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Modelling of air flow rate in significantly flattened rounded rectangular ventilation ducts

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Abstract. Paper presents new mathematical model for air flow velocity distribution in rounded rectangular ducts and its experimental verification. In papers [1, 2] an mathematical model based on modified Prandtl equation for power power-law velocity profile was determined. It works very well for smaller cross sections. During the study of larger cross sections new phenomena in flowing air have been observed, it forced the search for a new model. The new model is based on a rounded rectangular division into two parts: slot and rounded square.

1 Introduction

An important element in the design of ventilation systems (first of all connected with air conditioning) is the selection of appropriate ducts. So far ducts of circular or rectangular shape are commonly used, sometimes of oval cross section area. For some time a new ventilation system has been developed, in which channels have a rounded rectangular cross section area.

This new system combines the advantages of circular and rectangular channels. It provides as well as rectangular ducts good use of space in rooms, allows good cleansing of them (no sharp corners), and has good flow properties.

Λ		W [mm]										
		300	400	500	600	800	1000	1200	1400	1600	1800	2000
H [mm]	200	0.667	0.500	0.400	0.333	0.250						
	250	0.833	0.625	0.500	0.417	0.313	0.250					
	300	1.000	0.750	0.600	0.500	0.375	0.300	0.250				
	400	1.333	1.000	0.800	0.667	0.500	0.400	0.333	0.286			
	500	1.667	1.250	1.000	0.833	0.625	0.500	0.417	0.357	0.313		
	600	2.000	1.500	1.200	1.000	0.750	0.600	0.500	0.429	0.375	0.333	0.300
	800	2.667	2.000	1.600	1.333	1.000	0.800	0.667	0.571	0.500	0.444	0.400
	1000	3.333	2.500	2.000	1.667	1.250	1.000	0.833	0.714	0.625	0.556	0.500
	1200	4.000	3.000	2.400	2.000	1.500	1.200	1.000	0.857	0.750	0.667	0.600

P		W [mm]										
		300	400	500	600	800	1000	1200	1400	1600	1800	2000
R [mm]	100	0.333	0.250	0.200	0.167	0.125	0.100	0.083	0.071	0.063	0.056	0.050

Fig. 1 Determination of dimensionless parameters.

Although the theory of air flow in ducts is reasonably well understood, theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe [3]. Therefore, we must rely on experimental results and empirical relations for most air flows in ducts rather than closed-form analytical solutions. Noting that the experimental results of ventilation ducts are obtained under carefully controlled laboratory conditions and that no two systems are exactly alike, we must not be so naive as to view the results obtained as "exact." An error of 10 percent (or more) in friction factors calculated using values presented in different design manuals is the "norm" rather than the "exception."

On the other hand, mathematical modelling of the air flow distribution in cross-sectional area of the duct is necessary. In Fig. 1, all dimension series of the developed new system ducts are shown, 79 cells are filled in this table. For each dimension there are about 40 different elements, this means that more than 3,000 items should be tested. This is not possible from a time point of view.

For this reason, it is necessary to develop a certain empirical model of air flow in cross section based on experimental research. The first step in the development of this model was the adoption of dimensionless parameters for characterization of the duct cross-section shape Λ and P, see Fig.1, Fig. 2.

2 Geometrical similarity

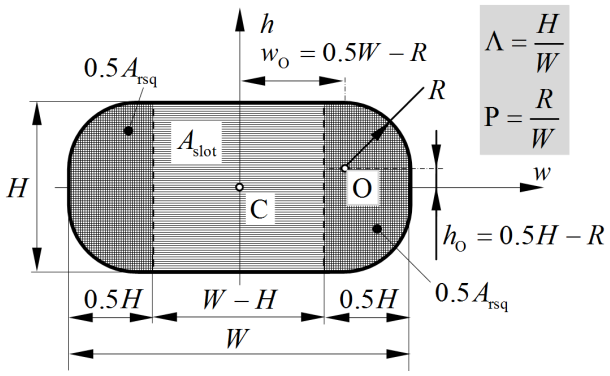

Fig. 2 Basic parameters of cross sectional area and idea it division into two basic parts.

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Cross sectional area is characterized by three parameters: width W, height H, and rounding radius R (Fig. 2). These parameters, during the analysis, are used in two non-dimensional forms: flatness of rectangle Λ and relative rounding P, where width W is the basic parameter.

