Analogous Gamow-Teller and $M1$ Transitions in $T_z = \pm 1/2$ Mirror Nuclei and in $T_z = \pm 1, 0$ Triplet Nuclei relevant to Low-energy Super GT state

Yoshitaka FUJITA[1,2,⋆], *Hirohiko* FUJITA[1], and *Yusuke* Tanumura[3]

[1] *Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan*
[2] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
[3] *Department of Physics, Tohoku University, Sendai 980-8578, Japan*

Abstract. Nuclei have spin- and isospin-degrees of freedom. Therefore, Gamow-Teller (GT) transitions caused by the $\sigma\tau$ operator (spin-isospin operator) are unique tools for the studies of nuclear structure as well as nuclear interactions. They can be studied in β decays as well as charge-exchange (CE) reactions. Similarly, $M1$ γ decays are mainly caused by the $\sigma\tau$ operator. Combined studies of these transitions caused by Weak, Strong, and Electro-Magnetic interactions provide us a deeper understanding of nuclear spin-isospin-type transitions. We first compare the strengths of analogous GT and $M1$ transitions in the $A = 27$, $T_z = \pm 1/2$ mirror nuclei ^{27}Al and ^{27}Si. The comparison is extended to the $T_z = \pm 1, 0$ nuclei. The strength of GT transition from the ground state (g.s.) of ^{42}Ca to the 0.611 MeV first $J^\pi = 1^+$ state in ^{42}Sc is compared with that of the analogous $M1$ transition from the 0.611 MeV state to the $T = 1$, 0^+ g.s. (isobaric analog state: IAS) in ^{42}Sc. The 0.611 MeV state has the property of Low-energy Super GT (LeSGT) state, because it carries the main part of the GT strength of all available transitions from the g.s. of ^{42}Ca (and ^{42}Ti) to the $J^\pi = 1^+$ GT states in ^{42}Sc.

1 Introduction: GT and $M1$ operators

Gamow-Teller (GT) transitions caused by the $\sigma\tau$-type operator are "allowed transitions" in β decays [1]. They are also selectively excited in charge-exchange (CE) reactions, such as (p, n) or (^{3}He, t) reactions, performed at intermediate incident energies of more than 100 MeV/nucleon and 0° [2–4]. Note that $\sigma\tau$-type nucleon-nucleon (N-N) interaction is strong under such conditions [5].

The $M1$ transitions studied in γ decays or (e, e') reactions are the simplest "magnetic-type transitions" caused by the electro-magnetic (EM) interaction [1, 4, 6]. In spite of the name $M1$, the operator that causes $M1$ transition is similar to that of GT in the sense that the main contribution is expected from the $\sigma\tau$ operator.

Our aim here is to compare the strengths of analogous GT and $M1$ transitions and derive one-step deeper structure information that are not obtained in individual studies of GT and $M1$ transitions.

2 Reduced GT and $M1$ transition strengths

The strengths of GT and $M1$ transitions can be compared by means of the "reduced transition strength (probability)" B(GT) and $B(M1)$. The expression for these values reduced in spin (J) can be found in text books (see *e.g.* [6, 7]). In order to compare transition strengths for analogous GT and/or $M1$ transitions, however, it is important to reduce the matrix element in terms of isospin as well [8, 9]. Therefore, we start here with the reduced matrix elements in spin (J) but not in isospin and follow the convention of Edmonds [10].

⋆e-mail: fujita@rcnp.osaka-u.ac.jp

2.1 Reduced GT transition strength

The reduced GT transition strength B(GT) for the transition from the initial state with spin J_i, isospin T_i, and z-component of isospin T_{zi} to the final state with J_f, T_f, and T_{zf} is given by [11]

$$B^{\pm 1}(\mathrm{GT}) = \frac{1}{2J_i + 1}\left|\langle J_f T_f T_{zf} || \frac{1}{\sqrt{2}} \sum_{j=1}^{A} (\boldsymbol{\sigma}_j \tau_j^{\pm 1}) || J_i T_i T_{zi}\rangle\right|^2, \quad (1)$$

