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Pairing and $(\frac{9}{2})^n$ configuration in nuclei in the ^{208}Pb region

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Abstract. Excited states in low-energy spectra in nuclei near ^{208}Pb are considered. The pure $(j = 9/2)^n$ configuration approximation with delta-force is used for ground state multiplet calculations. The multiplet splitting is determined by the pairing energy, which can be defined from the even-odd straggering of the nuclear masses. For the configurations with more than two valence nucleons, the seniority scheme is used. The results of the calculations agree with the experimental data for both stable and exotic nuclei within 0.06-6.16%. Due to simplicity and absence of the fitted parameters, the model can be easily applied for studies of nature of the excited states in a wide range of nuclei.

1 Introduction

Pair interaction of identical nucleons significantly effects on the properties of atomic nuclei [1]. A manifestations of the nucleon pairing is the mass surface splitting and the zero angular momentum of the even-even nuclei ground state. The breaking of two valence identical nucleons pair leads to formation of a characteristic set of the excited states with the positive parity and even spin, the so-called ground state multiplet (GSM). The GSM splitting is determined by the pairing energy (or by the doubled even-odd staggering effect): $\Delta_{nn(pp)} = 2\Delta_{n(p)}$ [2]. The classical example of GSM is the set of low-lying states in the ^{210}Pb and ^{210}Po spectra, which can be considered as the ^{208}Pb core with the pair of the valence neutrons or protons. Since ^{208}Pb is the heaviest double magic stable nucleus, this approximation is quite accurate. Therefore, it is interesting to trace the GSM formation in the systematics of isotopes spectra in the $(N, Z) = (126, 82)$ region with sequential filling of the $2g_{9/2}$ subshell with neutrons and the $1h_{9/2}$ subshell with protons. As an example, figure 1 shows the experimental excitation spectra of the even-even isotones $N = 128$. It can be seen that the neutron pairing energy systematically corresponds to the excited state $J^P = 8^+$, and the proton pairing energy corresponds to the state $J^P = 10^+$. A similar systematic behavior is observed in other chains of isotones and isotopes in the ^{208}Pb region [6].

2 Ground state multiplets and seniority scheme

Since the nucleon force, including the residual pairing interaction, has a short-range attractive character, the δ-potential $V_{12} = -V_0\delta(r_1 - r_2)$ is a good approximation for its description [7]. In this

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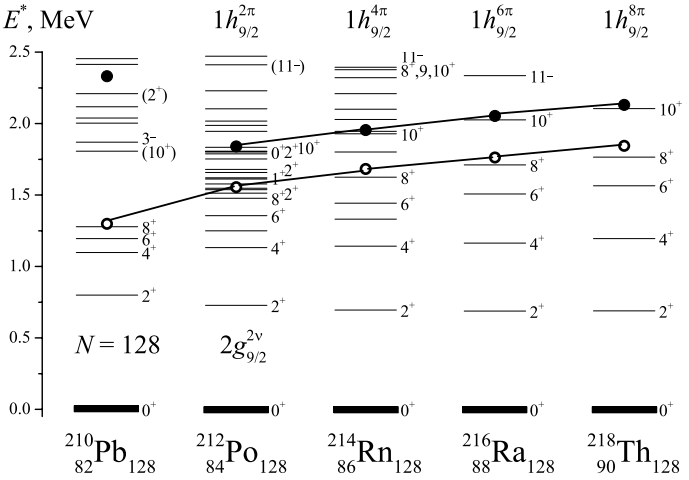

Figure 1: Experimental spectra of low-lying excitations in isotones $N = 128$. The values of the pairing energy of protons $\Delta_{pp}(\bullet)$ and neutrons $\Delta_{nn}(\circ)$ are calculated using nuclear masses [3]. Here and below, the experimental data are from [4, 5]. The figure is from [6].

approximation, the energy of the state with a total angular momentum J depends on V_0 and the radial Slater integral $F_0(nl)$ [7, 8]:

$$\Delta E_{jj:J} = -\frac{1}{2} V_0 F_0(nl)(2j+1)^2 \begin{pmatrix} j & j & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2. \quad (1)$$

Since the values of V_0 and $F_0(nl)$ do not depend on J, the structure of the GSM can be determined from the ratio of the energies of levels with different values of J, to the known experimental value of the excitation energy, for example, $E(2^+)$ [9] or $E(J_{max})$, where $J_{max} = 2j - 1$ [10]. However, the GSM structure can be calculated using the ground state energy shift ΔE_0, defined by the even-odd staggering effect: $\Delta E_0 = \Delta_{nn(pp)} = 2\Delta_{n(p)}$ [11]. Thus, the energies of all the excited GSM states are determined by the values of the nuclear masses.

