4D $\mathcal{N} = 1$ SYM supercurrent on the lattice in terms of the gradient flow

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Abstract. The gradient flow [1–5] gives rise to a versatile method to construct renormalized composite operators in a regularization-independent manner. By adopting this method, the authors of Refs. [6–9] obtained the expression of Noether currents on the lattice in the cases where the associated symmetries are broken by lattice regularization. We apply the same method to the Noether current associated with supersymmetry, i.e., the supercurrent. We consider the 4D $\mathcal{N} = 1$ super Yang–Mills theory and calculate the renormalized supercurrent in the one-loop level in the Wess–Zumino gauge. We then re-express this supercurrent in terms of the flowed gauge and flowed gaugino fields [10].

1 Introduction

Lattice gauge theory provides a non-perturbative definition of quantum field theory (QFT) and a powerful tool of simulating it. In this framework, QFT is regularized by discretizing the spacetime and hence the continuum spacetime symmetries are explicitly broken. Although these symmetries are often expected to be restored in the continuum limit, this fact complicates the construction of the Noether current associated with those spacetime symmetries, e.g. the energy-momentum tensor.

In general, composite operators such as Noether currents potentially have UV divergences and hence need regularization. In order to construct the composite operators in a regularization-independent manner, we consider the gradient flow which is defined for the gauge field by

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x), \tag{1}$$

where

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + \left[B_\mu(t,x), B_\nu(t,x)\right], \qquad D_\mu = \partial_\mu + \left[B_\mu, \cdot\right]. \tag{2}$$

It was proved in Ref. [4] that composite operators composed of the flowed gauge field are UV finite and therefore independent of regularization (see also Ref. [11]). Also, a small flow-time limit $t \to 0$ of a bare composite operator at $t > 0$ can be expanded by renormalized composite operators at $t = 0$ [4]. By using these facts, one can express for example the chiral condensate $\langle \bar{q}(x) q(x) \rangle$ in massless QCD in terms of a $t \to 0$ limit of a composite operator at $t > 0$ [8, 9]. Also, the renormalized energy–momentum tensor can be expressed in terms of flowed bare composite operators [6, 7]. The validity of these representations has been numerically confirmed in Refs. [12–15].

In the present study, we apply the same method to construct the supercurrent in the 4D $\mathcal{N} = 1$ super Yang–Mills theory (SYM).[1] Supersymmetry can be a crucial element in theories beyond the standard

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[1] In Ref. [16], the gradient flow in this system is studied from a quite different perspective.

model, providing a solution to the fine-tuning problem in the Higgs mass. It is thus interesting to explore the prediction of supersymmetric models by using the lattice simulation. However, since supersymmetry is a spacetime symmetry, it is explicitly broken by lattice regularization and, as a consequence, one has to tune the bare parameters towards the supersymmetric point. For this tuning, a priori knowledge on the correct supercurrent which restores the conservation law in the continuum limit will be quite helpful.[2]

2 Renormalized supercurrent in 4D $\mathcal{N} = 1$ SYM

In order to construct a Noether current using the gradient flow, we proceed in two steps. The first is a construction of the Noether current in a regularized theory; the second is to express it by flowed bare composite operators. In this section, we carry out the first step. That is, we find the expression of the correctly-normalized supercurrent in the one-loop level by using dimensional regularization.

The Euclidean action of the 4D $\mathcal{N} = 1$ SYM is given by

$$S = \frac{1}{4g_0^2} \int d^D x\, F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) + \frac{1}{2} \int d^D x\, \bar{\psi}^a(x) \not{D}^{ab} \psi^b(x). \tag{3}$$

We adopt dimensional regularization with $D = 4 - 2\epsilon$. The gaugino $\psi^a(x)$ is a Majorana fermion in the adjoint representation satisfying $\bar{\psi}(x) = \psi^T(x)(-C^{-1})$, where C is the charge-conjugation matrix such that $C^{-1}\gamma_\mu C = -\gamma_\mu^T$. The gauge-field strength $F_{\mu\nu}(x)$ and the covariant derivative D_μ are defined by

