

# Studies of positive-parity low-spin states in the $A = 150$ region

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**Abstract.** A systematic investigation of low-lying levels of nuclides in the mass 150 region has been undertaken at iThemba LABS. An extensive set of data on the low-lying, positive-parity bands in the nuclides between  $N = 88$  and  $92$  and  $Sm$  to  $Yb$  has been obtained from  $\gamma$ - $\gamma$  coincidence measurements following fusion-evaporation reactions optimized of the population of low-spin states. The energies and electromagnetic properties of the so-called  $\beta$ - and  $\gamma$ -bands of nuclei in this region have been compared with the solutions of a five dimensional collective Hamiltonian for quadrupole vibrational and rotational degrees of freedom, with moments-of-inertia and mass parameters determined by constrained self-consistent relativistic mean-field calculations using the PC-F1 relativistic functional. Some of the results of this comparison are presented here.

## 1 Introduction

The Bohr Hamiltonian [1], one of the earliest models describing the lowest-lying excitations of deformed nuclei, predicted the existence of the so-called  $\beta$  and  $\gamma$  vibrational bands, based on  $K=0$  and  $K=2$  excitations.

While numerous examples of  $K=0$  and  $K=2$  bands have been identified, the identification of the lowest lying  $K=0$  band with the  $\beta$ -band of the Bohr Hamiltonian is under question [2]. The mass 150 region elicits experimental interest as a testing ground for alternate descriptions of low-lying  $0^+$  bands. The phenomenon of shape-coexistence (see e.g. [3,4]), pairing isomers [5] and X(5) symmetry [6] have been invoked to interpret the low-lying structures around  $^{152}Sm$  at  $N=90$ . However, the original Bohr-Hamiltonian approximated the nuclear potential with a harmonic oscillator in  $\beta$  and  $\gamma$ . A more realistic potential would be expected to be more complex, possibly including co-existing wells in

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the  $\beta - \gamma$  plane. Such a model would then automatically include shape-coexistence and vibrational bands under the one umbrella. Indeed a more realistic “Bohr Hamiltonian” has been developed - a five-dimensional Hamiltonian for quadrupole vibrational and rotational degrees of freedom, with collective parameters determined by constrained self-consistent covariant density functional calculations for triaxial shapes [7,8]. Rather than use a harmonic oscillator potential in  $\beta$  and  $\gamma$ , a realistic potential  $V_{coll}(\beta, \gamma)$  is determined from the potential energy surfaces calculated using relativistic mean field (RMF) theory.

Experimentally, a large body of data from in-beam  $\gamma\gamma$ -coincidence measurements, following fusion evaporation reactions, has been collected in experiments performed at iThemba LABS, Argonne National Laboratory and the University of Jyväskylä. In most even-even nuclides between  $N = 88$  and  $92$  and between Sm and Yb, the  $0_2^+$ , and gamma bands have been delineated up to at least spin  $12\hbar$ . Here, we will combine a subset of our results with data from the literature and compare the data with the predictions of the “realistic” five-dimensional Hamiltonian.

## 2 Theory

Detailed formalism of the five-dimensional collective Hamiltonian (5DCH) can be found in a number of articles, see, e.g. [7-10]. For completeness, a brief introduction is presented here. The five-dimensional collective Hamiltonian, which can simultaneously treat the quadrupole vibrational and rotational excitations, is expressed in terms of the two deformation parameters  $\beta$  and  $\gamma$ , and three Euler angles  $\Omega$  that define the orientation of the intrinsic principal axes in the laboratory frame,