where $\tau^{\pm 1} = \mp(1/\sqrt{2})(\tau_x \pm i\tau_y)$ and transforms as a tensor of rank one, and $T_z = (N - Z)/2$. By applying the Wigner-Eckart theorem in isospin space, we get

$$B(\mathrm{GT}) = \frac{1}{2J_i + 1}\,\frac{1}{2}\,\frac{C_{\mathrm{GT}}^2}{2T_f + 1}\,\left|\langle J_f T_f ||| \sum_{j=1}^{A} (\boldsymbol{\sigma}_j \boldsymbol{\tau}_j) ||| J_i T_i\rangle\right|^2$$

$$= \frac{1}{2J_i + 1}\,\frac{1}{2}\,\frac{C_{\mathrm{GT}}^2}{2T_f + 1}\Big[M_{\mathrm{GT}}(\sigma\tau)\Big]^2, \quad (2)$$

where C_{GT} is the isospin Clebsch-Gordan (CG) coefficient $(T_i T_{zi} 1 \pm 1 | T_f T_{zf})$, and the $M_{\mathrm{GT}}(\sigma\tau)$ is the isovector (IV) spin-type GT matrix element.

From this expression for the "reduced" GT transition strength, we see that B(GT) consists of the squared value of the matrix element of the IV spin operator $M_{\mathrm{GT}}(\sigma\tau)$ and spin and isospin geometrical factors. Therefore, even if the initial and final states are common, transitions in reversed directions have different B(GT) values. For example, a GT transition from a state having $|J\ T\ T_z\rangle$ of $|0\ T_0\ T_0\rangle$ to the $|1\ T_0-1\ T_0-1\rangle$ state has three times larger B(GT) than that in the reverse direction.

2.2 Reduced *M*1 transition strength

The operator $\boldsymbol{\mu}$ for $M1$ transitions and magnetic moments consists of an orbital part $g_\ell \boldsymbol{\ell}$ and a spin part $g_s \boldsymbol{s}$ [$=(1/2)g_s\boldsymbol{\sigma}$]. It can be rewritten as the sum of isoscalar (IS) and IV terms (for example, see [6]) as,

$$\begin{aligned}\boldsymbol{\mu} &= \Big[\sum_{j=1}^{A} g_\ell \boldsymbol{\ell}_j + g_s \boldsymbol{s}_j\Big]\mu_N \\ &= \Big[\sum_{j=1}^{A}\left(g_\ell^{\rm IS}\boldsymbol{\ell}_j + g_s^{\rm IS}\boldsymbol{s}_j\right) - \sum_{j=1}^{A}\left(g_\ell^{\rm IV}\boldsymbol{\ell}_j + g_s^{\rm IV}\boldsymbol{s}_j\right)\tau_{zj}\Big]\mu_N, \quad (3)\end{aligned}$$

where μ_N is the nuclear magneton, the z-component of the isospin operator $\tau_{zj}=1$ for neutrons and -1 for protons, and τ_{zj} is τ_j^0. The coefficients $g^{\rm IS}$ and $g^{\rm IV}$ are the IS and IV combinations of gyromagnetic factors (g factors): $g_\ell^{\rm IS}=\frac{1}{2}(g_\ell^\pi+g_\ell^\nu)$, $g_s^{\rm IS}=\frac{1}{2}(g_s^\pi+g_s^\nu)$, $g_\ell^{\rm IV}=\frac{1}{2}(g_\ell^\pi-g_\ell^\nu)$, and $g_s^{\rm IV}=\frac{1}{2}(g_s^\pi-g_s^\nu)$. For bare protons and neutrons, the orbital and spin g factors are $g_l^\pi=1$ and $g_l^\nu=0$, and $g_s^\pi=5.586$ and $g_s^\nu=-3.826$, respectively. Therefore, we get $g_\ell^{\rm IS}=0.5$, $g_s^{\rm IS}=0.880$, $g_\ell^{\rm IV}=0.5$, and $g_s^{\rm IV}=4.706$.