The cross-sectional area of ducts varies from $A_{rr\_\min} = 0.051\,\mathrm{m}^2$ to $A_{\mathrm{rr}\_\max} = 2.323\,\mathrm{m}^2$. This wide scope of areas imposes high demands relating to the measurement stand. Rounding radius R is substantial for the degree of the rounded rectangular reduction area with respect to the rectangular cross section.

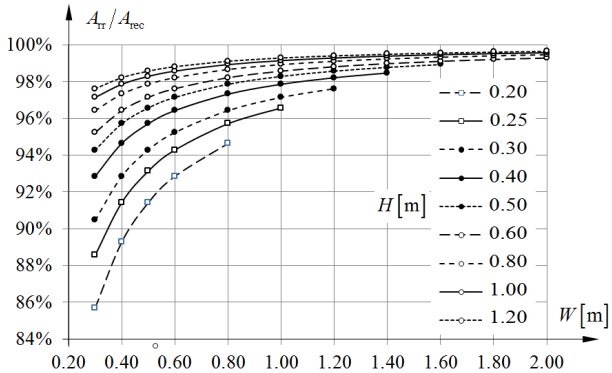

Fig. 3 Degree of use of full rectangular cross section in ducts with rounded rectangulars

In Fig. 3 impact of the rounding radius on the size of rounded rectangular the cross sectional area related to full rectangular cross section. This effect was determined by the formula:

$$\frac{A_{\mathrm{rr}}}{A_{\mathrm{rec}}} = \frac{W \cdot H - (4-\pi)R^2}{W \cdot H} = 1 - \frac{(4-\pi)\mathrm{P}^2}{\Lambda} \tag{1}$$

where: A_{rr} – rounded rectangular area, A_{rec} – full rectangular area (without rounded corners)

For technological reasons only one rectangle rounding radius $R = 0.1\,\mathrm{m}$ was tested.

3 Modelling of volume flow rate

Modelling of volume flow rate is concerned mainly to rectangular cross sections for $\Lambda < 0.5$ or $\Lambda > 2.0$. Therefore, a new model of velocity distribution in the cross section was to be found. The analysis below is only case-sensitive $\Lambda < 0.5$, because the cross–sections for $\Lambda > 2.0$ are only rotate the section by angle 90°, what is irrelevant from the mathematical point of view. It was noted that within a wide range of coordinates around the central point C, the local flow velocity was not changed. This suggested a presumption that velocity distribution in this area is similar to the velocity distribution in a slot. This observation confirms the well-known fact that the Reynolds number based on the hydraulic diameter D_{h} for cross sections different from circular better fit the similarity of flows, if the cross section is closer to the circle., e.g. square, hexagon and so on. Therefore, the new model was taken over that the flow in the channel is twofold: one–dimensional as in slot and two–dimensional as in circular cross-section (Fig 2).

Cross–section marked in the figure Fig. 2 as A_{slot} is a rectangle of dimensions $w \times h = (W-H)H$. The flow in this section is similar to the flow in an infinitely broad slot of height h. This means that the flow velocity depends only on the dimension (high) h and is independent of w. Across the slot axis the velocity is maximum and is equal $\left[v(h,w)_{h=0}\right]_{w=-\frac{W-H}{2}}^{w=\frac{W-H}{2}} = v_{\mathrm{C}}$. Two side parts of the duct cross section together they form a square with rounded corners of side H, marked in the figure Fig. 2 as A_{rsq}. For a parameter $R = 0(\mathrm{P}=0)$ this cross section becomes a square, while for $R = 0.5H$ the cross section becomes a circle.

Volume flow rate in the slot

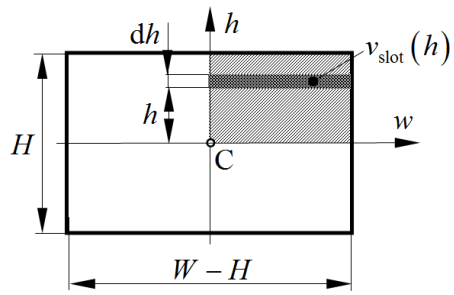

Fig. 4 Idea of the rectangular slot

The volume flow rate through the elementary cross section is defined by the formula:

$$\mathrm{d}\dot{V}_{\mathrm{slot}} = 0.5(W-H)v_{\mathrm{slot}}(h)\mathrm{d}h \tag{2}$$

