

# Hadronic vacuum polarization and $e^+e^- \rightarrow \mu^+\mu^-$ cross section Reanalysis with new precise data for $\sigma_h$ with $4\pi$ final states included

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**Abstract.** The interference effect between leptonic radiative corrections and hadronic polarization functions is calculated via optical theorem for  $\mu$ -pair production in vicinity of narrow resonances. Within seven most dominant exclusive channels of the production cross section  $\sigma_h(e^+e^- \rightarrow \text{hadrons})$  one achieves high accuracy which is necessary for the comparison with experiments. The result is compared with KLOE and KLOE2 experiments for  $\mu^-\mu^+$  and  $\mu^-\mu^+\gamma$  productions at  $\phi$  and  $\omega/\rho$  meson energy.

## 1 Introduction

Comparisons between theory and experiment are used to test Standard Theory for decades. For an accurate measurement the studies require consistent account of leptonic as well as hadronic virtual corrections. The hadronic contribution to photon vacuum polarization function plays particularly important role, since it is the main source of uncertainties in theoretical calculation of muon anomalous magnetic moment  $a_\mu$ . The last precise measurement of  $a_\mu$ , together with the last decades data for electro-hadron production, leads to an evidence of tension between Standard Theory and experiments [1, 2]. Similar confrontation of the theoretical technique with the experimental accuracy is offered by long time known [3, 4] interference effect between leptonic and hadronic vacuum polarization functions in close vicinity of narrow resonances:  $\omega$  and  $\phi$  as well as heavier quarkonia the  $J/\Psi$ ,  $\Psi$  and  $\Upsilon$ 's. There, the hadronic vacuum polarization is enhanced several orders of magnitude above remaining hadronic background. In practice, the effect is explored in the so called B-factories like BABAR, BELLE or BES or more earlier in Frascati with detectors and accelerator tuned at the  $\phi$  meson mass. Most recent precise measurements of muon production  $e^+e^- \rightarrow \mu^+\mu^-$  by SND, CMD-2,3 and the KLOE/KLOE2 detectors allow to another test of the Standard theoretical assumptions. With incomes of many new precise data for hadroproduction cross section  $\sigma_h$  we reanalyzed the calculation for [5] within the new data used. Having extracted hadron polarization function, we provide independent comparison of theory and recent KLOE2 [6] measurement of the fine structure coupling constant.

## 2 $\sigma_{\mu\mu}$ for KLOE 2004

In the first part I will present the comparison between calculated cross section  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  and the high precision measurement obtained by KLOE detector [5].

The method of extraction of hadronic polarization function  $\Pi_h$  will be briefly mentioned. As we shall see the recent knowledge of  $\sigma_h$  allows theoretical determination of  $\sigma_{\mu\mu}$  with theoretical error comparable to KLOE  $\sigma_{stat}$  for  $\sigma_{\mu\mu}$ . Thus the error of theory due to the uncertainty in  $\sigma_h$  becomes 10 times smaller then the experimental error of 2004 KLOE measurement. The result presented here represents a precise prediction for a future more accurate experiments.

The details of theory of  $e^+e^- \rightarrow \mu^+\mu^-$  for KLOE detector can be found in [7]. The integral cross section formula is proportional to the square of the fine structure constant  $\alpha(s)$ , which reads

$$\alpha(s) = \frac{\alpha}{1 - \Pi(s)}, \quad (1)$$

with  $\alpha = \alpha(0) = 1/137.0359991390$  and the polarization function  $\Pi(s) = \Pi_l(s) + \Pi_h(s)$  is made of the leptonic  $l$  and the hadronic  $h$  part.

Purely QED contributions, represented by  $\Pi_l$ , are well known from perturbation theory and relevant terms are listed in [7], however  $\Pi_h$  which is partially known in the spacelike domain [8] is not directly calculable from the equation of motions in the Minkowski space. Instead of this, with certain assumptions, it is obtained via Unitarity relation within the help of of many other experiments  $e^+e^- \rightarrow \text{hadrons}^*$ , where the stars \* means that the final state photons should be included as well, while in opposite, the initial ones should be subtracted. In fact, the evaluation of  $\Pi_h$  relies on numerical evaluation of the following singular integral [9, 10]:

$$\Pi_h(s) = \frac{s}{4\pi^2\alpha} \int_{m_\pi^2}^{\infty} d\omega \frac{\sigma_h(\omega) \left[ \frac{\alpha}{\alpha(\omega)} \right]^2}{\omega - s + i\epsilon}. \quad (2)$$