Starting from the reduced matrix elements in spin but not in isospin, and following the convention of Edmonds [10], the reduced $M1$ transition strength B(M1) can be written as [11]

$$\begin{aligned}B(M1) &= \frac{1}{2J_i+1}\,\frac{3}{4\pi}\left|\langle J_f T_f T_{zf}\|\boldsymbol{\mu}\|J_i T_i T_{zi}\rangle\right|^2 \\ &= \frac{1}{2J_i+1}\,\frac{3}{4\pi}\mu_N^2\Big|\langle J_f T_f T_{zf}\|\sum_{j=1}^{A}\left(g_\ell^{\rm IS}\boldsymbol{\ell}_j + g_s^{\rm IS}\frac{1}{2}\boldsymbol{\sigma}_j\right) \\ &\qquad - \sum_{j=1}^{A}\left(g_\ell^{\rm IV}\boldsymbol{\ell}_j + g_s^{\rm IV}\frac{1}{2}\boldsymbol{\sigma}_j\right)\tau_j^0\|J_i T_i T_{zi}\rangle\Big|^2, \quad (4)\end{aligned}$$

where $T_z=T_{zf}=T_{zi}$ for $M1$ transitions. By again applying the Wigner-Eckart theorem in isospin space, we get

$$\begin{aligned}B(M1) &= \frac{1}{2J_i+1}\,\frac{3}{4\pi}\mu_N^2\Big[\left(g_\ell^{\rm IS}M_{M1}(\ell)+g_s^{\rm IS}\frac{1}{2}M_{M1}(\sigma)\right) \\ &\quad - \frac{C_{M1}}{\sqrt{2T_f+1}}\left(g_\ell^{\rm IV}M_{M1}(\ell\tau)+g_s^{\rm IV}\frac{1}{2}M_{M1}(\sigma\tau)\right)\Big]^2. \quad (5)\end{aligned}$$

The isospin CG coefficient $C_{M1}=(T_iT_{zi}10|T_fT_{zf})$ comes out explicitly by the use of reduced matrix elements, where $T_{zf}=T_{zi}=T_z$. The matrix elements $M_{M1}(\ell)$, $M_{M1}(\sigma)$, $M_{M1}(\ell\tau)$, $M_{M1}(\sigma\tau)$ correspond to the four terms of the $\boldsymbol{\mu}$ operator, respectively.

Since the coefficient $g_s^{\rm IV}$ ($=4.706$) is the largest, the contribution from the term of IV spin matrix $M_{M1}(\sigma\tau)$ is often the largest [7, 12]. On the other hand, it is expected that the contributions of the IS terms are small [12].

2.3 Meson exchange current

In the comparison of GT transitions with the analogous $M1$ transitions, a simple relationship is obtained for the transition strengths if the IV spin term is dominant in the $M1$ transitions. From the comparison of Eq. (2) and Eq. (5), the "quasi proportionality" between B(GT) and $B(M1)$ is expressed as

$$B(M1) \approx \frac{3}{8\pi}(g_s^{\rm IV})^2\mu_N^2\frac{C_{M1}^2}{C_{\rm GT}^2}B({\rm GT}). \quad (6)$$

However, it is known that the contributions from meson-exchange currents (MEC) enhance $B(M1)$ strength over the corresponding B(GT) strength [8, 9]. The enhancement is traced back to larger and additive contributions of the vector MEC over the axial-vector MEC which are active in $M1$ and GT transitions, respectively [13]. Taking the enhancement factor $R_{\rm MEC}$ into account, we have now the relationship

$$B(M1) \approx \frac{3}{8\pi}(g_s^{\rm IV})^2\mu_N^2\frac{C_{M1}^2}{C_{\rm GT}^2}R_{\rm MEC}B({\rm GT}). \quad (7)$$

In order to remove the constructive and destructive contributions of both IS and orbital terms, it was proposed to sum up the strengths over a wide range of excitation energy [14] and such cancellation of orbital contributions was predicted in shell-model calculations [15]. The analyses performed for the even-even self-conjugate nuclei ^{24}Mg and ^{28}Si suggested that the $R_{\rm MEC}$ value is in the range of 1.20 - 1.85 [14, 16, 17].