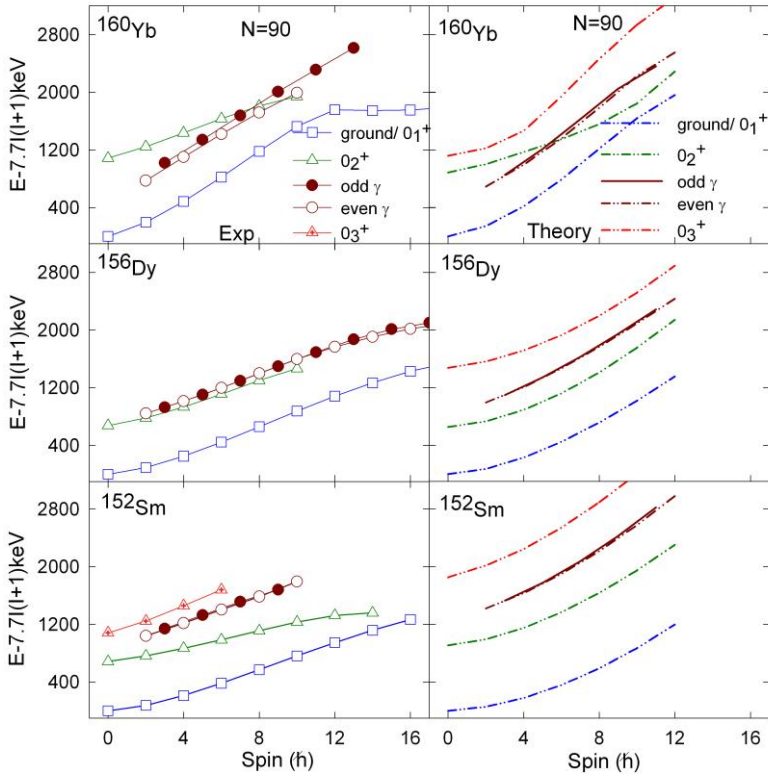
$$H_{coll}(\beta, \gamma) = T_{vib}(\beta, \gamma) + T_{rot}(\beta, \gamma, \Omega) + V_{coll}(\beta, \gamma) \quad (1)$$

The three terms in collective Hamiltonian are respectively the vibrational kinetic energy, rotational kinetic energy, and collective potential. There are seven collective parameters, including the mass parameters  $B_{\beta\beta}$ ,  $B_{\beta\gamma}$ ,  $B_{\gamma\gamma}$ , the moments of inertia  $I_k$  ( $k=1, 2, 3$ ) and the collective potential  $V_{coll}$ , which are all functions of the intrinsic deformations  $\beta$  and  $\gamma$ . They in total govern the dynamics of the collective Hamiltonian. The eigenvalue problem of the Hamiltonian (1) is solved using an expansion of eigenfunctions in terms of a complete set of basis functions that depend on the five collective coordinates  $\beta$ ,  $\gamma$ , and  $\Omega$ . Using the collective wave functions thus obtained, various observables such as transition probabilities can be calculated.

In recent years, the five-dimensional collective Hamiltonian based on the covariant density functional theory (5DCH-CDFT) has been developed and extensively applied to describe the nuclear collective properties, such as the phase transitions, shape evolutions as well as the low-lying spectra along with the isotopic and isotonic chains in different mass regions [7,8,11-13]. In the framework of 5DCH-CDFT, the microscopic collective parameters of 5DCH are all determined from the CDFT. The moments of inertia are calculated with Inglis-Belyaev formula and the mass parameters from the cranking approximation.  $V_{coll}$  is obtained by subtracting the zero-point energy corrections from the total energy that corresponds to the solution of constrained triaxial CDFT.

## 3 Results and Discussion

To highlight the salient points of the work in these proceedings, we now focus on the data and calculations of a subset of the region, along  $N=90$ . The energies of the experimental ground,  $0_2^+$ , gamma and  $0_3^+$  bands for  $^{152}\text{Sm}$  [14, 15],  $^{156}\text{Dy}$  [16] and  $^{160}\text{Yb}$  [17] are plotted, less a rigid-rotor reference, as a function of spin in Fig. 1.



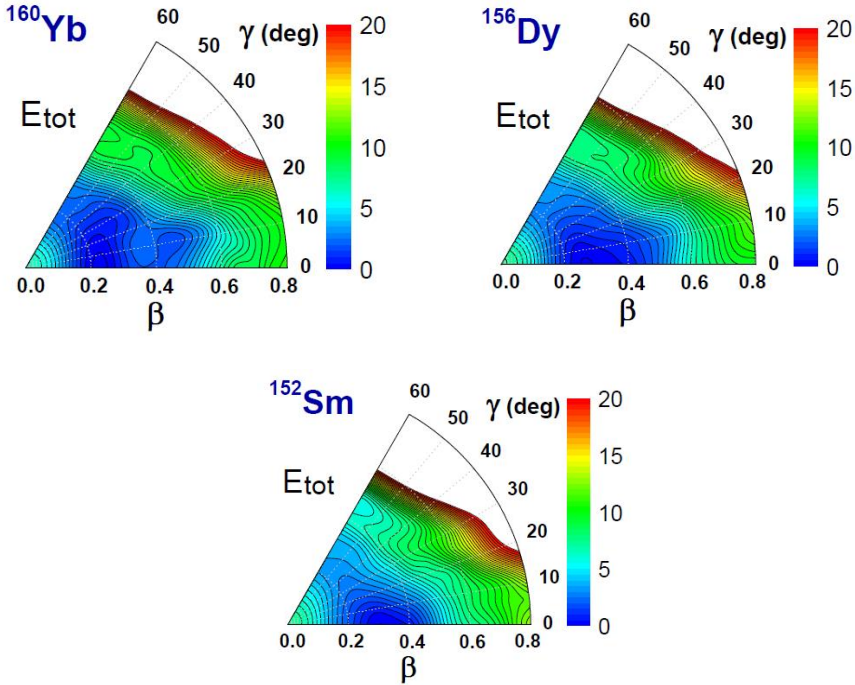
**Fig. 1.** Experimental bands left, compared with calculated bands, right.