Recall, the use of experimental data straightforwardly would lead to a large numerical noise and lost of required

accuracy. I found the usual way (for the method of clusters see [11, 12]) produces cusps and spikes. They cause the error stemming from such interpolation, averaging and integration procedure is hard to estimate due to the presence of Principal value integration (recall for clarity, such numerical problem is avoided in the case of  $a_\mu$  evaluation as the integral is regular).

Therefore, instead of direct use of experimental data, the fit (including errors) is made for each combinations of measurements. I used the fine selection method of the data, which is based on the following simple criterion:

$$\sigma_{syst}^2 + \sigma_{stat}^2 < \epsilon_h^2, \quad (3)$$

where on the left side there is sum of statistical and systematic error of the data points and on the right side a suited choice of error function evaluated at data points is employed. The data satisfying the inequality (3) are used to establish the fits, wherein the experimental errors are replaced by the inflated error function (IEF)  $\epsilon_h$ . While all data points not satisfying the rule (3) are not used for making a fits.

We stress here, that with the choice of a given IEF  $\epsilon_h$ , the condition (3) does not automatically ensure the existence of a good global fit satisfying  $\chi^2 < 1$ . It actually happens in cases when one combine various experiments with the error underestimated by the experimental group, which usually happens when the systematic error is not completely known (not named explicitly, the older threshold data extracted by ISR method are a typical example). Impossibility of minimizing  $\chi^2$  such that  $\chi^2 < 1$  (note  $\chi^2 \simeq 1$  is valid only for non-inflated error) indicates the badness, or rather say the incompatibility of the data. In this case we are either force to inflate further the IEF (by changing the prescription for  $\epsilon_h$ ) or we discard the problematic data set from the fine selection.

There is an obvious price to pay and one must to perform cross section fits explicitly. To this point, well established interpolating fits to the existing experimental data for  $\sigma_h$  at each exclusive channel have been found during several last years. Using these, the large number of generated quasidata points makes systematic error from principal value integration in (2) immaterial and the error of theory for  $\sigma_{\mu\mu}$  is almost solely due to the propagation of "inflated" error  $\epsilon_h$ . More explicitly, the systematic error due to the integration procedure has been minimized with the relative precision smaller then 0.005/40 for the muon production cross section (the absolute value is approximately hundred times smaller then the experimental error [5]).

The functional form of the error function  $\epsilon_h$  can be taken arbitrary, however the choice, which does not reduce n.d.f. too much and is simultaneously simple enough, is preferred. For this purpose we have used the following IEF

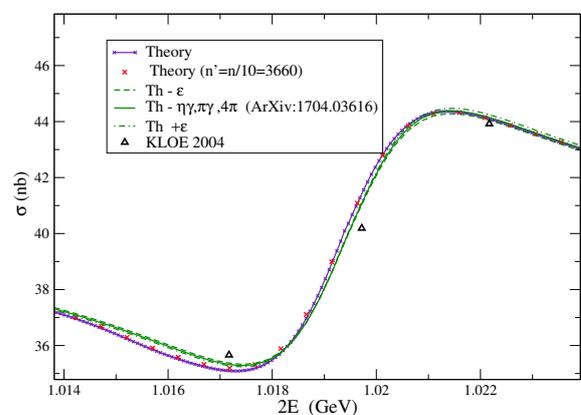
$$\epsilon_h(s) = c_l \sqrt{\sigma_h(s)}, \quad \epsilon_h(s) = c_s \sigma_h(s), \quad (4)$$

where the left equation in (4) is used for  $\sigma_h(s)$  larger the  $1nb$ , while the right one is used for small  $\sigma_h(s) < 1nb$ . It is enough to take constant values for parameters  $c_{l,s}$ , noting that values  $c_l = 0.8nb^{1/2}$ ,  $c_s = 1/3$  in Eq. (4) were most recently used in almost all the hadronic channels.

