action is defined by

$$\ln \mathcal{Z}[A, A_\star] \equiv \ln \int \prod_x [d\psi(x) d\bar{\psi}(x)] \exp\left[-a^4 \sum_x \bar{\psi}(x) \hat{\mathcal{D}}_\chi \psi(x) \right], \tag{5}$$

where A and $A_\star$ are regarded as independent non-dynamical variables. To investigate the (de)coupling, two infinitesimal variations δ and $\delta_\star$ are introduced such that δ acts only on A but not on $A_\star$,

$$\delta A \neq 0, \qquad \delta A_\star \equiv 0, \tag{6}$$

and $\delta_\star$ acts in an opposite way,

$$\delta_\star A \equiv 0, \qquad \delta_\star A_\star \neq 0. \tag{7}$$

Then, we will find that in the continuum limit a double variation of the effective action is given as

$$\delta \delta_\star \ln \mathcal{Z}[A, A_\star] = - \int d^4x\, \mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star), \tag{8}$$

where $\mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star)$ is a *local* polynomial of its arguments and their spacetime derivatives.

To find a possible implication of Eq. (8), we take *gauge variations* as δ and $\delta_\star$:

$$\delta^\omega A_\mu(x) \equiv \partial_\mu \omega(x) + [A_\mu(x), \omega(x)], \qquad \delta^\omega A_{\star\mu}(x) = 0, \tag{9}$$

$$\delta_\star^\omega A_{\star\mu}(x) \equiv \partial_\mu \omega(x) + [A_{\star\mu}(x), \omega(x)], \qquad \delta_\star^\omega A_\mu(x) = 0. \tag{10}$$

Since, as a property of the gradient flow, the two gauge fields A and $A_\star$ transform in the same way under the gauge transformation, the gauge invariance of the effective action implies

$$(\delta^\omega + \delta_\star^\omega) \ln \mathcal{Z}[A, A_\star] = 0 \Rightarrow \delta(\delta^\omega + \delta_\star^\omega) \ln \mathcal{Z}[A, A_\star] = 0. \tag{11}$$

Therefore, using Eq. (8), we can obtain

$$\delta\delta^{\omega}\mathcal{Z}[A,A_\star] = -\delta\delta^{\omega}_{\star}\mathcal{Z}[A,A_\star] = \int d^4x\,\mathcal{L}(A,A_\star;\delta A,\delta^{\omega}_{\star}A_\star). \tag{12}$$

Now, let us assume that $A_\star$ becomes pure gauge under the gradient flow with infinite flow time (see Appendix A):

$$A_\star = g^{-1}dg. \tag{13}$$

Then the gauge transformation $A^{g^{-1}}$ makes $A_\star = 0$, where

$$A^{g} = g^{-1}(d+A)g. \tag{14}$$

That is, we can impose the $A_\star = 0$ gauge on Eq. (12)

$$\delta\delta^{\omega}\ln\mathcal{Z}[A,0] = \int d^4x\,\mathcal{L}(A,A_\star=0;\delta A,\delta^{\omega}_{\star}A_\star|_{A_\star=0}). \tag{15}$$

We will see below that the right-hand side does not vanish even if the anomaly cancellation condition is met.

It will be shown in the next section that $\ln\mathcal{Z}[A,0]$ has the term

$$\ln\mathcal{Z}[A,0] = \int d^4x\,\frac{f_0}{2a^2}\,\mathrm{tr}\,A_\mu A_\mu + \cdots, \tag{16}$$

thus the mass term $\mathrm{tr}\,A_\mu A_\mu$ is produced in the one-loop level. The propagator of the gauge potential in this $A_\star = 0$ thus has the structure,

$$\left\langle A^a_\mu(x)A^a_\nu(y)\right\rangle = g_0^2\,\delta^{ab}\int\frac{d^4p}{(2\pi)^4}\,e^{ip(x-y)}\left(\delta_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)\frac{1}{p^2+m_A^2+\cdots}, \tag{17}$$

where we have defined the mass parameter m_A as

$$m_A^2 = g_0^2\frac{f_0}{2a^2}. \tag{18}$$

Therefore, the perturbative spectrum is modified in a *weird* way; this would not be what we want to obtain for chiral gauge theories. Since these effects in the one-loop effective action (16) should be removed by local counterterms, the formulation of Grabowska and Kaplan will be undesirable as a non-perturbative formulation of chiral gauge theories. Then their formulation with the abrupt transition should be improved in some possible way.

