

## Interaction of real and virtual $N\bar{N}$ pairs in $J/\psi$ decays

Alexander Milstein<sup>1</sup> and Sergey Salnikov<sup>1,2,3,\*</sup>

<sup>1</sup>Budker Institute of Nuclear Physics, 630090, Novosibirsk, Russia

<sup>2</sup>Novosibirsk State University, 630090, Novosibirsk, Russia

<sup>3</sup>L.D. Landau Institute for Theoretical Physics, 142432, Chernogolovka, Russia

**Abstract.** The differential decay rates of the processes  $J/\psi \rightarrow p\bar{p}\pi^0$ ,  $J/\psi \rightarrow p\bar{p}\eta$ ,  $J/\psi \rightarrow p\bar{p}\omega$ ,  $J/\psi \rightarrow p\bar{p}\rho$ , and  $J/\psi \rightarrow p\bar{p}\gamma$  close to the  $p\bar{p}$  threshold are calculated with the help of the  $N\bar{N}$  optical potential. We use the potential which has been suggested to fit the cross sections of  $N\bar{N}$  scattering together with all other  $N\bar{N}$  experimental data available. The  $p\bar{p}$  invariant mass spectra of  $J/\psi$  decays are in agreement with the available experimental data. The anisotropy of the angular distributions of the decays  $J/\psi \rightarrow p\bar{p}\pi^0(\eta)$ , which appears due to the tensor forces in the  $N\bar{N}$  interaction, is predicted close to the  $p\bar{p}$  threshold. This anisotropy is large enough to be investigated experimentally. Such measurements would allow one to check the accuracy of the model of  $N\bar{N}$  interaction. Using our potential and the Green's function approach we also describe the peak in the  $\eta'\pi^+\pi^-$  invariant mass spectrum in the decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  in the energy region near the  $N\bar{N}$  threshold.

### 1 Introduction

Investigation of the nucleon-antinucleon interaction in the low-energy region is an actual topic today. Unusual behavior of the cross sections of several processes has been discovered in recent years. For instance, the cross sections of the processes  $e^+e^- \rightarrow p\bar{p}$  and  $e^+e^- \rightarrow n\bar{n}$  reveal an enhancement near the threshold [1–4]. The enhancement near the  $p\bar{p}$  threshold is also observed in the decays  $J/\psi(\psi') \rightarrow p\bar{p}\pi^0(\eta)$  [5–7],  $J/\psi(\psi') \rightarrow p\bar{p}\omega(\gamma)$  [5, 8–11]. The sharp peak in the vicinity of  $N\bar{N}$  threshold has been observed in the cross sections of several processes, i.e.,  $e^+e^- \rightarrow 6\pi$  [12–16] and  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  [17]. These observations led to numerous speculations about a new resonance [5],  $p\bar{p}$  bound state [18–20], or even a glueball state [21–23] with the mass about double proton mass. Another possibility, which we are studying, is the nucleon-antinucleon interaction in the final or intermediate states.

We describe the nucleon-antinucleon interaction by means of an optical potential model. Several optical nucleon-antinucleon potentials [24–26] are usually used to describe the interaction in the low-energy region. All these nucleon-antinucleon potentials have been proposed to fit the nucleon-antinucleon scattering data. These data include elastic, charge-exchange, and annihilation cross sections of  $p\bar{p}$  scattering, as well as some single-spin observables. There were attempts to describe the processes of  $N\bar{N}$  production in  $e^+e^-$  annihilation using these potential models. For instance, using the Paris [27] and Jülich [28]

models, it has been shown that the near-threshold enhancement of the cross sections of these processes can be explained by the final-state nucleon-antinucleon interaction. The strong dependence of the ratio of electromagnetic form factors of the proton on the energy in the timelike region near the threshold has been explained by the influence of the tensor part of the nucleon-antinucleon interaction.

In our recent papers [29, 30], to fit the parameters of the potentials, we have suggested to include all available experimental data in addition to the nucleon-antinucleon scattering data. A simple potential model of  $N\bar{N}$  interaction in the partial waves  $^3S_1 - ^3D_1$ , coupled by the tensor forces, has been suggested [29]. The parameters of this model has been obtained by fitting simultaneously the nucleon-antinucleon scattering data, the cross sections of  $p\bar{p}$  and  $n\bar{n}$  production in  $e^+e^-$  annihilation, and the ratio of electromagnetic form factors of the proton in the timelike region. This model has allowed us to calculate also the contribution of virtual  $N\bar{N}$  intermediate state to the processes of meson production in  $e^+e^-$  annihilation and to describe the sharp dip in the cross section of  $6\pi$  production in the vicinity of the  $N\bar{N}$  threshold [29]. Similar results have also been obtained in Ref. [31] within the chiral model [26] but without the tensor  $N\bar{N}$  interaction taken into account.

The potential [29] has also been used to explain the enhancement observed in the  $p\bar{p}$  invariant mass spectra of the decays  $J/\psi(\psi') \rightarrow p\bar{p}\pi^0(\eta)$  near the  $p\bar{p}$  threshold [32]. Note that in these decays in the near-threshold region the most important contribution is also given by the partial waves  $^3S_1 - ^3D_1$ . The spectra of these decays, as well as the decays  $J/\psi(\psi') \rightarrow p\bar{p}\omega(\rho, \gamma)$ , have also been studied in Refs. [33, 34] using the chiral model [26].