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + f^{abc} A_\mu^b(x) A_\nu^c(x), \tag{4}$$

$$D_\mu^{ab} = \delta^{ab}\partial_\mu + f^{acb} A_\mu^c. \tag{5}$$

The super transformation in the Wess–Zumino gauge is given by

$$\delta_\xi A_\mu^a(x) = g_0 \bar{\xi} \gamma_\mu \psi^a(x), \qquad \delta_\xi \psi^a(x) = -\frac{1}{2g_0} \sigma_{\mu\nu} \xi F_{\mu\nu}^a(x), \qquad \delta_\xi \bar{\psi}^a(x) = \frac{1}{2g_0} \bar{\xi} \sigma_{\mu\nu} F_{\mu\nu}^a(x), \tag{6}$$

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. This transformation leaves the action S invariant.[3] We can read off the classical form of the supercurrent $s_\mu(x)$ by making the parameter local $\xi \to \xi(x)$ (the Noether method). The result is

$$s_\mu(x) = -\frac{1}{2g_0} \sigma_{\rho\sigma} \gamma_\mu \psi^a(x) F_{\rho\sigma}^a(x). \tag{9}$$

We next consider the Ward–Takahashi (WT) relation associated with supersymmetry and find the expression of the correctly-normalized supercurrent in the one-loop level. For perturbation theory, we

[2] In the 4D $\mathcal{N} = 1$ SYM, one may employ the chiral symmetry to carry out the tuning of the gaugino mass [18, 19]. See also Ref. [20].

[3] In order to prove the invariance, one has to use the Fierz identity,

$$(\bar{\psi}_1 \gamma_\mu \psi_2)(\bar{\psi}_3 \gamma_\mu \psi_4) = (\bar{\psi}_1 \gamma_\mu \psi_4)(\bar{\psi}_3 \gamma_\mu \psi_2). \tag{7}$$

If we use this relation, the variation of the action vanishes:

$$\delta_\xi S = -\frac{1}{2} g_0 \int d^D x\, f^{abc} \bar{\xi} \gamma_\mu \psi^a(x) \bar{\psi}^b(x) \gamma_\mu \psi^c(x) = 0. \tag{8}$$

However, the Fierz identity is broken with dimensional regularization and $\delta_\xi S$ cannot be neglected in quantum level; we have to take this effect into account in the Ward–Takahashi relation.

introduce the gauge-fixing term and the Faddeev–Popov ghost term,

$$S_{\mathrm{gf}} = \frac{\lambda_0}{2g_0^2}\int d^D x\,\partial_\mu A_\mu^a(x)\partial_\nu A_\nu^a(x), \tag{10}$$

$$S_{c\bar{c}} = -\frac{1}{g_0^2}\int d^D x\,\bar{c}^a(x)\partial_\mu D_\mu^{ab} c^b(x). \tag{11}$$

We set $\lambda_0 = 1$ (the Feynman gauge) in what follows. These terms are not invariant under the super transformation,

$$\delta_\xi S_{\mathrm{gf}} = -\int d^D x\,\bar{\xi} X_{\mathrm{gf}}(x), \qquad X_{\mathrm{gf}}(x) = \frac{\lambda_0}{g_0}\partial_\mu\partial_\nu A_\nu^a(x)\gamma_\mu\psi^a(x), \tag{12}$$

$$\delta_\xi S_{c\bar{c}} = -\int d^D x\,\bar{\xi} X_{c\bar{c}}(x), \qquad X_{c\bar{c}}(x) = \frac{1}{g_0} f^{abc}\partial_\mu\bar{c}^a(x)c^b(x)\gamma_\mu\psi^c(x). \tag{13}$$