2 Explicit forms of $\mathcal{L}$ and $\delta^{\omega}\ln\mathcal{Z}$

In this section, we show the results of the continuum limit of $\mathcal{L}(A,A_\star;\delta A,\delta_\star A_\star)$.[3] In what follows, we use the variables

$$C_\mu \equiv A_{\star\mu} - A_\mu, \tag{19}$$

$$\bar{A}_\mu \equiv \frac{1}{2}(A_\mu + A_{\star\mu}), \tag{20}$$

$$\bar{D}_\mu \equiv \partial_\mu + [\bar{A}_\mu,\cdot], \tag{21}$$

[3]For details of the computation of Eq. (8), see Ref. [12] and our work [17].

and the field strength

$$\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + [\bar{A}_\mu, \bar{A}_\nu]. \tag{22}$$

We also define the following lattice integrals:

$$f_0(am) \equiv \int_p \left(-\frac{1}{4t} - \frac{s_\rho^2}{4t} - \frac{cc_\rho}{4t} \right), \tag{23}$$

$$f_1(am) \equiv \int_p \left(\frac{1}{64t^2} - \frac{c_\rho c_\sigma}{128t} + \frac{s_\rho^2 s_\sigma^2}{32t^2} \right), \tag{24}$$

$$f_2(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{7 s_\rho^2 s_\sigma^2}{64t^2} + \frac{c s_\rho^2 c_\sigma}{32t^2} + \frac{c^2 c_\rho c_\sigma}{64t^2} \right), \tag{25}$$

$$f_3(am) \equiv \int_p \left(-\frac{c_\rho c_\sigma}{32t} + \frac{3 s_\rho^2 s_\sigma^2}{32t^2} - \frac{s_\rho^2}{32t^2} - \frac{cc_\rho}{32t^2} \right), \tag{26}$$

$$f_4(am) \equiv \int_p \left(\frac{1}{96t} + \frac{s_\rho^2}{96t} + \frac{cc_\rho}{96t} + \frac{1}{16t^2} \right), \tag{27}$$

$$f_5(am) \equiv \int_p \left(\frac{1}{16t} + \frac{c_\rho c_\sigma}{32t} + \frac{7}{32t^2} - \frac{c^2}{32t^2} + \frac{cc_\rho}{16t^2} + \frac{s_\rho^2}{32t^2} \right), \tag{28}$$

where

$$s_\rho \equiv \sin p_\rho, \qquad c_\rho \equiv \cos p_\rho, \tag{29}$$

$$c \equiv \sum_\mu (c_\mu - 1) + am, \qquad t \equiv \sum_\mu s_\mu^2 + c^2, \tag{30}$$

$$\int_p \equiv \int_{-\pi}^{\pi} \frac{d^4 p}{(2\pi)^4}. \tag{31}$$

$f_i(am)$ $(i = 0, \ldots, 5)$ as the function of am are plotted in Figs. 1–6.

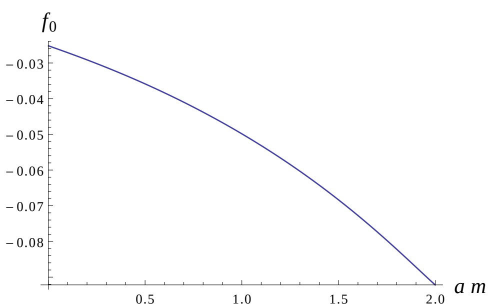

Figure 1. $f_0(am)$

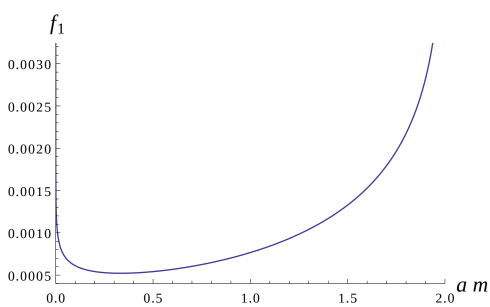

Figure 2. $f_1(am)$

The local functional $\mathcal{L}$ has three parts, according to the parity and Lorentz symmetry: (i) the parity-odd and Lorentz-preserving part, (ii) the parity-even and Lorentz-preserving part, and (iii) the

