

# Triaxiality in the odd- $A$ nuclei $^{109-117}\text{I}$ studied through a microscopic rotation-particle coupling

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**Abstract.** A systematic study of ground state spectrum with the triaxial deformation  $\gamma$  for odd- $A$  Iodine isotopes  $^{109-117}\text{I}$  is carried out with the nonadiabatic quasiparticle approach. The rotation-particle coupling is accomplished microscopically such that the matrix elements of a particle-plus-rotor system are written in terms of the rotor energies. The  $5/2^+$  state is confirmed as ground state for odd- $A$   $^{111-117}\text{I}$  and also coming out as lowest in energy for  $^{109}\text{I}$ .

## 1 Introduction

Iodine with  $Z = 53$  lies in the transitional region between near-spherical Sn ( $Z = 50$ ) and deformed Ce ( $Z = 58$ ) nuclei. The transitional nuclei exhibit many interesting phenomena like shape coexistence, triaxiality, rapid variation in shape with changing proton and neutron number, band crossing, etc. Ground state structure of odd- $A$  Iodine isotopes ( $^{109-117}\text{I}$ ) from the proton drip line to the region near the  $\beta$ -stability line are analysed in this work with the microscopic nonadiabatic quasiparticle approach. Study of Iodine chain is important to see the effect of a successive increment of neutron numbers on the shape of the isotopes.  $^{109}\text{I}$  is a ground state proton emitter and well studied in Refs. [1, 2]. The deformed shell model predicts that the odd proton lies in the  $d_{5/2}$  and  $g_{7/2}$  orbitals near the Fermi level [3]. These states are coupled to the states of the corresponding even-even core to study the rotational bands.

## 2 Formalism

In the microscopic rotation particle-coupling, the wave function of an odd- $A$  nucleus with the particle at position  $\vec{r}$ , and orientation  $\omega$  of the rotor is given by

$$\Psi_{IM}(\vec{r}, \omega) = \sum_{IjR\tau} \frac{\phi_{IjR\tau}^I(r)}{r} |IjR\tau, IM\rangle, \quad (1)$$

where  $(I, M, K)$  are quantum numbers for the particle-plus-rotor system,  $(l, j, \Omega)$  are related to the particle and  $(R, K_R, M_R)$  are rotor quantum numbers.  $\phi_{IjR\tau}^I(r)$  and  $|IjR\tau, IM\rangle$  are the radial and angular parts of the wave function, respectively. The total Hamiltonian is written as

$$H = H_{\text{avg}} + H_{\text{pair}} + H_{\text{rot}}, \quad (2)$$

where  $H_{\text{avg}}$  corresponds to the intrinsic energy of the odd particle. The pairing interaction is given by  $H_{\text{pair}}$ , and  $H_{\text{rot}}$  is the rotor Hamiltonian.

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The matrix element of  $H_{\text{rot}}$  can be written in  $K$  representation by utilizing the experimental rotor energies  $E_{TRI}$  and the calculated rotor wave functions  $c_{K_R}^{RI}$  as [2]

$$\begin{aligned} & \langle Ij\Omega_p, K' | IM | H_{\text{rot}} | Ij\Omega_p, K | IM \rangle \\ &= \sum_{R, K_R, K'_R} A_{j\Omega_p, RK'_R}^{IK'} \sum_i c_{K_R}^{RI} E_{TRI} c_{K'_R}^{RI} A_{j\Omega_p, RK_R}^{IK} \\ &= W_{j\Omega_p, \Omega_p}^{K'K}. \end{aligned} \quad (3)$$

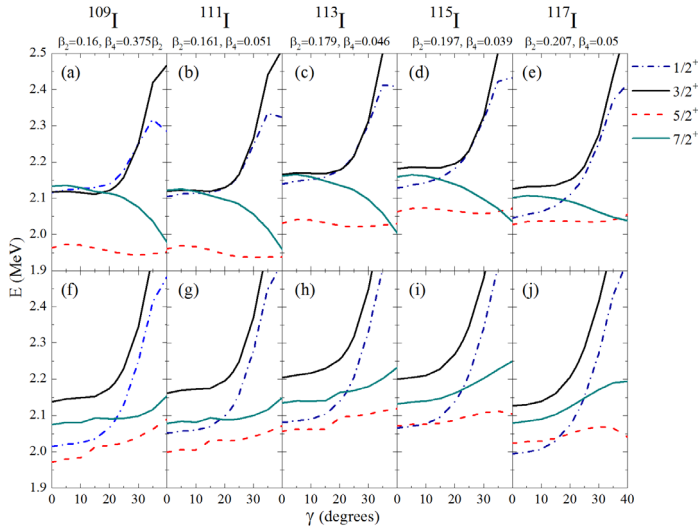
Here, the amplitude,  $A_{j\Omega_p, RK_R}^{IK}$  is used to transform the wave function from  $R$  to  $K$  representation and vice versa. The matrix element of the total Hamiltonian  $H$  can be written as