\*e-mail: S.G.Salnikov@inp.nsk.su

In the paper [30] we follow our idea and construct a simple optical potential model of the  $N\bar{N}$  interaction in the  $^1S_0$  partial wave. This partial wave gives the most important contribution to the final-state  $p\bar{p}$  interaction in the decays  $J/\psi(\psi') \rightarrow p\bar{p}\omega(\rho, \gamma)$  in the energy region close to the  $p\bar{p}$  threshold. The parameters of this potential has been obtained by fitting simultaneously the nucleon-antinucleon scattering data and the  $p\bar{p}$  invariant mass spectra of the decays  $J/\psi(\psi') \rightarrow p\bar{p}\gamma$  and  $J/\psi \rightarrow p\bar{p}\omega$ . We have showed that it is possible to describe the pronounced peak in the  $p\bar{p}$  invariant mass spectrum of the decay  $J/\psi \rightarrow p\bar{p}\gamma$  using a simple model of the  $N\bar{N}$  interaction. Moreover, in contrast to the results of Ref. [33], our model doesn't predict such peak in the spectrum of the decay  $J/\psi \rightarrow p\bar{p}\rho$  which has not been observed yet.

We have used our model to calculate the contribution of virtual  $N\bar{N}$  pair to the  $J/\psi \rightarrow \eta'\pi^+\pi^-$  decay rate in the energy region near the  $N\bar{N}$  threshold. Our model describes a peak in the  $\eta'\pi^+\pi^-$  invariant mass spectrum. It has been pointed out in Ref. [17] that a contribution of virtual  $p\bar{p}$  state may be one of possible origins of the peak in the spectrum. However, in Ref. [17] any models of the  $N\bar{N}$  interaction have not been applied.

The present paper is a compilation of the results of Refs. [30, 32], and some parts are identical to the corresponding sections of these papers.

## 2 Decay amplitudes

We give only the resulting formulas here and the reader is referred to the Refs. [30, 32] for more details.

At first let us discuss the  $J/\psi$  decay into a  $p\bar{p}$  pair and a pseudoscalar meson. Possible states for a  $p\bar{p}$  pair in the decays  $J/\psi \rightarrow p\bar{p}\pi^0$  and  $J/\psi \rightarrow p\bar{p}\eta$  have quantum numbers  $J^{PC} = 1^{--}$  and  $J^{PC} = 1^{+-}$ . The dominating mechanism of the  $p\bar{p}$  pair creation is the following. The  $p\bar{p}$  pair is created at small distances in the  $^3S_1$  state and acquires an admixture of  $^3D_1$  partial wave at large distances due to the tensor forces in the nucleon-antinucleon interaction. The  $p\bar{p}$  pairs have different isospins for the two final states under consideration ( $I = 1$  for the  $p\bar{p}\pi^0$  state, and  $I = 0$  for the  $p\bar{p}\eta$  state), that allows one to analyze two isospin states independently. Therefore, these decays are easier to investigate theoretically than the process  $e^+e^- \rightarrow p\bar{p}$ , where the  $p\bar{p}$  pair is a mixture of different isospin states.

We derive the formulas for the decay rate of the process  $J/\psi \rightarrow p\bar{p}x$ , where  $x$  is one of the pseudoscalar mesons  $\pi^0$  or  $\eta$ . The following kinematics is considered:  $\mathbf{k}$  and  $\boldsymbol{\varepsilon}_k$  are the momentum and the energy of the  $x$  meson in the  $J/\psi$  rest frame,  $\mathbf{p}$  is the proton momentum in the  $p\bar{p}$  center-of-mass frame,  $M$  is the invariant mass of the  $p\bar{p}$  system. The following relations hold:

$$\begin{aligned} p = |\mathbf{p}| &= \sqrt{\frac{M^2}{4} - m_p^2}, \\ k = |\mathbf{k}| &= \sqrt{\varepsilon_k^2 - m^2}, \\ \varepsilon_k &= \frac{m_{J/\psi}^2 + m^2 - M^2}{2m_{J/\psi}}, \end{aligned} \quad (1)$$

where  $m$  is the mass of the  $x$  meson,  $m_{J/\psi}$  and  $m_p$  are the masses of a  $J/\psi$  meson and a proton, respectively, and  $\hbar = c = 1$ . Since we consider the  $p\bar{p}$  invariant mass region  $M - 2m_p \ll m_p$ , the proton and antiproton are non-relativistic in their center-of-mass frame, while  $\varepsilon_k$  is about 1 GeV.