Further detailed analysis has been performed for the analogous $B(M1)$ and B(GT) values in the $A=27$ mirror nuclei ^{27}Al and ^{27}Si [8, 9] (also see Sec. 3). The cumulative sums of $B(M1)$ and B(GT) values were calculated for the states with reliable B(GT) values and good correspondence in the region up to the excitation energy $E_x=8.2$ MeV. From the ratio of these cumulative sums, an enhancement factor $R_{\rm MEC}=1.4$ has been derived [8, 9]. Since the sum is for a limited region in excitation energy, it is not appropriate to extract a definite conclusion for the value of $R_{\rm MEC}$. We simply mention that this value 1.4 is consistent with the previous results.

2.4 Isoscalar and orbital contributions in *M*1 transitions

From Eq. (7), we now see that the value

$$B(M1_{\sigma\tau}) = 2.644\mu_N^2\frac{C_{M1}^2}{C_{\rm GT}^2}R_{\rm MEC}B({\rm GT}). \quad (8)$$

is the $\sigma\tau$ contribution expected in the $M1$ transition.

The interference of IS and IV orbital terms with the IV spin term in an $M1$ transition can be studied for the jth analogous $M1$ and GT transitions in isobars by examining the ratio defined by

$$R_{\rm ISO}^j = \frac{B^j(M1)}{B^j(M1_{\sigma\tau})}. \quad (9)$$

By comparing Eq. (2) and Eq. (5), it is seen that $R_{\rm ISO} > 1$ usually indicates that the IS term and/or the IV orbital term make a constructive contribution to the IV spin term, while $R_{\rm ISO} < 1$ shows a destructive contribution. As discussed in [9, 18], the contribution of the IS term is usually minor. Therefore, it is expected that the deviation of $R_{\rm ISO}$ from unity mainly shows a contribution of the IV orbital term in each $M1$ transition.

3 Orbital contribution in $M1$ transitions in $T = 1/2$, $A = 27$ nuclei

We examine the $M1$ γ decays in the $T_z = 1/2$, ^{27}Al nucleus and compare the $M1$ transition strengths with the GT strengths obtained from the ^{27}Al(^{3}He, t)^{27}Si reaction. The jth state with $J^\pi_j = 3/2^+, 5/2^+$, and $7/2^+$ can decay directly to the $J^\pi_0 = 5/2^+$ g.s. by $M1$ transitions.

3.1 Analog states and analogous transitions

A pair of isospin $T = 1/2$ mirror nuclei is characterized by $T_z = \pm 1/2$. All other quantum numbers of corresponding states are the same. Thus, with the assumption that isospin is a good quantum number, for every state in one of the mirror nuclei, an analog state should be found in the other nucleus (see Fig. 1). The energy levels should be almost identical in the pair nuclei, although small differences are expected from the state-dependent differences in the Coulomb displacement energies. In addition, the Coulomb displacement energy itself allows the g.s. of the $T_z = -1/2$ nucleus to undergo β decay to the g.s. as well as to several low-lying states of the $T_z = +1/2$ nucleus.

As a result of the analogous nature of corresponding states in the mirror nuclei, following analogous transitions are expected. They are transitions from an initial to a final state in the $T_z = +1/2$ and -1/2 nuclei and all other transitions in which initial and/or the final state is replaced by the respective analog state. Transitions reversing the initial and final states are also possible. Analogous $M1$ and GT transitions from or to the g.s. with spin value $J_{\rm g.s.} \neq 0$ and with isospin $T = 1/2$ are schematically shown in Fig. 1.

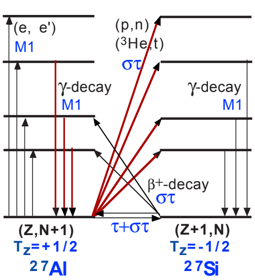

Figure 1. Schematic level schemes are shown for $T = 1/2$, odd-A mirror nuclei. Analogous transitions connecting the ground states in the mirror nuclei with excited states in the same nucleus and the conjugate nucleus are indicated. The type of reaction or decay and the relevant interactions causing each transition are shown along the arrows indicating the transitions.