A first observation is that in all cases, the gamma-bands have a very similar slope to that of the ground bands – indicating similar moments-of-inertia for the two bands. The same cannot be said for the  $0_2^+$  bands, where only in the case of  $^{156}\text{Dy}$  does the  $0_2^+$  band run parallel to the ground band. In  $^{160}\text{Yb}$  the  $0_2^+$  band, has a higher-excitation energy and a larger moment-of-inertia than in  $^{156}\text{Dy}$ , which causes it to cross the gamma-band. In  $^{152}\text{Sm}$  the  $0_2^+$  band again has larger moment-of-inertia than the ground band, but no crossing occurs with the gamma-band due to the lowering of the  $0_2^+$  excitation energy.

To understand the possible role deformation plays in the moments-of-inertia of these bands, we inspect the RMF potential energy surfaces for these nuclei in Fig. 2. The surface of  $^{156}\text{Dy}$  shows a single minimum in the  $\beta - \gamma$  plane that is slightly extended towards higher  $\beta$  and mild triaxiality. This extension develops into a well-deformed triaxial superdeformed minimum in  $^{160}\text{Yb}$  at  $(\beta, \gamma) \sim (0.45, 10^\circ)$ .

The corresponding solutions to the 5DCH are shown on the right of Fig. 1, where they are compared to the experimental bands on the left. Note that in all cases, the predicted gamma-bands follow the ground band with a similar moment-of-inertia. Inspection of the odd-even spin staggering in the gamma-bands in both experiment and theory indicates a tendency towards gamma-rigid rotation along  $N=90$  and  $92$ , and vibrational character along  $N=88$ . For the  $0_2^+$  bands, the theory predicts that sometimes they can be interpreted as collective excitations built on the ground band and in other cases they are the result of a co-existing minimum in the potential energy surface. In  $^{160}\text{Yb}$  the calculations predict that the  $0_2^+$  band should have a very different moment-of-inertia to the ground and gamma-bands.

This results in a crossing of the gamma-band by the  $0_2^+$  band, in good agreement with the data. Inspection of the probability density distribution of the wavefunction of the  $0_2^+$  band confirms that it is localized in the minimum near  $\beta=0.45$ . In other words, the  $\beta$ -band of  $^{160}\text{Yb}$  should rather be identified as a superdeformed triaxial band.



**Fig. 2.** RMF potential energy surfaces for  $^{160}\text{Yb}$ ,  $^{156}\text{Dy}$  and  $^{152}\text{Sm}$

The crossing of the gamma band by the  $0_2^+$  band is a general feature observed in the data for the  $N=88$  to  $92$  isotopes of Yb and Er [17]. The crossings are well reproduced by the theory in which the so-called  $\beta$ -band is identified with a coexisting triaxial minimum.

In  $^{156}\text{Dy}$  the calculated  $0_2^+$  band is in reasonable agreement with experiment – while the  $0_2^+$  band lies slightly lower in energy than the experimental one, it has a similar moment-of-inertia to the ground and gamma-bands. This band is better understood as a vibration.

The PES surface for  $^{152}\text{Sm}$  has a single minimum on the prolate axis that is soft in the  $\beta$ -direction, reminiscent of the flat  $X(5)$  potential[5]. The feature of softness in the PES in the  $\beta$ -direction is also a feature for the  $N=90$  nucleus  $^{154}\text{Gd}$  [17]. Three  $0^+$  bands are known experimentally in  $^{152}\text{Sm}$ , but their theoretical counterparts lie higher in excitation energy. Theoretically, the  $0_3^+$  band is understood as a two-phonon band; the low excitation energy of the experimental band may point to a role for other degrees of freedom such as pairing[14].

Many other quantities have been calculated using the 5DCH, including E2 transition rates and the staggering in the gamma-bands. These quantities are generally in good agreement with the data. The E0 transition rates are an exception, as they are generally calculated to be stronger than those observed experimentally. A full report of this work will be published elsewhere [17].

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