The δ-potential approximation allows one to describe GSM in the cases of several pairs or an odd number of valence nucleons using the seniority scheme. The seniority model makes it possible to express multiplets of states in the case of the nucleon number $N > 2$ on the basis of the known spectrum of states of a nucleon pair through the sets of coefficients of fractional parentage (CFP) $\langle j^{N-1} s_1 J_1; j | \} j^N sJ \rangle$ [12]:

$$\Delta E_J = \sum_{J_1} \langle j^{N-1} s_1 J_1; j | \} j^N sJ \rangle^2 \Delta E_{J_1}, \quad (2)$$

where seniority s is the number of unpaired nucleons, $s_1 = s - 1$, J_1 is the total spin corresponding to s_1, the sum runs over all values of J_1. This approach is actively used for the GSM description in the ^{40}Ca region, because the seniority is a good quantum number for any two-particle interaction for $j \leq 7/2$. Moreover this approach can be used for higher values of j for a wide range of two-particle interactions, including the δ-potential [9].

Table 1: Possible values of the total angular momentum J for the $j = 9/2$ subshell.

Seniority s	J
2	$2, 4, 6, 8$
3	$3/2, 5/2, 7/2, 11/2, 13/2, 15/2, 17/2, 21/2$
4	$0, 2, 3, 4^2, 5, 6^2, 7, 8, 9, 10, 12$

Table 1 shows the possible values of the total angular momentum J for the subshell $j = 9/2$ for different values of the seniority s. With increasing j, the number of possible states essentially increases. Usually in the experimental spectra the whole set of states for $s = 4$ is not observed, but the levels with high values of $J = 10$ and 12 are well identified.

3 Results

The neutron-rich isotopes of lead are ideal for testing of the properties of nucleon-nucleon interaction and the seniority scheme since they have the closed shell core ^{208}Pb. Owing to the development of experimental possibilities, it became possible to determine the position of excited states in the neutron-rich lead isotopes up to ^{216}Pb. In figure 2 the results of our calculations for the seniority $s = 2$ are presented, assuming the filling of the subshell $2g_{9/2}$ by neutrons.

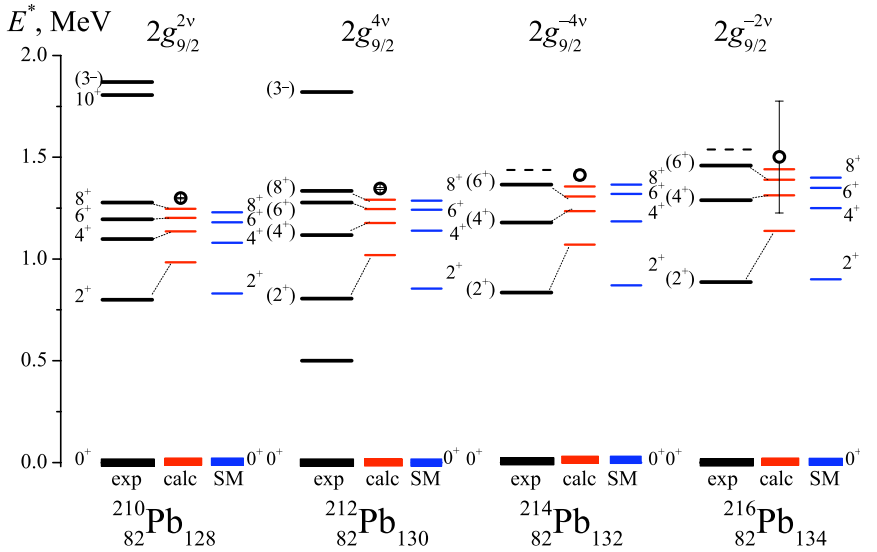

Figure 2: The spectra of the $^{210-216}$Pb isotopes. Black lines are the experimental spectra (exp) from [4, 5], red lines are our calculations (calc) based on Δ_{nn} (∘), blue lines are the results from [13] (SM).