3.2 Comparison of $M1$ and GT transition strengths

By comparing the analogous $B(M1)$ and B(GT) strengths in $T = 1/2$ mirror nuclei, it is possible to extract the combined contribution of the IS term and the IV orbital term in the $B(M1)$ strength using Eq. (9). Since the effect of MEC should be independent of the wave function of the individual state, the value $R_{\rm MEC} = 1.4$ derived from the analysis of the $A = 27$ system is used (for details, see Ref. [8]). The $R_{\rm ISO}$ ratios were calculated for the fourteen pairs of $M1$ and GT transitions up to the excitation energy of 9.7 MeV in ^{27}Al, where pairs well corresponding in E_x values were selected.

The results are shown in Fig. 2 for the $M1$ transitions in the $T_z = +1/2$ ^{27}Al nucleus as a function of $B(M1)\uparrow$ value [the $B(M1)$ value from the g.s. to the jth final excited state]. It is interesting to note that the $R_{\rm ISO}$ value tends to deviate from unity by more than a factor of two when the $B(M1)\uparrow$ is less than approximately 0.1. This shows that the "combined IS and orbital contribution" is rather large in weaker transitions and the quasi proportionality [Eq. (7)] of the $B(M1)$ values for $\Delta T = 0$ $M1$ transitions and the analogous B(GT) values is lost. This finding is interpreted as follows; since the IS term is small and the IV orbital term cannot be large for the s or d orbits with smaller orbital angular momentum ℓ, the dominance of the IV spin term of the $M1$ operator is guaranteed if the transitions are at least of average strength. However, the contribution of the IV spin term can also be small. Then the relative contribution of the IS term and the IV orbital term becomes significant although the transition itself is weak [8, 12, 18, 19]. A similar discussion applies to the $M1$ transitions in $T_z = -1/2$ mirror nuclei [9].

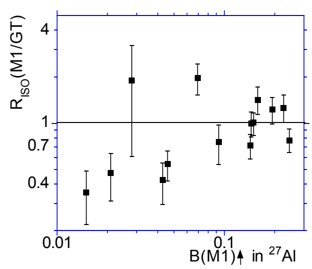

Figure 2. The ratio $R_{\rm ISO}$ for the $M1$ transitions in ^{27}Al as a function of $B(M1)\uparrow$. (Note the log-log scale of the figure.) The ratio is sensitive to the combined contribution of IS term and IV orbital term to each $M1$ transition. Values of $R_{\rm ISO} > 1 (< 1)$ suggest constructive (destructive) interference of these terms with the IV spin term. For the definition of $R_{\rm ISO}$, see text.

4 Comparison of analogous $M1$ and GT transition strengths in $A = 42$ nuclei

In a simple shell model (SM) picture, GT transitions are allowed among the LS-partner orbits $j_>$ and $j_<$. As a result, GT excitations are expected in the low-energy region (nominally at $E_x = 0$) and the region around $E_x = 3 - 6$ MeV (the energy difference of the LS partner orbits).

Table 1. The matrix elements M_{GT} for the configurations contributing to the GT transitions exciting $T = 0$, $J^{\pi} = 1^{+}$ low-lying states in ^{42}Sc from the 0^{+} g.s. of ^{42}Ca. The pf-shell SM calculation was performed [23] using the GXPF1J interaction [27]. The notation $f7 \to f7$, for example, stands for the transition with the $\nu f_{7/2} \to \pi f_{7/2}$ type. The summed value of the matrix elements is denoted by ΣM_{GT} and its squared value is the B(GT), where the B(GT) values do not include the quenching factor of the SM calculation.