The spin-1 wave function of the  $p\bar{p}$  pair in the center-of-mass frame has the form [35]

$$\psi'_\lambda = \mathbf{e}_\lambda u'_1(0) + \frac{u'_2(0)}{\sqrt{2}} [\mathbf{e}_\lambda - 3\hat{\mathbf{p}}(\mathbf{e}_\lambda \cdot \hat{\mathbf{p}})], \quad (2)$$

where  $\hat{\mathbf{p}} = \mathbf{p}/p$ ,  $\mathbf{e}_\lambda$  is the polarization vector of the spin-1  $p\bar{p}$  pair,  $u'_1(r)$  and  $u'_2(r)$  are the components of two independent solutions of the coupled-channels radial Schrödinger equations for  $^3S_1$ – $^3D_1$  partial waves (see Ref. [32] for more details). The dimensionless amplitude of the decay with the corresponding isospin of the  $p\bar{p}$  pair can be written as

$$T'_{\lambda\lambda'} = \frac{\mathcal{G}_I}{m_{J/\psi}} \psi'_\lambda [\mathbf{k} \times \boldsymbol{\varepsilon}_{\lambda'}]. \quad (3)$$

Here  $\mathcal{G}_I$  is an energy-independent dimensionless constant and  $\boldsymbol{\varepsilon}_{\lambda'}$  is the polarization vector of  $J/\psi$ .

The decay rate of the process  $J/\psi \rightarrow p\bar{p}x$  can be written in terms of the dimensionless amplitude  $T'_{\lambda\lambda'}$  as (see, e.g., [36])

$$\frac{d\Gamma}{dM d\Omega_p d\Omega_k} = \frac{pk}{2^9 \pi^5 m_{J/\psi}^4} |T'_{\lambda\lambda'}|^2, \quad (4)$$

where  $\Omega_p$  is the proton solid angle in the  $p\bar{p}$  center-of-mass frame and  $\Omega_k$  is the solid angle of the  $x$  meson in the  $J/\psi$  rest frame.

Substituting the amplitude (3) in Eq. (4) and averaging over the spin states, we obtain the  $p\bar{p}$  invariant mass and angular distribution for the decay rate

$$\begin{aligned} \frac{d\Gamma}{dM d\Omega_p d\Omega_k} &= \frac{\mathcal{G}_I^2 p k^3}{2^{11} \pi^5 m_{J/\psi}^4} \left\{ |u'_1(0) + \frac{1}{\sqrt{2}} u'_2(0)|^2 + \right. \\ &\quad \left. + |u'_1(0) - \sqrt{2} u'_2(0)|^2 (\mathbf{n} \cdot \hat{\mathbf{k}})^2 + \right. \\ &\quad \left. + \frac{3}{2} \left[ |u'_2(0)|^2 - 2\sqrt{2} \operatorname{Re}(u'_1(0)u'_2(0)) \right] \times \right. \\ &\quad \left. \times [(\mathbf{n} \cdot \hat{\mathbf{p}})^2 - 2(\mathbf{n} \cdot \hat{\mathbf{k}})(\mathbf{n} \cdot \hat{\mathbf{p}})(\hat{\mathbf{p}} \cdot \hat{\mathbf{k}})] \right\}. \end{aligned} \quad (5)$$

The invariant mass distribution can be obtained by integrating Eq. (5) over the solid angles  $\Omega_p$  and  $\Omega_k$ :

$$\frac{d\Gamma}{dM} = \frac{\mathcal{G}_I^2 p k^3}{2^5 3\pi^3 m_{J/\psi}^4} \left( |u'_1(0)|^2 + |u'_2(0)|^2 \right). \quad (6)$$

The sum in the brackets is the so-called enhancement factor which equals to unity if the  $p\bar{p}$  final-state interaction is turned off.

More information about the properties of  $N\bar{N}$  interaction can be extracted from the angular distributions. Integrating Eq. (5) over  $\Omega_p$  we obtain

$$\frac{d\Gamma}{dM d\Omega_k} = \frac{\mathcal{G}_I^2 p k^3}{2^9 \pi^3 m_{J/\psi}^4} \left( |u'_1(0)|^2 + |u'_2(0)|^2 \right) [1 + \cos^2 \vartheta_k], \quad (7)$$

where  $\vartheta_k$  is the angle between  $\mathbf{n}$  and  $\mathbf{k}$ . However, the angular part of this distribution does not depend on the features of the  $p\bar{p}$  interaction. The proton angular distribution in the  $p\bar{p}$  center-of-mass frame is more interesting. To obtain this distribution we integrate Eq. (5) over the solid angle  $\Omega_k$ :

$$\frac{d\Gamma}{dM d\Omega_p} = \frac{\mathcal{G}_1^2 p k^3}{2^7 3\pi^4 m_{J/\psi}^4} \left( |u_1'(0)|^2 + |u_2'(0)|^2 \right) \times \left[ 1 + \gamma' P_2(\cos \vartheta_p) \right], \quad (8)$$

where  $\vartheta_p$  is the angle between  $\mathbf{n}$  and  $\mathbf{p}$ ,  $P_2(x) = \frac{3x^2-1}{2}$  is the Legendre polynomial, and  $\gamma'$  is the parameter of anisotropy:

$$\gamma' = \frac{1}{4} \frac{|u_2'(0)|^2 - 2\sqrt{2} \operatorname{Re} [u_1'(0)u_2'^*(0)]}{|u_1'(0)|^2 + |u_2'(0)|^2}. \quad (9)$$