$$\begin{aligned} \langle q' K' | IM | H | q K | IM \rangle &= \epsilon_q \delta_{K'K} \delta_{q'q} + \\ & \sum_{Ij\Omega_p} W_{j\Omega_p, \Omega_p}^{K'K} \times \int dr f_{uw} \phi_{Ij\Omega_p}^{IK'}(r) \phi_{Ij\Omega_p}^{IK}(r), \end{aligned} \quad (4)$$

where  $q$  defines particle state and  $\epsilon_q$  is the quasiparticle energy obtained through the single-particle energy [4]. The quantity  $f_{uw}$  is used to transform the matrix element from single-particle states to quasiparticle states.

## 3 Results and discussions

We analyze the systematics of the energy levels of even  $N$  iodine isotopes  $^{109}\text{I}$  to  $^{117}\text{I}$  with different parameter sets for the Woods-Saxon potential. The corresponding rotor ( $^{108}\text{Te}$  to  $^{116}\text{Te}$ ) states [5] are taken as input in the nonadiabatic quasiparticle approach. The positive parity spectrum of Iodine isotopes is presented in Fig. 1 as a function of  $\gamma$ . In Fig. 1(a-e), the Esbensen-Davids parameter set [6] is used to calculate the energy levels of the considered iodine isotopes. The value of deformation parameters  $\beta_2$  and  $\beta_4$  are taken from Ref. [7]. Our calculations with the Esbensen-Davids parameters suggest that the state with  $I^\pi = 5/2^+$  is the ground state of  $^{109}\text{I}$  to  $^{117}\text{I}$ , in consistency with Ref. [1] where the experimental results are presented for  $^{111}\text{I}$  to  $^{117}\text{I}$  along with a prediction for  $^{109}\text{I}$ . The



**Figure 1.** The yrast states of  $^{109}\text{I}$  to  $^{117}\text{I}$  as a function of  $\gamma$ . In the upper panel (a-e) the results are shown calculated with the Esbensen-Daids [6] parameter set and in the lower panel (f-j) the Chepurnov [8] parameter set is utilised. Rotor ( $^{108,110,112,114,116}\text{Te}$ ) energies are taken from Ref. [5].

other lower angular momentum states ( $1/2^+$  and  $3/2^+$ ) are nearly degenerate with  $7/2^+$  at lower  $\gamma$ . This degeneracy is lifted with an increase in the neutron number. In Ref. [1], the energy gap between  $I^\pi = 5/2^+$  and  $I^\pi = 7/2^+$  is reported to be increasing with neutron number which exhibits decrement in triaxiality with our results. From Fig. 1(a), it can be seen that the calculated results for  $^{109}\text{I}$  following the same trend of the results for other isotopes.

To check the impact of the parameters of the mean field potential on the above inferences, we repeat the calculations with the Chepurnov parameter set [8] for which the results are presented in Fig. 1(f-j). The major difference between above two parameter sets is the strength of the spin-orbit potential. With the Chepurnov parameter set also, the energy of  $I^\pi = 5/2^+$  turns out to be the lowest for  $^{109,111,113}\text{I}$  but not for  $^{115,117}\text{I}$  at low  $\gamma$  ( $< 15^\circ$ ).

We infer that the results from Esbensen-Daids parameter set are consistent with the data and thus this parameter set is more reliable than the Chepurnov parameter set. The state  $5/2^+$  is lowest in energy for all these isotopes but since  $^{109}\text{I}$  is the ground state proton emitter, the analysis of decay width proves  $3/2^+$  state as its proton emitting state and the ground state [2].

## 4 Summary

A systematic study of a chain of nuclei is important to understand the change in nuclear interaction with the one species of nucleons. Iodine isotopes  $^{109-117}\text{I}$  with even- $N$  are analysed with the triaxial degree of freedom. Despite the triaxiality decreasing with increasing  $N$ , the ground state calculated with the Esbensen-Daids parameter set

for  $^{111}\text{I}$  to  $^{117}\text{I}$  shows a good agreement with the experimental data compared to the Chepurnov parameter set. For the proton emitter  $^{109}\text{I}$ , the decay analysis suggests  $3/2^+$  as the ground state.

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