As noted in the footnote 3, $\delta_\xi S$ does not vanish in $D = 4 - 2\epsilon$,

$$\delta_\xi S = \int d^D x\left[\partial_\mu\bar{\xi}(x)s_\mu(x) - \bar{\xi}(x)X_{\mathrm{Fierz}}(x)\right], \tag{14}$$

where

$$X_{\mathrm{Fierz}}(x) = \frac{1}{2}g_0 f^{def}\gamma_\mu\psi^d(x)\bar{\psi}^e(x)\gamma_\mu\psi^f(x). \tag{15}$$

With the above three breaking terms, we have a WT relation,

$$\begin{aligned}&\left\langle\left[\partial_\mu s_\mu(x) + X_{\mathrm{Fierz}}(x) + X_{\mathrm{gf}}(x) + X_{c\bar{c}}(x)\right]A_\alpha^b(y)\bar{\psi}^c(z)\right\rangle\\ &= -\left\langle\delta(x-y)g_0\gamma_\alpha\psi^b(y)\bar{\psi}^c(z)\right\rangle - \left\langle\delta(x-z)A_\alpha^b(y)\frac{1}{2g_0}\sigma_{\rho\sigma}F_{\rho\sigma}^c(z)\right\rangle.\end{aligned} \tag{16}$$

The effect of $X_{\mathrm{Fierz}}(x)$ can be taken into account as

$$\begin{aligned}&\left\langle\left[\partial_\mu s_\mu(x) + X_{\mathrm{gf}}(x) + X_{c\bar{c}}(x)\right]A_\alpha^b(y)\bar{\psi}^c(z)\right\rangle'\\ &= -\left\langle\delta(x-y)g_0\gamma_\alpha\psi^b(y)\bar{\psi}^c(z)\right\rangle' - \left\langle\delta(x-z)A_\alpha^b(y)\frac{1}{2g_0}\sigma_{\rho\sigma}F_{\rho\sigma}^c(z)\right\rangle',\end{aligned} \tag{17}$$

where the prime (′) implies that the expectation values are computed with respect to the action with a counterterm $S' \equiv -\frac{1}{(4\pi)^2}C_2(G)\frac{1}{6}\int d^D x\,F_{\mu\nu}^a(x)F_{\mu\nu}^a(x)$. This follows from the fact that the one-loop level expectation value $\langle X_{\mathrm{Fierz}}(x)A_\alpha^b(y)\bar{\psi}^c(z)\rangle$ can be compensated by the variation of the counterterm $\langle\delta_\xi S' A_\alpha^b(y)\bar{\psi}^c(z)\rangle$.

We want to find the correctly-normalized supercurrent which induces the renormalized super transformation on *renormalized* (elementary) fields. For this, we first replace all the fields and couplings in the WT relation (17) by renormalized ones. With the notation $\Delta \equiv \frac{g^2}{(4\pi)^2}C_2(G)\frac{1}{\epsilon}$, bare fields/couplings and the renormalized ones are related in the one-loop level as

$$g_0 = \mu^\epsilon \left(1 - \frac{3}{2}\Delta\right) g, \tag{18}$$

$$\lambda_0 = (1 - \Delta)\lambda, \tag{19}$$

$$A^a_\mu(x) = (1 - \Delta)A^a_{\mu R}(x), \qquad \psi^a(x) = \left(1 - \frac{1}{2}\Delta\right)\psi^a_R(x), \tag{20}$$

$$c^a(x) = \left(1 - \frac{5}{4}\Delta\right)c^a_R(x), \tag{21}$$

$$F^a_{\mu\nu}(x) = \left(1 - \frac{5}{2}\Delta\right)\left[\partial_\mu A^a_{\nu R}(x) - \partial_\nu A^a_{\mu R}(x)\right] + \left(1 - \frac{11}{4}\Delta\right) f^{abc}\left[A^b_\mu(x)A^c_\nu(x)\right]_R. \tag{22}$$