Averaging (5) over the direction of  $\mathbf{n}$  gives the distribution over the angle  $\vartheta_{pk}$  between  $\mathbf{p}$  and  $\mathbf{k}$ :

$$\frac{d\Gamma}{dM d\Omega_{pk}} = \frac{\mathcal{G}_1^2 p k^3}{2^7 3\pi^4 m_{J/\psi}^4} \left( |u_1'(0)|^2 + |u_2'(0)|^2 \right) \times \left[ 1 - 2\gamma' P_2(\cos \vartheta_{pk}) \right]. \quad (10)$$

Note that this distribution can be written in terms of the same anisotropy parameter (9). The mass spectrum (6) and the anisotropy parameter (9) are sensitive to the tensor part of the  $N\bar{N}$  potential and, therefore, give the possibility to verify the potential model.

Now let us discuss other  $J/\psi$  decays where a  $p\bar{p}$  pair and a vector particle ( $\rho$ ,  $\omega$ ,  $\gamma$ ) are produced. Due to the  $C$ -parity conservation law, possible states for a  $p\bar{p}$  pair in such decays are  $^1S_0$  and  $^3P_J$ . The  $S$ -wave state dominates in the near-threshold region where the relative velocity of the nucleons is small. The  $p\bar{p}$  pairs have different isospins for the final states containing a vector meson ( $I = 1$  for the  $p\bar{p}\rho$  state, and  $I = 0$  for the  $p\bar{p}\omega$  state). In the case of  $p\bar{p}\gamma$  final state, the  $p\bar{p}$  pair is a mixture of two isospin states.

The same kinematics relations (1) hold, as well as the relation between the decay rate and the amplitude (4). The dimensionless amplitude of these decays can be written via the radial wave function of the  $p\bar{p}$  pair corresponding to the  $^1S_0$  wave,  $\psi_R^I(r)$ , as

$$T_{\lambda\lambda'}^I = \frac{\mathcal{G}_I}{m_{J/\psi}} \mathbf{e}_{\lambda} \cdot [\mathbf{k} \times \mathbf{e}_{\lambda'}] \psi_R^I(0). \quad (11)$$

Here  $\mathcal{G}_I$  is an energy-independent dimensionless constant,  $\mathbf{e}_{\lambda}$  and  $\mathbf{e}_{\lambda'}$  are the polarization vectors of the final vector particle and  $J/\psi$ , respectively.

Substituting the amplitude (11) in Eq. (4), averaging over the spin states and integrating over the solid angles, we obtain the  $p\bar{p}$  invariant mass distribution

$$\frac{d\Gamma}{dM} = \frac{\mathcal{G}_I^2 p k^3}{2^4 3\pi^3 m_{J/\psi}^4} |\psi_R^I(0)|^2. \quad (12)$$

The wave function module squared is the so-called enhancement factor which equals to unity if the  $p\bar{p}$  final-state interaction is turned off.

The optical  $N\bar{N}$  potential can also be used to calculate the decay rates of the processes with a virtual  $N\bar{N}$  pair in the intermediate state. In Ref. [29] it is shown that the total cross section of  $N\bar{N}$  production, which is a sum of the cross section of real  $N\bar{N}$  pair production (the elastic cross section) and the cross section of the meson production via annihilation of a virtual  $N\bar{N}$  pair (the inelastic cross section), can be written in terms of the Green's function of the  $N\bar{N}$  pair. According to Ref. [29], in order to switch from the elastic cross section to the total one, we should replace  $|\psi_R^I(0)|^2$  by  $(-\operatorname{Im} \mathcal{D}^I(0, 0|E)/m_p p)$ , where  $\mathcal{D}^I(r, r'|E)$  is the Green's function of the Schrödinger equation. Therefore, the contribution of the  $N\bar{N}$  intermediate state with quantum numbers  $^1S_0$  to the decay rate of the process  $J/\psi \rightarrow N\bar{N}x \rightarrow \text{particles} + x$  (particles in the final state can be nucleons or mesons) has the form

$$\frac{d\Gamma_{\text{tot}}}{dM} = -\frac{\mathcal{G}_1^2 k^3}{2^4 3\pi^3 m_p m_{J/\psi}^4} \operatorname{Im} \mathcal{D}^I(0, 0|E), \quad (13)$$

where  $M$  is the invariant mass of the mesons,  $E = M/2 - m_p$ . The Green's function can be written in terms of regular,  $\psi_R^I(r)$ , and non-regular,  $\psi_N^I(r)$ , solutions of the Schrödinger equation:

$$\mathcal{D}^I(r, r'|E) = -m_p p \left\{ \theta(r' - r) \psi_R^I(r) \psi_N^I(r') + \theta(r - r') \psi_N^I(r) \psi_R^I(r') \right\}, \quad (14)$$