When the integration of flow rate over the entire surface we obtain:

$$\begin{aligned}\dot{V}_{\mathrm{slot}} &= 4\int_0^{0.5H} 0.5(W-H)v_{\mathrm{slot}}\mathrm{d}h \\ &= 2(W-H)\int_0^{0.5H} v_{\mathrm{slot}}\mathrm{d}h\end{aligned} \tag{3}$$

Based on empirical studies, it can be assumed that for large sections the model resulting from the modified Prandtl formula is superior to that of its original model. According to the modified Prandtl formula (power-law velocity profile), the flow velocity distribution along the coordinate is defined by the formula [1]

$$v_{\mathrm{slot}}(h) = v_{\mathrm{C}}\left(1-\left(\frac{h}{0.5H}\right)^2\right)^{\frac{1}{n}} \tag{4}$$

After substituting (4) to (3) we obtain

$$\dot{V}_{\mathrm{slot}} = 2(W-H)v_{\mathrm{C}}\int_0^{0.5H}\left(1-\left(\frac{h}{0.5H}\right)^2\right)^{\frac{1}{n}}\mathrm{d}h \tag{5}$$

The integral defined by the formula (5) can be determined after the substitution $h^2 = 0.25H^2t$, or $h = 0.5H\sqrt{t}$, so we get:

$$\mathrm{d}h = 0.5H\frac{1}{2\sqrt{t}} = 0.25Ht^{-\frac{1}{2}}\mathrm{d}t \tag{6}$$

After substituting (6) to (5) we obtain

$$Q_{\text{slot}} = 0.5(W-H)Hv_{\text{C}}\int_0^1 t^{\frac{1}{2}-1}(1-t)^{\frac{1}{n}+1-1}\,\mathrm{d}h \qquad (7)$$

The integral expression in formula (7) is a beta function $\mathrm{B}\left(\frac{1}{2},\frac{1}{n}+1\right)$, also called the Euler integral of the first kind. A key property of the Beta function is its relationship to the Gamma function Γ

$$\mathrm{B}\left(\frac{1}{2},\frac{1}{n}+1\right) = \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{n}+1\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} \qquad (8)$$

Finally, we can write

$$\dot{V}_{\text{slot}} = 0.5(W-H)H\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{n}+1\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)}v_{\text{C}} \qquad (9)$$

or using non-dimensional parameter Λ of cross sectional area

$$\dot{V}_{\text{slot}} = 0.5(1-\Lambda)\Lambda\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{n}+1\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)}W^2 v_{\text{C}} \qquad (10)$$

2.2 Volume flow rate through the rounded square

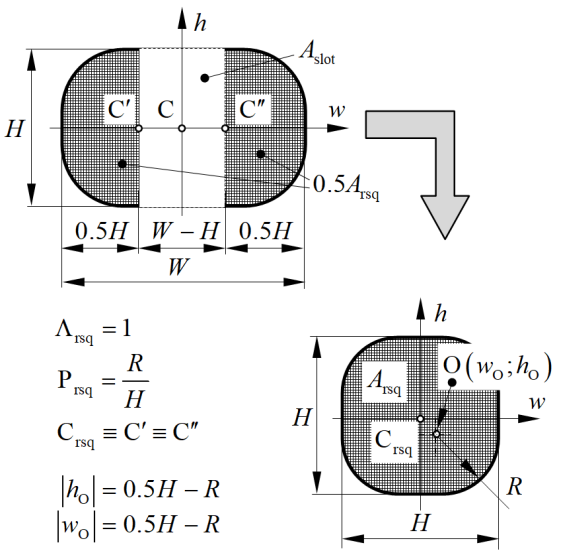

Fig. 5 Idea of virtual rounded square duct

Two outer parts of duct cross section together form a square of side H, because of the symmetry and the same velocities on the perpendicular to the axis w dividing lines, passing through points C' and C''. Virtual both parts can be connected to a rounded square duct with centre of symmetry C_{rsq}. Note that

$$v_{\mathrm{C}_{\text{rsq}}} = v_{\mathrm{C}'} = v_{\mathrm{C}''} = v_{\text{C}} \qquad (11)$$