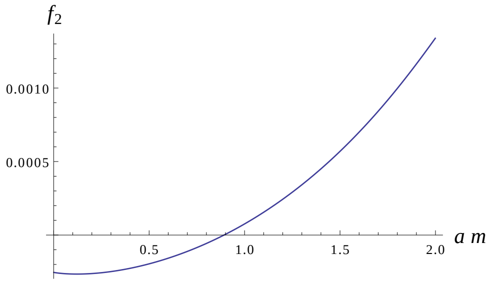

Figure 3. $f_2(am)$

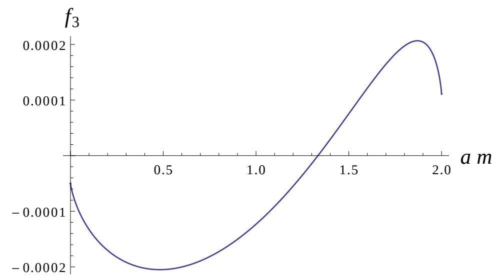

Figure 4. $f_3(am)$

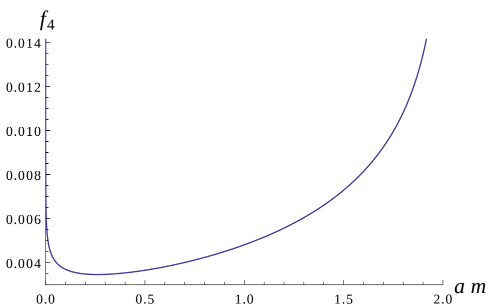

Figure 5. $f_4(am)$

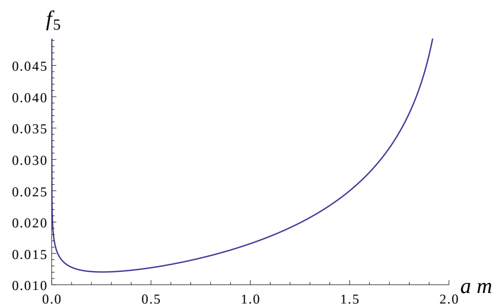

Figure 6. $f_5(am)$

parity-even and Lorentz-violating part. First, the parity-odd part of $\mathcal{L}$ is given by

$$\begin{aligned}
&\mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star)|_{\text{parity-odd}} \\
&= -\frac{1}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}\Bigg[\left(\bar{F}_{\mu\nu} + \frac{1}{12}[C_\mu, C_\nu]\right)\{\delta A_\rho, \delta_\star A_{\star\sigma}\} \\
&\qquad\qquad - \frac{1}{3}C_\mu\left(\{\delta A_\nu, \bar{D}_\rho\delta_\star A_{\star\sigma}\} + \{\delta_\star A_{\star\nu}, \bar{D}_\rho\delta A_\sigma\}\right)\Bigg],
\end{aligned} \tag{32}$$

where and in what follows the symbol tr is assumed to be omitted. This part is proportional to the gauge anomaly coefficient; thus this vanishes if the anomaly cancellation condition is met. Second, we have the parity-even and Lorentz-preserving part of $\mathcal{L}$,