States in ^{42}Sc		Configurations						Transition Strengths	
E_x (MeV)	T	$f7 \to f7$	$f7 \to f5$	$f5 \to f7$	$p3 \to p3$	$p3 \to p1$	$p1 \to p3$	ΣM_{GT}	B(GT)
0.33	0	1.383	0.548	0.063	0.031	0.024	0.016	2.07	4.28
4.41	0	0.719	-0.742	-0.085	-0.079	-0.073	-0.048	-0.31	0.09
7.41	0	0.193	-0.788	-0.090	0.142	0.060	0.040	-0.44	0.19

Note that the overview of GT responses can be studied by CE reactions [3, 4]. Against the above expectation for the GT strength distribution, these studies showed that the distribution in each nucleus can be largely different and dependent on the specific nuclear structure.

4.1 Low-energy Super Gamow-Teller transition observed in ^{42}Ca(^{3}He, t)^{42}Sc reaction

The most famous structure formed by GT excitation is the GT resonance (GTR) situated in high E_x regions of 9-15 MeV. In 1980s, GTRs were intensively studied by pioneering (p, n) reactions [3]. They were observed in almost all nuclei with mass $A > 60$ and $N > Z$. It was found that GTRs consume $\approx$ 60% of the GT strength predicted by the Ikeda sum rule [20].

Various random-phase-approximation (RPA) calculations can reproduce the high E_x values and the concentration of the available GT strengths in GTRs by introducing IV-type effective residual interactions (ERIs). The "repulsive" IV-type ERIs push GT strengths up in energy than is expected in a simple SM picture and concentrate the GT strengths in GTRs. In addition, available configurations contributing to the excitation of GTRs are "in phase" [21], showing the collective nature of them. It is known that IV-type ERIs are active in configurations with particle−hole (p−h) nature in GT excitations. Note that this condition is always realized in $N >> Z$ nuclei.

On the other hand, high resolution (^{3}He, t) study on ^{42}Ca [22, 23], and recently on ^{18}O [24], showed that the GT strength can also be concentrated in the lowest-energy GT state. As we see in Fig. 3, the 1^{+}, 0.611 MeV GT state in ^{42}Sc is dominant in the 0-degree spectrum. A total B(GT) value of 2.7(4) is in discrete states up to 13 MeV [about half of the Ikeda sum-rule value of $3(N - Z) = 6$], and $\approx$ 80% of that [B(GT) = 2.17] is concentrated in the lowest 0.611 MeV GT state. We call it the Low-energy Super GT state (LeSGT state) [22, 23]. In RPA calculations, it was found that LeSGT states are formed by the contribution of IS-type ERIs [22, 23, 25] that are active in proton particle−neutron particle (πp−νp) configurations [25, 26] on top of the LS-closed core of ^{40}Ca.

It was found that the SM calculation for ^{42}Ca $\to^{42}$Sc GT transitions using the GXPF1J interaction [23, 27] reproduces the concentration of the GT transition strength to the lowest $T = 0$, 1^{+} state (see Table 1). Note that not only the main configurations, i.e., $\nu f_{7/2} \to \pi f_{7/2}$ and $\nu f_{7/2} \to \pi f_{5/2}$, but also p-shell configurations contribute all in phase to the excitation of the lowest 1^{+} state at 0.33 MeV, i.e., the LeSGT state. On the other hand, contributions of the configurations are out of phase in the other two 1^{+} states.

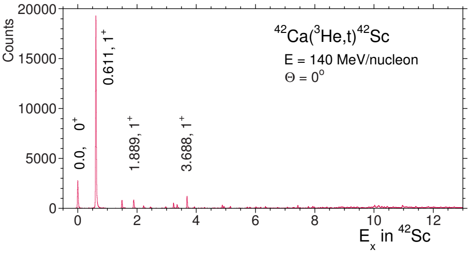

Figure 3. The 0°, ^{42}Ca(^{3}He, t)^{42}Sc spectrum. Two prominent peaks observed in the low-energy region are the $J^{\pi} = 0^{+}$ IAS (g.s.) and the 0.611 MeV, 1^{+} LeSGT state excited with $\Delta L = 0$ transitions. No GTR structure, usually expected at $\approx$ 10 MeV, is observed.