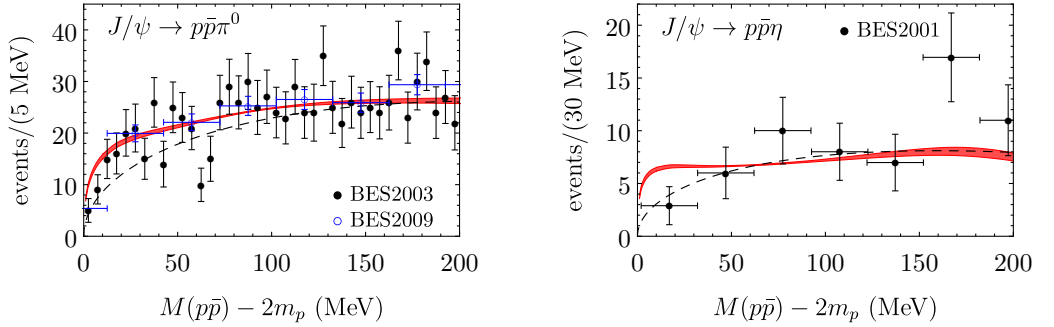
where  $\theta(x)$  is the Heaviside function.

## 3 Results and Discussion

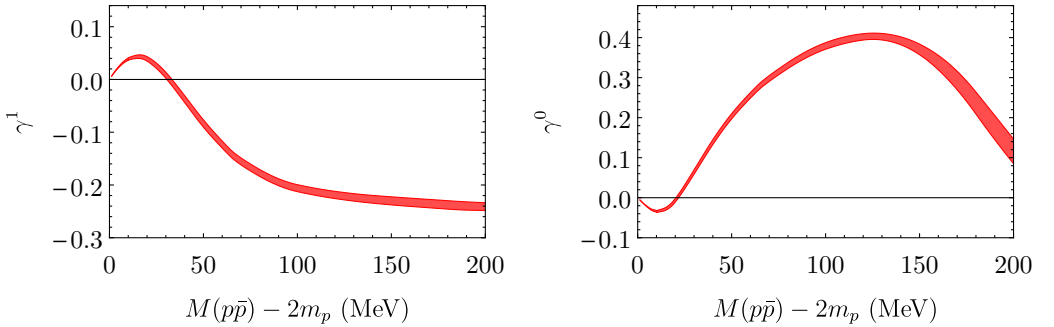
### 3.1 The decays $J/\psi \rightarrow p\bar{p}\pi^0(\eta)$

In the present work we use the potential model suggested in Ref. [29] to describe the  $p\bar{p}$  interaction in the  $^3S_1 - ^3D_1$  coupled channels. The parameters of this model have been fitted using the  $p\bar{p}$  scattering data, the cross section of  $N\bar{N}$  pair production in  $e^+e^-$  annihilation near the threshold, and the ratio of the electromagnetic form factors of the proton in the timelike region. We have slightly refitted the parameters of the model in order to achieve a better description of the invariant mass spectra of the decays considered. By means of this model and Eq. (6), we predict the  $p\bar{p}$  invariant mass spectra in the processes  $J/\psi \rightarrow p\bar{p}\pi^0$  and  $J/\psi \rightarrow p\bar{p}\eta$ . The isospin of the  $p\bar{p}$  pair is  $I = 1$  and  $I = 0$  for, respectively, a pion and  $\eta$  meson in the final state. The model [29] predicts the enhancement of the decay rates of both processes near the threshold of  $p\bar{p}$  pair production (see Fig. 1). The invariant mass spectra predicted by our model are similar to those predicted in Ref. [33] with the use of the chiral model.

An important prediction of our model is the angular anisotropy of these  $J/\psi$  decays. This anisotropy is the result of  $D$ -wave admixture due to the tensor forces in  $N\bar{N}$  interaction. The anisotropy (see Eqs. (8) and (10)) is characterized by the parameters  $\gamma^1$  and  $\gamma^0$  (9) for the  $p\bar{p}\pi^0$  and  $p\bar{p}\eta$  final states, respectively. The dependence of the parameters  $\gamma^I$  on the invariant mass of the  $p\bar{p}$  pair is shown



**Figure 1.** The invariant mass spectra of  $J/\psi$  decays to  $p\bar{p}\pi^0$  (left) and  $p\bar{p}\eta$  (right). The phase space behavior is shown by the dashed curve. The experimental data are taken from Refs. [5–7]. The measurement of Ref. [5] is adopted for the scale of the left plot.



**Figure 2.** The dependence of the anisotropy parameters  $\gamma^l$  in the decays  $J/\psi \rightarrow p\bar{p}\pi^0$  (left) and  $J/\psi \rightarrow p\bar{p}\eta$  (right) on the  $p\bar{p}$  invariant mass.

in Fig. 2. For  $p\bar{p}$  invariant mass about 100 – 200 MeV above the threshold, significant anisotropy of the angular distributions is predicted. Note that the anisotropy in the distribution over the angle  $\vartheta_{pk}$  is expected to be two times larger than in the distribution over the angle  $\vartheta_p$  (compare Eqs. (8) and (10)).

There are some data on the angular distributions in the decays  $J/\psi \rightarrow p\bar{p}\pi^0$  [6] and  $J/\psi \rightarrow p\bar{p}\eta$  [7]. However, these distributions are obtained by integration over the whole  $p\bar{p}$  invariant mass region. Unfortunately, our predictions are valid only in the narrow energy region above the  $p\bar{p}$  threshold. Therefore, we cannot compare the predictions with the available experimental data. The measurements of the angular distributions at  $p\bar{p}$  invariant mass close to the  $p\bar{p}$  threshold would be very helpful. Such measurements would provide another possibility to verify the available models of  $N\bar{N}$  interaction in the low-energy region.