Substituting these into Eq. (17) and evaluating UV divergences coming from 1PI one-loop diagrams containing composite operators, after rearrangements of various terms, we have [10]

$$\begin{aligned}&\left\langle\left[\partial_\mu s_{\mu R}(x) + X_{\mathrm{gf}R}(x) + X_{c\bar{c}R}(x)\right] A^b_{\alpha R}(y)\bar{\psi}^c_R(z)\right\rangle' \\ &= -\left\langle\delta(x-y) g\gamma_\alpha \psi^b_R(y)\bar{\psi}^c_R(z)\right\rangle' \\ &\quad - \left\langle\delta(x-z)A^b_{\alpha R}(y)\frac{1}{2g}\sigma_{\rho\sigma}\left[\partial_\rho A^c_{\sigma R}(z) - \partial_\sigma A^c_{\rho R}(z) + f^{cde}(A^d_\rho(x)A^e_\sigma(z))_R\right]\right\rangle'. \end{aligned}\tag{23}$$

For the definition of various renormalized composite operators, see Ref. [10].

Equation (23) tells us that the finite combination $\partial_\mu s_{\mu R}(x) + X_{\mathrm{gf}R}(x) + X_{c\bar{c}R}(x)$ induces the renormalized super transformation on renormalized fields in the one-loop level. We can further show that the combination $X_{\mathrm{gf}}(x) + X_{c\bar{c}}(x)$ vanishes in on-shell correlation functions of gauge-invariant operators [10]. Thus, in such correlation functions, the correctly-normalized supercurrent to the one-loop level is given by

$$\mathcal{S}_{\mu R}(x) = s_{\mu R}(x) = -\frac{1}{2g_0}\sigma_{\rho\sigma}\gamma_\mu\psi^a(x)F^a_{\rho\sigma}(x) + O(g_0^3). \tag{24}$$

3 Supercurrent in terms of the flowed fields

In the previous section, we found that $\mathcal{S}_{\mu R}(x) = -\frac{1}{2g_0}\sigma_{\rho\sigma}\gamma_\mu\psi^a(x)F^a_{\rho\sigma}(x) + O(g_0^3)$ gives rise to the correctly-normalized supercurrent. We now express this composite operator in terms of flowed fields $B_\mu(t,x)$ and $\chi(t,x)$ for a small flow time.

We adopt the flow equations in Ref. [5]:

$$\partial_t B^a_\mu(t,x) = D^{ab}_\nu G^b_{\nu\mu}(t,x), \qquad B^a_\mu(t=0,x) = A^a_\mu(x), \tag{25}$$

$$\partial_t \chi^a(t,x) = \left(D^2\right)^{ab}\chi^b(t,x), \qquad \chi^a(t=0,x) = \psi^a(x), \tag{26}$$

$$\partial_t \bar{\chi}^a(t,x) = \bar{\chi}^b(t,x)\left(\overleftarrow{D}^2\right)^{ba}, \qquad \bar{\chi}^a(t=0,x) = \bar{\psi}^a(x), \tag{27}$$

where

$$D^{ab}_\mu = \delta^{ab}\partial_\mu + f^{acb}B^c_\mu(t,x), \tag{28}$$

$$G^a_{\mu\nu}(x) = \partial_\mu B^a_\nu(t,x) - \partial_\nu B^a_\mu(t,x) + f^{abc}B^b_\mu(t,x)B^c_\nu(t,x). \tag{29}$$