Use of Δ_{nn} as the GSM splitting leads to a good agreement for the state $J^P = 6^+$ and allows us to estimate the energy of the state $J^P = 8^+$. The pure-configuration $(2g_{9/2})^n$ approximation with δ-force cannot correctly reproduce positions of the $J^P = 2^+, 4^+$ levels. The regard for the configuration mixing leads to the substantial improvement of the experimental data description. Figure 2 shows the

results of the calculations performed in the shell model (SM) in the $(g_{9/2}i_{11/2}d_{3/2}d_{5/2}g_{7/2}s_{1/2}j_{15/2})^n$ configuration space [13]. The wide configuration space in combination with the Kuo-Herlig potential modified for heavy nuclei [14] makes it possible to describe the experimental spectra with an accuracy of about 100 keV [13].

Isotones with $N = 126$ represent an excellent example of the applicability of the senority scheme with $s > 2$ [15]. Calculations in the entire range of proton configurations using the modified Kuo-Herling interaction showed a good agreement with the experimental data and confirmed the seniority conservation for the isotones up to $Z = 91$. The results were obtained in a wide configuration space, and it would be interesting to verify correlation between the experimental spectra and calculations in the pure $(1h_{9/2})^n$ configuration. In figure 3a the experimental spectrum of the even-odd ^{211}At isotope are presented. Our calculations in the configuration approximation $(1h_{9/2})^{3\pi}$ show a good agreement with experiment except for the first state $(7/2)^-$. For comparison, in figure 3a the results of the calculations in the shell model using an effective potential based on the Bonn potential A (SM2) [16] are also present. The states shown in the figure were obtained with the configuration $(1h_{9/2})^{3\pi}$, except for the first state $(7/2)^-$, for which the contribution of the $(1h^2_{9/2}2f_{7/2})^{\pi}$ configuration dominates.

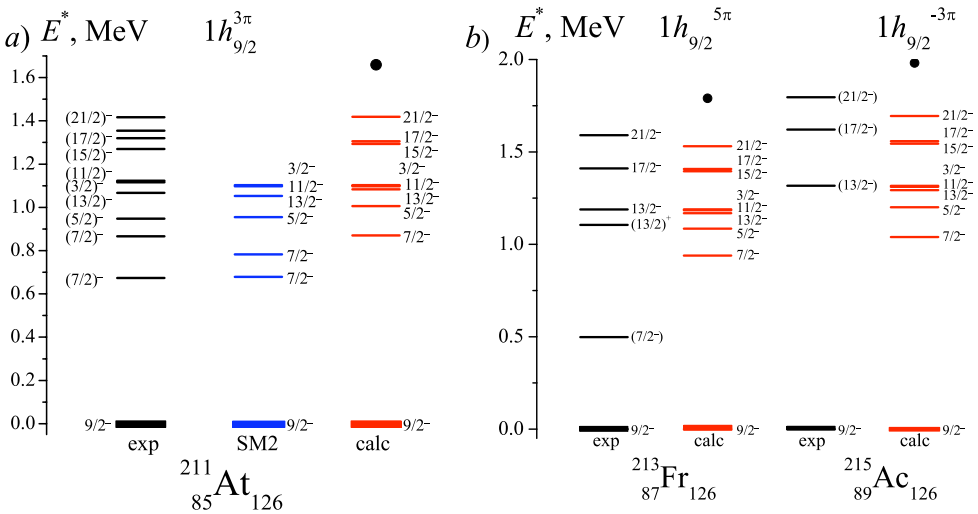

Figure 3: The spectra of the odd isotopes a) ^{211}At, b) ^{213}Fr and ^{215}Ac with Δ_{pp} (•).