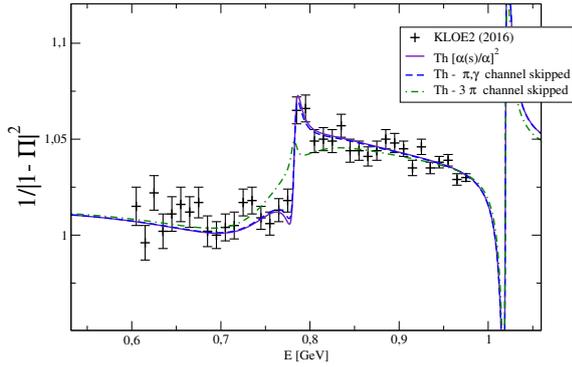
Recent measurements are taken completely into account and the complete list of selected experimental data and their fits will be presented in an updated version of [7]. Without specifications of channels, the choice (4) fully accept the last experiments, eg. CMD-3, BES-III, KLOE, also most CMD-2 measurements as well as large-s BABAR data are fully taken, it cuts partially some data from SND, CMD-2, it also cuts on resonance data from BABAR, while in practice we can freely discard the data from and old experiments completely (CMD,DM,NA7,OLYA,TOF).

In order to evaluate  $\Pi_h$ , the main exclusive channels:  $\eta\gamma, \pi\gamma, \pi\pi, K^+K^-, K_L K_S$  and  $\pi\pi\pi$  as well as  $4\pi$  have been considered. Contributions from final states, like  $KK\pi\pi$  and the one with higher multiplicity were estimated and neglected as their amounts is fairly below the systematic error of  $\sigma_{\mu\mu}$  cross section. The effect of well established vector charmonia and bottomonia was found to be nonnegligible and has been included through  $\sigma_h$  by using their BW forms with PDG experimentally determined values.

The result is shown in the Fig. 1, where the old analyses (set of green lines) is compared with the new one (violet line with crosses). The new analysis includes in addition the data from KLOE, BES-III and BABAR for KK channels, which have no practical effect to the final curve. However, more importantly, it newly includes  $\eta\gamma, \pi\gamma$  and  $4\pi$  channels of  $\sigma_h$ , which have been neglected in the previous analyses of  $\phi$ - meson study. The green solid line states for the old result shows the effect of neglecting the three mentioned channels. The band between  $\pm\epsilon$  (dashed and dot dashed line) reflects the propagation of inflated error  $\epsilon_h$  into the muon pair cross section and was obtained with  $c_l = 1nb^{1/2}$ ,  $c_s = 1$  in the previous analyses. The error band for a newest analyses is even more tight and not shown in the figure. The KLOE data points are represented by triangles, noting the statistical deviations roughly correspond with the size of the triangle. For interest, we also show the systematic deviation which stems from one order decrement of the number of integration points in Eq. (2), these points are labeled by red crosses with no line.



**Figure 1.** Muon pair cross section. The comparison between theory and experiment as described in text.



**Figure 2.** Square of fine structure constant. To see the portion of  $3\pi$  and  $\pi, \gamma$  contributions we also show the results with these channels subtracted from  $\sigma_h$ .

To conclude the first part of the talk, the observed  $\phi$ -meson interference effect in  $\mu\mu$  spectrum is roughly reproduced by the Standard Theory dispersion relation. Remind that the detector measurement provided three points with very small statistical error ( $\sigma_{stat} = 0.1nb$ ), unhappily the total error was governed by systematic error due to the luminosity and detection uncertainties ( $\delta_{syst} = 1.2\%$ ). Small  $1.7\sigma_{tot}$  difference from SM prediction does not represent large tension between theory and experiment. Lowering the systematic error would be not only experimental challenge for precise experimental facilities like KLOE2, CMD3, but also for the Standard Theory.

### 3 $\alpha_{QED}$ at KLOE2

As a bonus we get also similar interference effect in the  $\omega$  energy region. This, somehow smaller,  $3nb$  sized zig-zag structure has been measured by radiative return method only very recently by the KLOE2 collaboration [6]. In

this case the experiment is in complete agreement with the theory, noting the relative errors are much larger in the  $\rho/\omega$  region. The comparison with KLOE2 experiment and the way the  $\rho/\omega$  peak is pronounced in the QED running coupling is shown in Fig. 2. More precise comparison remains challenging for both the Standard Theory and new generation experiments as well.

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