Our goal is to rewrite the operator $\psi^a(x)F^a_{\mu\nu}(x)$ in the supercurrent (24) by the flowed fields. For this, we first expand the flowed composite operator $\chi^a(t,x)G^a_{\mu\nu}(t,x)$ in terms of unflowed composite operators assuming that the flow time t small, i.e., we consider a small flow-time expansion of $\chi^a(t,x)G^a_{\mu\nu}(t,x)$. Noting that the flow time t has mass-dimension -2 and the flow equations are Lorentz/gauge covariant, the small flow-time expansion takes the form

$$\begin{aligned}\chi^a(t,x)G^a_{\mu\nu}(t,x) &= \zeta_1(t)\psi^a(x)F^a_{\mu\nu}(x)\\ &\quad+\zeta_2(t)\left[\gamma_\mu\gamma_\rho\psi^a(x)F^a_{\rho\nu}(x)-\gamma_\nu\gamma_\rho\psi^a(x)F^a_{\rho\mu}(x)\right]\\ &\quad+\zeta_3(t)\sigma_{\rho\sigma}\sigma_{\mu\nu}\psi^a(x)F^a_{\rho\sigma}(x)+O(t).\end{aligned}\tag{30}$$

We compute the three coefficients $\zeta_i(t)$ by perturbation theory; perturbation theory is justified for the small flow-time limit $t\to 0$ by the asymptotic freedom. In Eq. (30), the off-diagonal operator mixings arise only through loop corrections. Thus, to the one-loop order, we have

$$\begin{aligned}\psi^a(x)F^a_{\mu\nu}(x) &= \left[1-\zeta_1^{(1)}(t)\right]\chi^a(t,x)G^a_{\mu\nu}(t,x),\\ &\quad+\zeta_2^{(1)}(t)\left[\gamma_\mu\gamma_\rho\chi^a(t,x)G^a_{\rho\nu}(t,x)-\gamma_\nu\gamma_\rho\chi^a(t,x)G^a_{\rho\mu}(t,x)\right]\\ &\quad+\zeta_3^{(1)}(t)\sigma_{\rho\sigma}\sigma_{\mu\nu}\chi^a(t,x)G^a_{\rho\sigma}(t,x)+O(t),\end{aligned}\tag{31}$$

where $\zeta_i^{(1)}$ are one-loop quantities. Substituting this into the expression of the supercurrent (24), we have

$$\begin{aligned}S_{\mu R}(x) &= -\frac{1}{2g_0}\left[1-\zeta_1^{(1)}(t)-2(D-3)\zeta_2^{(1)}(t)+(D-9)(D-4)\zeta_3^{(1)}(t)\right]\sigma_{\rho\sigma}\gamma_\mu\chi^a(t,x)G^a_{\rho\sigma}(t,x)\\ &\quad-\frac{1}{2g_0}\left[4(D-4)\zeta_2^{(1)}(t)-4(D-5)(D-4)\zeta_3^{(1)}(t)\right]\gamma_\rho\chi^a(t,x)G^a_{\rho\mu}(t,x)\\ &\quad+O(t)+O(g_0^3).\end{aligned}\tag{32}$$

For the computation of the coefficients $\zeta_i^{(1)}(t)$, it is convenient to utilize the background field method [21]. We thus decompose all the fields into the background fields (indicated by the hat (ˆ)) and the quantum fields as

$$A^a_\mu(x)=\hat{A}^a_\mu(x)+a^a_\mu(x),\qquad B^a_\mu(t,x)=\hat{B}_\mu(t,x)^a+b^a_\mu(t,x),\tag{33}$$

$$\psi^a(x)=\hat{\psi}^a(x)+p^a(x),\qquad \chi^a(t,x)=\hat{\chi}^a(t,x)+k^a(t,x),\tag{34}$$

$$\bar{\psi}^a(x)=\hat{\bar{\psi}}^a(x)+\bar{p}^a(x),\qquad \bar{\chi}^a(t,x)=\hat{\bar{\chi}}^a(t,x)+\bar{k}^a(t,x).\tag{35}$$