This duct of cross section A_{rsq} perimeter is created by side walls where the air velocity is zero (Fig. 5). Instead of deriving formulas for $\dot{V}_{\text{rsq}}$ as for slot, may be used equation (6) presented in [2]:

$$v_{\text{avg}} = \frac{\Lambda-(4-\pi)\mathrm{P}^2 v_{\text{C}}}{\Lambda\frac{n}{n+1}\left(\frac{n}{n+1}\left(1-\left(\frac{4\mathrm{P}^2}{\Lambda}\right)^{\frac{n+1}{n}}\right)+\pi 4^{\frac{1}{n}}\left(\frac{\mathrm{P}^2}{\Lambda}\right)^{\frac{n+1}{n}}\right)} \qquad (12)$$

Above mentioned equation (12) was is derived for a rounded rectangle, to get the equation for rounded square, just we can substitute $\Lambda = 1$ (note that $\Lambda = 1$ is only for virtual square) in this case, because $W = H$:

$$v_{\text{avg}} = \frac{1-(4-\pi)\mathrm{P}^2 v_{\text{C}}}{\frac{n}{n+1}\left(\frac{n}{n+1}\left(1-\left(4\mathrm{P}^2\right)^{\frac{n+1}{n}}\right)+\pi 4^{\frac{1}{n}}\mathrm{P}^{\frac{2(n+1)}{n}}\right)} \qquad (13)$$

and for volume flow rate $\dot{V}_{\text{rsq}}$ we can write

$$\dot{V}_{\text{rsq}} = \frac{\left(1-(4-\pi)\mathrm{P}^2\right)\Lambda^2\left(1-(4-\pi)\right)W^2 v_{\text{C}}}{\frac{n}{n+1}\left(\frac{n}{n+1}\left(1-\left(4\mathrm{P}^2\right)^{\frac{n+1}{n}}\right)+\pi 4^{\frac{1}{n}}\mathrm{P}^{\frac{2(n+1)}{n}}\right)} \qquad (14)$$

2.3 Volume flow rate through the rounded rectangular

Volume flow rate through the rounded rectangular is sum of flow rates through slot and rounded square $\dot{V}_{\text{rr}} = \dot{V}_{\text{slot}} + \dot{V}_{\text{rsq}}$, than

$$\dot{V}_{\text{rr}} = \left(0.5(1-\Lambda)\Lambda\frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{1}{n}+1\right)}{\Gamma\left(\frac{3}{2}+\frac{1}{n}\right)} + \frac{\left(1-(4-\pi)\mathrm{P}^2\right)\Lambda^2\left(1-(4-\pi)\right)}{\frac{n}{n+1}\left(\frac{n}{n+1}\left(1-\left(4\mathrm{P}^2\right)^{\frac{n+1}{n}}\right)+\pi 4^{\frac{1}{n}}\mathrm{P}^{\frac{2(n+1)}{n}}\right)}\right)W^2 v_{\text{C}} \qquad (15)$$

4 Measurement results

Received model that allows to determine volume flow rate through the rounded rectangular $\dot{V}_{\text{rr}}$ verified by measurements on a test bench. Test bench consist of flowmeter "testo", fan, tubular air deflector [4], and ventilation duct of $20\times W$ meters long. Initial part of test bench for measurement volume flow rate $\dot{V}_{\text{rr}}$ is presented in Fig. 6. Flowmeter "testo" was mounted on the fan inlet. This solution allows to use this measurement device for many measurements. Of course, it is assumed that the amount of incoming air is equal to the amount of exhaust air from duct line.

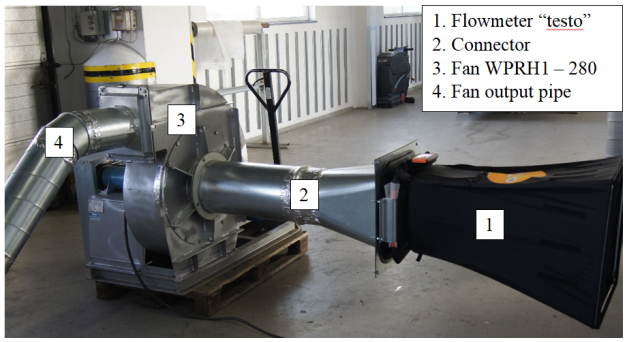

Fig. 6 Initial part of test bench for measurement volume flow rate

Flowmeter "testo" allows to measure the volume flow rate of air in the range $\dot{V} \in \langle 0.12 \div 1.2 \rangle \mathrm{m^3 s^{-1}}$. It has own tubular defector to calm the input flow. The fan rotational speed was controlled by the inverter.