In figure 3b, the spectra of the even-odd isotopes ^{213}Fr and ^{215}Ac calculated assuming the pure $(1h_{9/2})^n$ configuration for seniority $s = 1$ and 3 are compared with the experiment. The experimental spectra themselves no longer contain all the levels predicted by the seniority scheme. It should be noted that these isotopes belong to the region of neutron-rich short-lived nuclei and the precise determination of their spectra of excited states is a complex experimental task. Low-lying excitations $J^P = 13/2^-, 17/2^-, 21/2^-$ are defined with good reliability and interpreted as the $(1h_{9/2})^{5\pi}$ configuration in ^{213}Fr [17] and the $(1h_{9/2})^{-3\pi}$ configuration in ^{215}Ac [18]. Our calculations show the authenticity of this interpretation. However, the calculated energies of levels with $J^P = 21/2^-$ lie below the experimental values. Thus, the deviation of the calculated values from the experimental values is 6%. This value corresponds to the accuracy of our model description of the experimental data considered in this paper.

The results of the calculations for $s = 4$ are shown in figure 4 for spectra of the ^{212}Rn and ^{214}Ra nuclei with 4 protons and 4 proton holes in the $1h_{9/2}$ shell. For comparison, the results of calculations [13, 19] and [16] (SM3) are given. One can see that use of pairing energy estimated from the nuclear mass data [3] provide a good description not only for GSM with the senority $s = 2$, but also for experimental states with $J^P = 12^+, 10^+$ with $s = 4$.

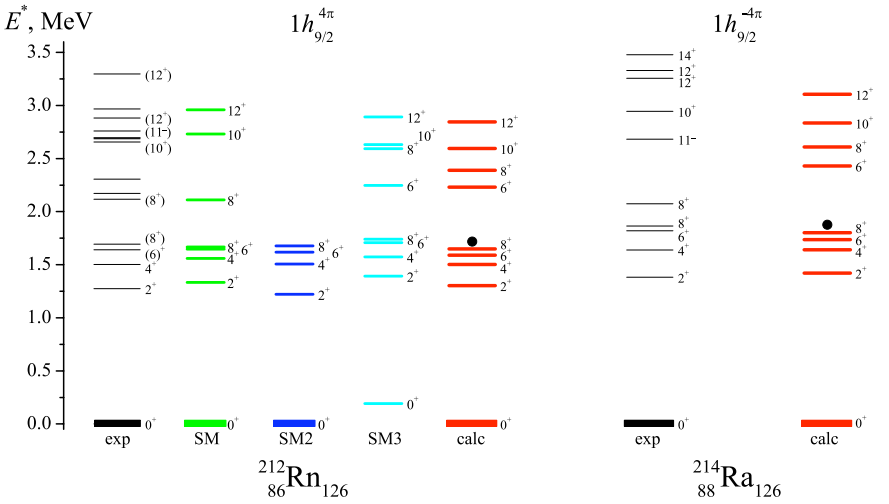

Figure 4: The spectra of even-even isotopes ^{212}Rn and ^{214}Ra. The values of Δ_{pp} are denoted by (•).

4 Conclusions

Low-lying excited states of neutron-rich lead isotopes and the isotopes of the $N = 126$ chain are considered. Basing on the nuclear mass data [3] multiplets for the seniority values $s = 2, 3, 4$ are calculated in the configuration approximation $(2g_{9/2})^n$ for the valence neutrons and $(1h_{9/2})^n$ for the valence protons. The results are compared with the shell model calculations with the effective potentials and the widest possible set of configurations for valence nucleons.

Since the ^{208}Pb core is a closed shell double magic nucleus, the approximation of pure configurations for the valence nucleons is appropriate, and the low-lying part of the spectrum obeys the seniority scheme. The approach of δ-potential considered in this paper is a simplified form of residual interaction and can not lead to an accurate description of the spectrum of excited states. Nevertheless, the application of the pairing energy estimated from the nuclear masses as the splitting of GSM allows us to obtain a agreement with accuracy 6% with the experimental data not only for the seniority $s = 2$, but also for the higher s values without involving fitting parameters.

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