4.2 Analogous $M1$ and GT transitions from and to the LeSGT state

Figure 4 shows the analogous $M1$ and GT transitions in the low-energy region of the $A = 42$ isobars. The $T_z = 0$ nucleus ^{42}Sc is unique in the sense that it has the $T = 1$ g.s. (the IAS of the g.s. of ^{42}Ca and ^{42}Ti). We notice that GT transitions from the ground states of ^{42}Ca and ^{42}Ti to the 0.611 MeV $T = 0$, 1^{+} state in ^{42}Sc are analogous. Using the experimental data evaluated in Ref. [28], B(GT) = 2.17 (5), the value mentioned earlier,

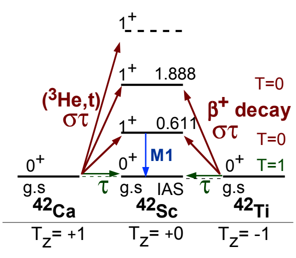

Figure 4. Gamow-Teller and $M1$ transitions in $A = 42$, $T_z = 1$, 0, -1 isobar nuclei ^{42}Ca, ^{42}Sc, and ^{42}Ti, where ^{42}Ca and ^{42}Ti are the mirror nuclei. The mirror GT^{-} and GT^{+} transitions that can be studied by the ^{42}Ca(^{3}He, t)^{42}Sc reaction and ^{42}Ti β^{+} decay, respectively, are caused by the $\sigma\tau$ operator. On the other hand, $M1$ γ transition in ^{42}Sc is caused by the $M1$ (μ) operator.

is obtained from the ^{42}Ti β^+ decay and the same value is expected in the ^{42}Ca(^{3}He, t)^{42}Sc reaction [23] on the basis of isospin symmetry between the mirror transitions. This large B(GT) value, as discussed, is due to the in-phase nature of all available f- and p-shell configurations contributing to this GT transition.

We also notice that the $M1$ γ decay from the 1^+, 0.611 MeV state in ^{42}Sc to the 0^+ g.s. is analogous to the GT transitions (see Fig. 4). For this $M1$ transition, a large $B(M1)$ of 6.2 (26) μ_N^2 [$B(M1)$ (W.u.) = 3.4 (15)] has been derived [28]. Large uncertainty is due to that in the half-life measurement of the 0.611 MeV state [$T_{1/2}$ = 28 (12) fs]. From the B(GT) = 2.17 (5) of the analogous GT transition and using Eq. (8), a value $B(M1_{\sigma\tau})\uparrow$ = 8.1 (24) μ_N^2, i.e., the $B(M1_{\sigma\tau})$ value from the T = 1 g.s. to the 1^+, 0.611 MeV state, is obtained, where $R_{\rm MEC}$ = 1.4 is assumed. The value $B(M1_{\sigma\tau})\downarrow$ that can be directly compared with the γ-decay $B(M1)$ is derived taking the differences of spin and isospin CG coefficients into account. We obtained a rather small value of 2.7 (8) μ_N^2. Relatively large uncertainty comes from that of the value of $R_{\rm MEC}$ = 1.2 - 1.85 that is not well known for pf-shell nuclei [14].

As discussed, under the assumption that strong $M1$ transitions are mainly caused by the $\sigma\tau$-part of the $M1$ operator, it is expected that analogous GT and $M1$ transitions have corresponding transition strengths. However, here, we hesitate to come to this simplified conclusion. Note that the contribution of the IV orbital term in Eq. (4) can be larger for larger ℓ orbits (ℓ = 3 for the $f_{7/2}$ and $f_{5/2}$ orbits). Looking for a clue to answer this question, we performed a three-body model calculation assuming valence p and n particles on top of a ^{40}Ca core [29], where both IS and IV pairing correlations were included. The calculation showed a $B(M1)$ value of 6.8 μ_N^2. It also suggested that contributions of the spin and orbital terms are constructive. The ratio of these contributions to the transition amplitude is approximately 1 : 0.47, which suggests that the $B(M1)$ can be approximately twice larger than the $B(M1_{\sigma\tau})$ value.