$$
\begin{aligned}
&\mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star)|_{\text{parity-even, Lorentz-preserving}} \\
&= \frac{f_0}{a^2}\delta A_\mu \delta_\star A_{\star\mu} \\
&\quad + \left(-\frac{3f_1}{2} + \frac{f_2}{2} - \frac{f_3}{2}\right)[(\bar{D}_\mu \delta A_\mu)C_\nu \delta_\star A_{\star\nu} - C_\mu \delta A_\mu (\bar{D}_\nu \delta_\star A_{\star\nu})] \\
&\quad - \left(\frac{f_1}{2} + \frac{f_2}{2} - \frac{3f_3}{2}\right)[C_\mu (\bar{D}_\nu \delta A_\mu)\delta_\star A_{\star\nu} - \delta A_\mu C_\nu (\bar{D}_\mu \delta_\star A_{\star\nu})] \\
&\quad - \left(\frac{f_1}{2} + \frac{f_2}{2} - \frac{3f_3}{2}\right)[C_\nu \delta A_\mu (\bar{D}_\mu \delta_\star A_{\star\nu}) - (\bar{D}_\nu \delta A_\mu) C_\mu \delta_\star A_{\star\nu}] \\
&\quad + \left(-\frac{7f_1}{2} + \frac{f_2}{2} + \frac{f_3}{2}\right)[(\bar{D}_\mu C_\mu)\delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu (\bar{D}_\mu C_\mu)\delta_\star A_{\star\nu}] \\
&\quad - \left(\frac{3f_1}{2} - \frac{f_2}{2} + \frac{f_3}{2}\right)[\delta A_\mu C_\mu (\bar{D}_\nu \delta_\star A_{\star\nu}) - C_\nu (\bar{D}_\mu \delta A_\mu)\delta_\star A_{\star\nu}] \\
&\quad + (13f_1 - 3f_2 - 3f_3)(\bar{D}_\mu \delta A_\mu)(\bar{D}_\nu \delta_\star A_{\star\nu}) \\
&\quad + (9f_1 - 3f_2 - f_3)(\bar{D}_\mu \delta A_\nu)(\bar{D}_\mu \delta_\star A_{\star\nu}) \\
&\quad + (-19f_1 + 5f_2 + 5f_3)(\bar{D}_\nu \delta A_\mu)(\bar{D}_\mu \delta_\star A_{\star\nu}) \\
&\quad + \left(\frac{11f_1}{6} - \frac{f_2}{6} - \frac{7f_3}{6}\right) C_\mu \delta A_\nu C_\mu \delta_\star A_{\star\nu} \\
&\quad + \left(-\frac{13f_1}{6} + \frac{11f_2}{6} - \frac{7f_3}{6}\right)(C_\mu \delta A_\mu C_\nu \delta_\star A_{\star\nu} + C_\nu \delta A_\mu C_\mu \delta_\star A_{\star\nu}) \\
&\quad + \left(-\frac{5f_1}{12} + \frac{19f_2}{12} - \frac{17f_3}{12}\right)(C_\nu C_\mu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\mu C_\nu \delta_\star A_{\star\nu}) \\
&\quad + \left(\frac{19f_1}{12} - \frac{5f_2}{12} - \frac{5f_3}{12}\right)(C_\mu C_\nu \delta A_\mu \delta_\star A_{\star\nu} + \delta A_\mu C_\nu C_\mu \delta_\star A_{\star\nu}) \\
&\quad + \left(-\frac{17f_1}{12} + \frac{19f_2}{12} - \frac{11f_3}{12}\right)(C_\mu C_\mu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\mu C_\mu \delta_\star A_{\star\nu}),
\end{aligned}
\tag{33}
$$

and finally the parity-even and Lorentz-violating part is given by

$$
\begin{aligned}
&\mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star)|_{\text{parity-even, Lorentz-violating}} \\
&= \frac{3}{2}\left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2}\right)[(\bar{D}_\nu C_\nu)\delta A_\nu \delta_\star A_{\star\nu} - \delta A_\nu (\bar{D}_\nu C_\nu)\delta_\star A_{\star\nu}] \\
&\quad - \left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2}\right)(\bar{D}_\nu \delta A_\nu)(\bar{D}_\nu \delta_\star A_{\star\nu}) \\
&\quad + \left(\frac{47f_1}{2} - \frac{7f_2}{2} - \frac{7f_3}{2} + \frac{f_4}{4} - \frac{7f_5}{4}\right) C_\nu \delta A_\nu C_\nu \delta_\star A_{\star\nu} \\
&\quad + \left(\frac{67f_1}{4} - \frac{11f_2}{4} - \frac{11f_3}{4} + \frac{5f_4}{8} - \frac{11f_5}{8}\right)(C_\nu C_\nu \delta A_\nu \delta_\star A_{\star\nu} + \delta A_\nu C_\nu C_\nu \delta_\star A_{\star\nu}).
\end{aligned}
\tag{34}
$$