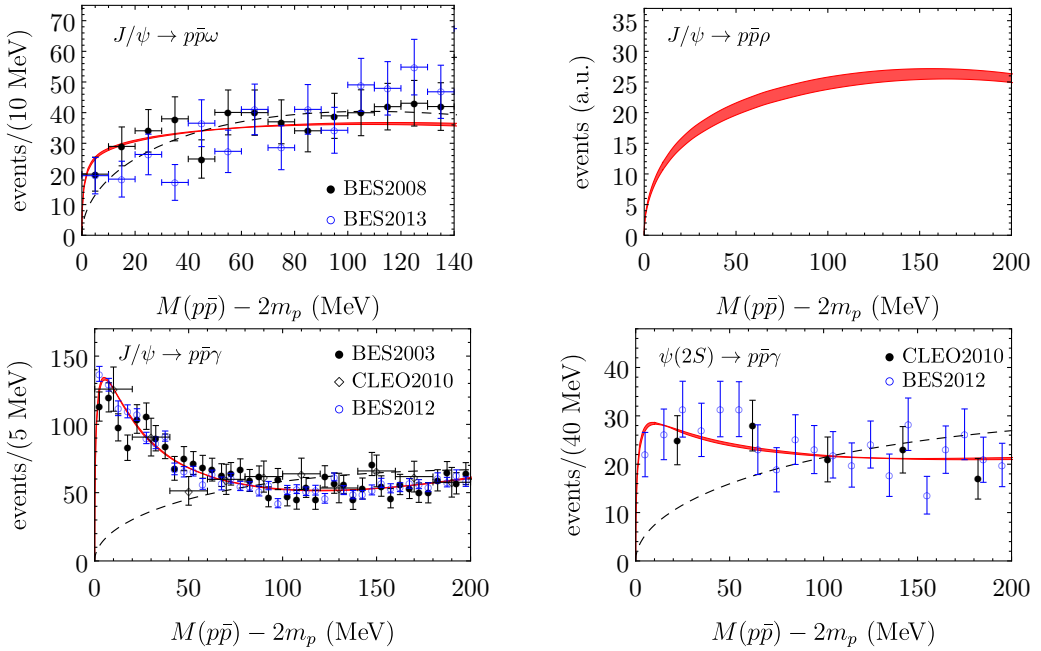
### 3.2 The decays $J/\psi \rightarrow p\bar{p}\gamma(\rho, \omega)$

In order to describe the  $p\bar{p}$  interaction in the decays  $J/\psi \rightarrow p\bar{p}\rho(\omega, \gamma)$  we use an  $N\bar{N}$  optical potential for the  $^1S_0$  partial wave proposed in Ref. [30]. The data used for fitting the parameters of the potential include the

partial contributions of  $^1S_0$  wave to the elastic, charge-exchange, and total cross sections of  $p\bar{p}$  scattering, and the  $p\bar{p}$  invariant mass spectra of the decays  $J/\psi \rightarrow p\bar{p}\omega$ ,  $J/\psi \rightarrow p\bar{p}\gamma$ , and  $\psi(2S) \rightarrow p\bar{p}\gamma$ . The partial cross sections of  $p\bar{p}$  scattering are calculated from the Nijmegen partial wave  $S$ -matrix (Table V of Ref. [25]).

By means of the potential model and Eq. (12), we calculate the  $p\bar{p}$  invariant mass spectra in the processes  $J/\psi \rightarrow p\bar{p}\omega$  and  $J/\psi \rightarrow p\bar{p}\rho$  (see Fig. 3). The isospin of the  $p\bar{p}$  pair is  $I = 0$  and  $I = 1$  for  $\omega$  meson and  $\rho$  meson in the final state, respectively. Therefore, the decay rates for these processes are given by Eq. (12) with the corresponding constants  $\mathcal{G}_I$  and wave functions  $\psi_R^I(0)$ . Our model fits the experimental data for the decay  $J/\psi \rightarrow p\bar{p}\omega$  quite well. There are no experimental data for the decay  $J/\psi \rightarrow p\bar{p}\rho$ , therefore, the predictions for the invariant mass spectrum are especially important. The  $p\bar{p}$  spectrum in the decay  $J/\psi \rightarrow p\bar{p}\rho$ , calculated in Ref. [33] with the use of the chiral model [26], has a pronounced peak close to the  $p\bar{p}$  threshold, while our model predicts a monotonically increasing spectrum without any peak.