As we have seen, the combined studies of GT and $M1$ transitions for the analogous transitions allow us to access "one-step deeper fields" in nuclear physics such as orbital and isoscalar contributions in $M1$ transitions or meson-exchange currents in $M1$ and GT transitions. Steady efforts for quantitative measurements, such as life-time measurements of excited states [30], and combined efforts of γ- and β-decay studies as well as studies using CE reactions are needed to approach such quantities.

Acknowledgments. YF acknowledges the support by JSPS KAKENHI, Japan, under Grants No.15540274, No.18540270 and No.JP15K05104. YT acknowledges the support from the Graduate Program on Physics for the Universe (GP-PU), Tohoku University.

References

[1] A. Bohr and B.R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. 1.
[2] F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992).
[3] J. Rapaport and E. Sugarbaker, Annu. Rev. Nucl. Part. Sci. **44**, 109 (1994).
[4] Y. Fujita, B. Rubio, and W. Gelletly, Prog. Part. Nucl. Phys. **66**, 549 (2011).
[5] W.G. Love and M.A. Franey, Phys. Rev. C **24**, 1073 (1981).
[6] H. Morinaga and T. Yamazaki, *In Beam Gamma-Ray Spectroscopy* (North-Holland, Amsterdam, 1976).
[7] S.S. Hanna, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), Chap. 12.
[8] Y. Fujita, *et al.*, Phys. Rev. C **59**, 90 (1999).
[9] Y. Fujita, *et al.*, Phys. Rev. C **62**, 044314 (2000).
[10] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1960).
[11] L. Zamick and D. C. Zheng, Phys. Rev. C **37**, 1675 (1988).
[12] E. K. Warburton and J. Weneser, in *Isospin in Nuclear Physics*, ed. D. H. Wilkinson, (North-Holland, Amsterdam, 1969) Chap. 5, and references therein.
[13] I. S. Towner and F. C. Khanna, Nucl. Phys. **A399**, 334 (1983).
[14] A. Richter, A. Weiss, O. Häusser, and B. A. Brown, Phys. Rev. Lett. **65**, 2519 (1990).
[15] M. Hino, K. Muto, and T. Oda, J. Phys. G **13**, 1119 (1987).
[16] Y. Fujita, *et al.*, Phys. Rev. C **55**, 1137 (1997).
[17] P. von Neumann-Cosel, A. Richter, Y. Fujita, and B. D. Anderson, Phys. Rev. C **55**, 532 (1997).
[18] Y. Fujita *et al.*, Phys. Rev. C **66**, 044313 (2002).
[19] G. Morpurgo, Phys. Rev. **110**, 721 (1958); *ibid.* **114**, 1075 (1959).
[20] K. Ikeda, S. Fujii, J.I. Fujita, Phys. Lett. **3**, 271 (1963).
[21] M.N. Harakeh, A. van der Woude, *Giant Resonances* Oxford Studies in Nucl. Phys. **24**, (Oxford University Press, Oxford, 2001).
[22] Y. Fujita, *et al.*, Phys. Rev. Lett. **112**, 112502 (2014).
[23] Y. Fujita, *et al.*, Phys. Rev. C **91**, 064316 (2015).
[24] H. Fujita, *et al.*, private communications.
[25] C.L. Bai, *et al.*, Phys. Rev. C **90**, 054335 (2014).
[26] *Fifty Years of Nuclear BCS, Pairing in Finite Systems* edited by R.A. Broglia and V. Zelevinsky (World Scientific, Singapore, 2013).
[27] M. Honma, T. Otsuka, T. Mizusaki, M. Hjorth-Jensen, and B.A. Brown, J. Phys.: Conf. Ser. **20**, 7 (2005).
[28] Jun Chen and Balraj Singh, Nucl. Data Sheets, **135**, 1 (2016).
[29] Y. Tanimura, H. Sagawa, and K. Hagino, Prog. Theor. Exp. Phys. **2015**, 053D02.
[30] A. Dewald, O. Möller, and P. Petkov, Prog. Part. Nucl. Phys. **67**, 786 (2012).