With these decompositions, Eq. (30) becomes

$$\begin{aligned}&[\hat{\chi}^a(t,x)+k^a(t,x)]\left[\hat{F}^a_{\mu\nu}(x)+\hat{D}^{ab}_\mu b^b_\nu(t,x)-\hat{D}^{ab}_\nu b^b_\mu(t,x)+f^{abc}b^b_\mu(t,x)b^c_\nu(t,x)\right]\\ &\quad-\left[\hat{\psi}^a(x)+p^a(x)\right]\left[\hat{F}^a_{\mu\nu}(x)+\hat{D}^{ab}_\mu a^b_\nu(x)-\hat{D}^{ab}_\nu a^b_\mu(x)+f^{abc}a^b_\mu(x)a^c_\nu(x)\right]\\ &=\zeta_1^{(1)}(t)\hat{\psi}^a(x)\hat{F}^a_{\mu\nu}(x)+\zeta_2^{(1)}(t)\left[\gamma_\mu\gamma_\rho\hat{\psi}^a(x)\hat{F}^a_{\rho\nu}(x)-\gamma_\nu\gamma_\rho\hat{\psi}^a(x)\hat{F}^a_{\rho\mu}(x)\right]\\ &\quad+\zeta_3^{(1)}(t)\sigma_{\rho\sigma}\sigma_{\mu\nu}\hat{\psi}^a(x)\hat{F}^a_{\rho\sigma}(x)+O(t).\end{aligned}\tag{36}$$

We compute the one-loop expectation value of the both sides by using propagators of quantum fields in the presence of the background fields (see Ref. [9]). Some calculation gives us

$$\zeta_1^{(1)}(t) = \frac{g_0^2}{(4\pi)^2} C_2(G) \frac{-2}{D-4} (8\pi t)^{2-D/2}, \tag{37}$$

$$\zeta_2^{(1)}(t) = \frac{g_0^2}{(4\pi)^2} C_2(G) \frac{2}{(D-4)(D-2)} (8\pi t)^{2-D/2}, \tag{38}$$

$$\zeta_3^{(1)}(t) = \frac{g_0^2}{(4\pi)^2} C_2(G) \frac{4}{(D-4)(D-2)} (8\pi t)^{2-D/2}. \tag{39}$$

Equation (30) then yields

$$\begin{aligned} \mathcal{S}_{\mu R}(x) = &-\frac{1}{2g_0}\left[1 + \frac{g_0^2}{(4\pi)^2} C_2(G) \frac{2(D-18)}{(D-2)D} (8\pi t)^{2-D/2}\right] \sigma_{\rho\sigma}\gamma_\mu \chi^a(t,x) G^a_{\rho\sigma}(t,x) \\ &- \frac{1}{2g_0}\frac{g_0^2}{(4\pi)^2} C_2(G) \frac{8(D-10)}{(D-2)D} (8\pi t)^{2-D/2} \gamma_\nu \chi^a(t,x) G^a_{\nu\mu}(t,x) + O(t) + O(g_0^3). \end{aligned} \tag{40}$$

We rewrite this expression by the renormalized gauge coupling g in the MS scheme and the modified flowed gaugino field $\mathring{\chi}(t,x)$[4]

$$\begin{aligned} \mathcal{S}_{\mu R}(x) = &-\frac{1}{2g}\left[1 + \frac{g^2}{(4\pi)^2} C_2(G) \left[-\frac{7}{2} - \frac{3}{2}\ln(8\pi\mu^2 t) + \frac{1}{2}\ln(432)\right]\right\} \sigma_{\rho\sigma}\gamma_\mu \mathring{\chi}^a(t,x) G^a_{\rho\sigma}(t,x) \\ &- \frac{g}{(4\pi)^2} C_2(G) 3\gamma_\nu \mathring{\chi}^a(t,x) G^a_{\nu\mu}(t,x) + O(t) + O(g^3). \end{aligned} \tag{42}$$