All measurements were provided near the end of duct. In Fig.7 is presented traversing system (left) and HWA probe placed 6 mm before the end of duct.

Fig. 7 Measurement of velocity distribution in duct

Usefulness (verification) of the new model is presented in Fig. 8. These are the results of measurements and calculations for the duct of $W = 0.6\,\mathrm{m}$, $H = 0.2\,\mathrm{m}$, and $R = 0{,}1\,\mathrm{m}$, therefore $\Lambda = 0.33$ and $\mathrm{P} = 0.167$.

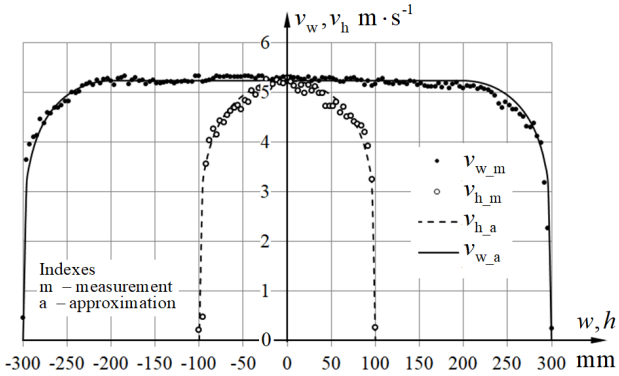

Fig. 8 An approximation of the measured velocity distribution using the new model

Presented in Fig. 8 approximation was made for $n = 5$ and velocity $v_C = 5.24\,\mathrm{m \cdot s^{-1}}$. Velocity v_C was determined as the mean value of the all measurements between $w = \pm 200\,\mathrm{mm}$. Value of measured average velocity, calculated as $v_{avg\_m} = \dot{V}_{rr} / A_{rr}$, was $4.65\,\mathrm{m \cdot s^{-1}}$. Value of counted average velocity was $4.54\,\mathrm{m \cdot s^{-1}}$ which is $97{,}6\%$ of v_{avg_m}. Cross sectional area of duct was $A_{rr} = 0.111\,\mathrm{m^2}$.

5 Conclusions

The new model is very useful especially when modelling large significantly flattened rounded rectangular ventilation ducts. In this case the models based on Prandtl equation for power power-law velocity fail. They needs big number n, e.g. $n > 20$. Unfortunately this model is more complicated. The main disadvantage is the gamma function, which requires application of advanced computational technique. However, in some applications it can be very useful, for example, developing a new type of flow meter, based on flow velocity measurement at the central point C.

This new model can also be used for smaller cross sectional, it is more universal.

The required measuring range of volume flow rate $\dot{V} \in \langle 0.257 \div 11.957 \rangle \mathrm{m^3 s^{-1}}$ exceeded the measuring range of the flowmeter. Therefore, the new model developed was used to determine the value of volume flow rate $\dot{V}$. This was necessary for the 57 ducts examined. Only for 22 cross sections it was possible use flowmeter "testo", therefore the new developed model has greatly helped in carrying out research.

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References

1. K. Peszyński, L. Olszewski, E. Smyk E, and D. Perczyński, (2016): Analysis of the Velocity Distribution in Different Types of Ventilation System Ducts, International Conference Experimental Fluid Mechanics 2016, Nov. 15th – 18th, 2016, Mariánské Lázně, Czech Republic, p. 577 – 580
2. K. Peszynski, L. Olszewski, E. Smyk, T. Kasprowicz (2017): Development of New Type Ventilation Ducts System, Proceedings of 23rd International Conference Engineering Mechanics 2017, Svratka, Czech Republic, p. 50 – 53
3. Z. Orzechowski, J. Prywer, R. Zarzycki: Mechanika płynów w inżynierii środowiska (1997), (Eng. Fluid Mechanics in Environmental Engineering), WNT, Warszawa
4. E. Smyk, P. Mrozik, S. Wawrzyniak, K. Peszynski (2017): Tabular Air Deflector in Ventilation Ducts, Proceedings of 23rd International Conference Engineering Mechanics 2017, Svratka, Czech Republic, p. 882 – 885