By using the above form of $\mathcal{L}(A, A_\star; \delta A, \delta_\star A_\star)$, one can deduce the gauge variation of $\ln \mathcal{Z}[A, 0]$, $\delta^\omega \ln \mathcal{Z}[A, 0]$ (see Appendix A of Ref. [17] for details). The parity-odd part of $\mathcal{L}$ gives rise to (leaving out the symbol $\int d^4x$ tr)

$$\delta^{\omega} \ln \mathcal{Z}[A,0]|_{\text{parity-odd}} = \frac{1}{24\pi^2}\epsilon_{\mu\nu\rho\sigma}(\partial_\mu\omega)\left(A_\nu\partial_\rho A_\sigma + \frac{1}{2}A_\nu A_\rho A_\sigma\right), \tag{35}$$

which is the consistent gauge anomaly associated with a left-handed fermion. It is impossible to rewrite this expression as the gauge variation of a local term. On the other hand, the parity-even part of $\delta^{\omega} \ln \mathcal{Z}$ can be written as the gauge variation of local terms:

$$\begin{aligned}
&\delta^{\omega} \ln \mathcal{Z}[A,0]|_{\text{parity-even}} \\
&= \delta^{\omega}\Bigg[\frac{f_0}{2a^2}A_\mu A_\mu \\
&\qquad + \frac{1}{2}(-13f_1 + 3f_2 + 3f_3)A_\mu\partial_\mu\partial_\nu A_\nu \\
&\qquad\quad + (5f_1 - f_2 - 2f_3)(A_\mu\partial_\nu\partial_\nu A_\mu - A_\mu A_\nu\partial_\mu A_\nu + A_\mu A_\nu\partial_\nu A_\mu) \\
&\qquad\qquad + \frac{2}{3}(f_1 + f_2 - 2f_3)A_\mu A_\mu A_\nu A_\nu + \frac{1}{12}(-11f_1 + f_2 + 7f_3)A_\mu A_\nu A_\mu A_\nu \\
&\qquad\qquad\quad + \frac{1}{2}\left(9f_1 - f_2 - f_3 - \frac{f_4}{2} - \frac{f_5}{2}\right)A_\mu\partial_\mu\partial_\mu A_\mu \\
&\qquad\qquad\qquad + \frac{1}{4}\left(19f_1 - 3f_2 - 3f_3 + \frac{f_4}{2} - \frac{3f_5}{2}\right)A_\mu A_\mu A_\mu A_\mu\Bigg].
\end{aligned} \tag{36}$$

The last two lines are not Lorentz invariant. This parity-even part does not vanish even if the gauge representation is anomaly-free. For example, the first term $\delta^{\omega}[(f_0/2a^2)A_\mu A_\mu]$ corresponds to the gauge variation of the mass term of the gauge field. The *regularization garbage* in Eq. (36) can be subtracted by local counterterms. However, such a necessity for counterterms will be undesirable from a perspective of a non-perturbative formulation of chiral gauge theories.

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A Gradient flow for infinite flow time

The gradient flow of the gauge field is defined by

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x). \tag{37}$$

In the abelian theory, we can solve this equation as

$$B_\mu(t,x) = \int d^4y \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)}\left[\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)e^{-tp^2} + \frac{p_\mu p_\nu}{p^2}\right]A_\nu(y). \tag{38}$$

This shows that after infinite flow time the configuration becomes pure gauge:

$$B_\mu(t,x) \overset{t\to\infty}{\to} g(x)^{-1}\partial_\mu g(x), \tag{39}$$

where

$$g(x) = \exp\left[-\int d^4y \int \frac{d^4p}{(2\pi)^4}\frac{e^{ip(x-y)}}{p^2}\partial_\mu A_\mu(y)\right]. \tag{40}$$

Note that $g(x)$ is a non-local functional of the original gauge field $A_\mu(y)$.

For the non-abelian theory, we cannot solve the flow equation in a closed form. However, we can show that the Euclidean action integral $S = \int d^4x \, \frac{1}{4g_0^2} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x)$ monotonically decreases along the flow. Since the minimum of the action integral in the topologically trivial sector is given by a pure gauge configuration, the flowed configuration in the topologically trivial sector approaches a pure gauge configuration. In fact, the pure gauge configuration

$$B_\mu(t, x) = g(x)^{-1} \partial_\mu g(x) \tag{41}$$

is a stationary solution of the flow equation, $\partial_t B(t, x) = 0$.

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