As Eq. (24) shows, the supercurrent does not depend on the renormalization scale μ when expressed by the running coupling constant. We then may set the renormalization scale as $\mu = 1/\sqrt{8t}$ to yield

$$\begin{aligned} \mathcal{S}_{\mu R}(x) = &-\frac{1}{2\bar{g}(1/\sqrt{8t})}\left\{1 + \frac{\bar{g}^2(1/\sqrt{8t})}{(4\pi)^2} C_2(G) \left[-\frac{7}{2} - \frac{3}{2}\ln\pi + \frac{1}{2}\ln(432)\right]\right\} \sigma_{\rho\sigma}\gamma_\mu \mathring{\chi}^a(t,x) G^a_{\rho\sigma}(t,x) \\ &- \frac{\bar{g}(1/\sqrt{8t})}{(4\pi)^2} C_2(G) 3\gamma_\nu \mathring{\chi}^a(t,x) G^a_{\nu\mu}(t,x) + O(t) + O(\bar{g}^3(1/\sqrt{8t})). \end{aligned} \tag{43}$$

Finally, by taking the $t \to 0$ limit, we obtain the desired expression,

$$\begin{aligned} &\mathcal{S}_{\mu R}(x) \\ &= \lim_{t\to 0}\left(-\frac{1}{2\bar{g}(1/\sqrt{8t})}\left\{1 + \frac{\bar{g}^2(1/\sqrt{8t})}{(4\pi)^2} C_2(G) \left[-\frac{7}{2} - \frac{3}{2}\ln\pi + \frac{1}{2}\ln(432)\right]\right\} \sigma_{\rho\sigma}\gamma_\mu \mathring{\chi}^a(t,x) G^a_{\rho\sigma}(t,x)\right. \\ &\qquad \left. - \frac{\bar{g}(1/\sqrt{8t})}{(4\pi)^2} C_2(G) 3\gamma_\nu \mathring{\chi}^a(t,x) G^a_{\nu\mu}(t,x)\right). \end{aligned} \tag{44}$$

[4]Unlike the flowed gauge field, the flowed fermion field requires the wave function renormalization. The modified flowed gaugino field $\mathring{\chi}(t,x)$ defined by

$$\begin{aligned} \mathring{\chi}(t,x) &= \sqrt{\frac{-\dim(G)}{(4\pi)^2 t^2 \langle \bar{\chi}(t,x)\gamma_\mu (D_\mu - \overleftarrow{D}_\mu)\chi(t,x)\rangle}}\chi(t,x) \\ &= \frac{1}{(8\pi t)^{\epsilon/2}}\left\{1 + \frac{g^2}{(4\pi)^2} C_2(G)\left[\frac{3}{2}\frac{1}{\epsilon} + \frac{3}{2}\ln(8\pi\mu^2 t) - \frac{1}{2}\ln(432)\right] + O(g^4)\right\}\chi(t,x) \end{aligned} \tag{41}$$

can avoid the explicit usage of the wave function renormalization constant.

4 Conclusion

In this work, we constructed the correctly-normalized supercurrent in the 4D $\mathcal{N} = 1$ SYM in the Wess–Zumino gauge by using the gradient flow and the small flow-time expansion. For this, we determined the renormalized supercurrent in dimensional regularization in the one-loop level (Sec. 2). Then we computed the small flow-time expansion of a composite operator (30) (Sec. 3). The obtained expression (44), being UV finite, is independent of the regularization method and hence in particular can be used with the lattice regularization. We hope our representation of the supercurrent will be useful in tuning the gaugino mass towards the supersymmetric point in the continuum limit.[5]

It is interesting to extend the present study to supersymmetric theories which contain matter multiplets; for those theories, the parameter tuning to the supersymmetric point will be quite demanding. In order to treat these extended theories, we have to consider also the flow of the scalar field $\phi(x)$. The simplest choice of the flow equation would be

$$\partial_t\varphi(t,x) = D_\mu D_\mu\varphi(t,x), \qquad \varphi(t=0,x) = \phi(x). \tag{45}$$

We are now studying the small flow-time representation of the supercurrent in the 4D $\mathcal{N} = 2$ SYM.

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