The decay amplitude of the process  $J/\psi \rightarrow p\bar{p}\gamma$  is a sum of two isospin contributions. Therefore, the decay



**Figure 3.** The invariant mass spectra of  $J/\psi$  decays to  $p\bar{p}\omega$ ,  $p\bar{p}\rho$ ,  $p\bar{p}\gamma$ , and  $\psi(2S)$  decay to  $p\bar{p}\gamma$ . The invariant mass spectra without  $N\bar{N}$  interaction taken into account are shown by the dashed curves. The experimental data are taken from Refs. [5, 8–11]. The earliest measurements are adopted for the scale of the plots. This Figure has been copied from Ref. [30].

rate reads

$$\frac{d\Gamma_{p\bar{p}\gamma}}{dM} = \frac{pk^3}{2^4 3\pi^3 m_{J/\psi}^4} |\mathcal{G}_{\gamma 0}\psi_R^0(0) + \mathcal{G}_{\gamma 1}\psi_R^1(0)|^2. \quad (15)$$

Our model describes with good accuracy the pronounced peak, seen fairly well in the experimental data for the decay  $J/\psi \rightarrow p\bar{p}\gamma$  (see Fig. 3). For the best fit, the ratio of the constants is  $\mathcal{G}_{\gamma 1}/\mathcal{G}_{\gamma 0} = -1.29 - 0.6i$ . We have investigated in details the origin of this peak and found out that it arises because of a significant compensation of two isospin amplitudes at energy above 10 MeV per nucleon, though each isospin amplitude has no peak. This leads to another interesting prediction. The decay rate of the process  $J/\psi \rightarrow n\bar{n}\gamma$ , given by the formula

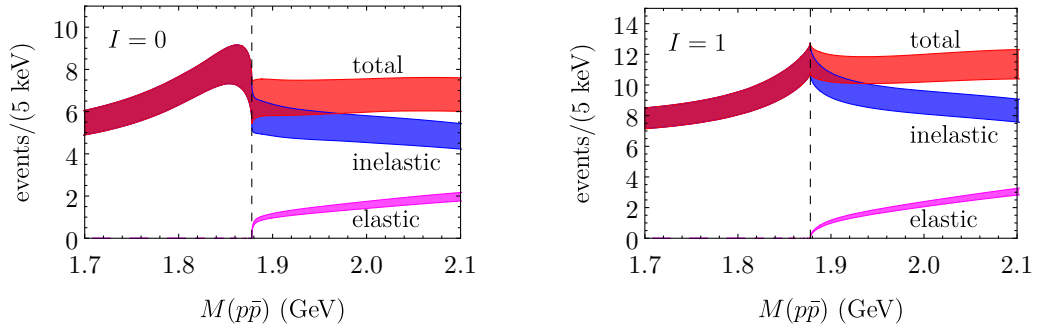
$$\frac{d\Gamma_{n\bar{n}\gamma}}{dM} = \frac{pk^3}{2^4 3\pi^3 m_{J/\psi}^4} |\mathcal{G}_{\gamma 0}\psi_R^0(0) - \mathcal{G}_{\gamma 1}\psi_R^1(0)|^2, \quad (16)$$

should be much larger than that for the process  $J/\psi \rightarrow p\bar{p}\gamma$ . The experimental investigation of the decay  $J/\psi \rightarrow n\bar{n}\gamma$  would provide important information about the  $N\bar{N}$  interaction. For completeness, we also consider the decay  $\psi(2S) \rightarrow p\bar{p}\gamma$  (the corresponding ratio of the constants is  $\mathcal{G}_{\gamma 1}/\mathcal{G}_{\gamma 0} = -1.06 - 0.09i$ ), see Fig. 3.

Making use of our potential model and Eq. (13), we obtain also the predictions for the decay rates of the processes with the interaction of virtual nucleon-antinucleon pairs in the intermediate state (see Fig. 4). A peak in the

total and inelastic invariant mass spectra exists near the  $p\bar{p}$  threshold, especially in the isoscalar channel. This behavior seems to be the consequence of the existence of a quasi-bound state near the  $p\bar{p}$  threshold. Our analysis shows that such state does exist in the isoscalar channel, and its energy is  $E_B = (22 - 33i)$  MeV. This is an unstable bound state in the classification of Ref. [37] because its energy moves to  $E_B = -3$  MeV when the imaginary part of the  $N\bar{N}$  potential is turned off.

Let us discuss the exotic behavior of the decay rate of the process  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  near the  $N\bar{N}$  threshold observed in Ref. [17]. An approximate description of the spectrum of this decay was suggested in Ref. [38]. However, due to poor accuracy of the experimental data available at that time, only general resonance structure of the spectrum has been discussed in Ref. [38], but not the sharp dip in the vicinity of  $N\bar{N}$  threshold. The new experimental data [17] allows us to investigate this spectrum in more details. The  $G$ -parity of the intermediate  $N\bar{N}$  state,  $G_{N\bar{N}} = C_{N\bar{N}}(-1)^I$ , should be equal to that of the final  $\eta'\pi^+\pi^-$  state,  $G_{\eta'\pi^+\pi^-} = 1$ . Taking into account  $C$ -parity conservation we obtain  $C_{N\bar{N}} = 1$ , thus the isospin of the  $N\bar{N}$  pair is  $I = 0$ . Possible  $N\bar{N}$  states with positive  $C$ -parity are  $^1S_0$  and  $^3P_0$ , and the former one is expected to dominate in the near-threshold region. Therefore, we believe that the peak in the  $\eta'\pi^+\pi^-$  invariant mass spectrum could occur because of the interaction of virtual nucleons in the isoscalar  $^1S_0$  intermediate state. The contribution



**Figure 4.** The  $p\bar{p}$  invariant mass spectra for the isospin components in the decays  $J/\psi \rightarrow \gamma p\bar{p}$  (elastic),  $J/\psi \rightarrow \gamma + p\bar{p} \rightarrow \gamma + \text{mesons}$  (inelastic), and total. Vertical dashed line is the  $p\bar{p}$  threshold. This Figure has been copied from Ref. [30].

of non- $N\bar{N}$  channels should be a smooth function in the vicinity of the  $N\bar{N}$  threshold. Therefore, we approximate the invariant mass spectrum of the decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  by the function

$$A \cdot d\Gamma_{\text{inel}}^0/dM + B \cdot E + C, \quad (17)$$

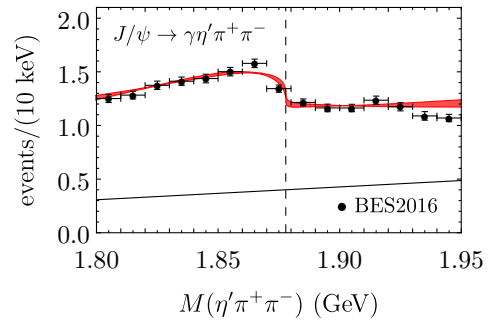
where  $A$ ,  $B$  and  $C$  are some fitting parameters. The comparison of the experimental data and our fitting formula in Fig. 5 demonstrates good agreement in the near-threshold region.

One can expect that the  $\gamma p\bar{p}$  intermediate state plays an important role in the decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  near the threshold of  $p\bar{p}$  pair production. Therefore, the branching ratio of this channel should be related to the branching ratio for the  $p\bar{p}$  annihilation at rest into the  $\eta'\pi^+\pi^-$  final state. To check the validity of this statement, we should take into account that the scale of Fig. 4 for the decay  $J/\psi \rightarrow p\bar{p}\gamma$  differs from that of Fig. 5 for the decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$  because of different total numbers of  $J/\psi$  events in the experiments [5] and [17]. After that we find the absolute value of the coefficient  $A$  in Eq. (17). The coefficient  $A \approx 4 \cdot 10^{-3}$  can be considered as the estimation of the branching ratio of the decay  $p\bar{p} \rightarrow \eta'\pi^+\pi^-$  at rest. This value is very close to the branching ratio  $3.46 \cdot 10^{-3}$  measured in the experiment [39].

## 4 Conclusions

Using the model proposed in Ref. [29], we have calculated the effects of  $p\bar{p}$  final-state interaction in the decays  $J/\psi \rightarrow p\bar{p}\pi^0(\eta)$ . Our results for the  $p\bar{p}$  invariant mass spectra close to the  $p\bar{p}$  threshold are in agreement with the available experimental data. The tensor forces in the  $p\bar{p}$  interaction result in the anisotropy of the angular distributions. The anisotropy in the decay  $J/\psi \rightarrow p\bar{p}\pi^0$  and especially in the  $J/\psi \rightarrow p\bar{p}\eta$  decay is large enough to be measured. The observation of such anisotropy close to the  $p\bar{p}$  threshold would allow one to refine the model of  $N\bar{N}$  interaction.

Using the model of  $N\bar{N}$  interaction in  $^1S_0$  partial wave we have calculated the effects of  $p\bar{p}$  final-state interaction



**Figure 5.** The  $\eta'\pi^+\pi^-$  invariant mass spectrum for the decay  $J/\psi \rightarrow \gamma\eta'\pi^+\pi^-$ . The thin line shows the contribution of non- $N\bar{N}$  channels. Vertical dashed line is the  $N\bar{N}$  threshold. The experimental data are taken from Ref. [17]. This Figure has been copied from Ref. [30].

in other  $J/\psi$  decays. Our model describes the  $p\bar{p}$  invariant mass spectra of the decays  $J/\psi \rightarrow p\bar{p}\omega$ ,  $J/\psi \rightarrow p\bar{p}\gamma$ , and  $\psi(2S) \rightarrow p\bar{p}\gamma$  with good precision. We have also obtained the predictions for the  $p\bar{p}$  invariant mass spectrum in the decay  $J/\psi \rightarrow p\bar{p}\rho$  which has not been measured yet. Our prediction for this spectrum differs from the theoretical results obtained earlier. Therefore, the experimental study of the decay rate of this process would help to discriminate different models of the nucleon-antinucleon interaction.

We have used the Green's function approach to calculate the contribution of the interaction of virtual  $N\bar{N}$  pairs in the  $^1S_0$  state to the cross sections of the processes. In particular we have calculated the contribution of the  $N\bar{N}$  intermediate state to the  $\eta'\pi^+\pi^-$  invariant mass spectrum for the decay  $J/\psi \rightarrow \gamma N\bar{N} \rightarrow \gamma\eta'\pi^+\pi^-$  in the energy region near the  $N\bar{N}$  threshold. Our results are in good agreement with the available experimental data and describe the peak in the invariant mass spectrum just below the  $N\bar{N}$  threshold.



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