

# Temperature dependence of bulk viscosity of $SU(3)$ -gluodynamics within lattice simulation

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**Abstract.** In this paper we present the results of the measurements of bulk viscosity temperature dependence in  $SU(3)$ -gluodynamics by means of lattice simulations. Our approach is based on the Kubo formula, which relates bulk viscosity and the correlator of the energy-momentum trace anomaly. We measured the correlation functions of the energy-momentum trace anomaly in the range of temperatures  $T/T_c \in [0.9, 1.5]$  and extracted the value of bulk viscosity using two methods. The first one was to apply the middle-point estimation. The second method is based on the Backus-Gilbert method allowing to extract the bulk viscosity from the lattice data nonparametrically. The results obtained within both methods are in agreement. Based on our results we conclude that the Gluon Plasma under study reveals the properties of a strongly interacting system, which cannot be described perturbatively.

## 1 Introduction

Modern heavy ion collision experiments like RHIC and LHC are aimed at study of quark-gluon plasma (QGP). Hydrodynamic description of QGP evolution proved to be efficient in understanding of experimental results [1, 2]. Despite this success hydrodynamics can be considered only as an effective theory which correctly accounts dynamics of infrared degrees of freedom. As the result there are unknown parameters such as shear viscosity, bulk viscosity, conductivity, etc. These parameters cannot be calculated within hydrodynamics but must be determined either from experiment or from the calculation based on the first principles.

Unfortunately today there is no analytical approach which fully accounts nonperturbative dynamics of QGP based on the first principles. For this reason the only way to calculate bulk viscosity of QGP is lattice simulation of QCD. Despite considerable progress in lattice study of QCD properties today it is not possible to calculate bulk viscosity of QGP with dynamical quarks. Even the study of bulk viscosity of gluodynamics is extremely complicated task. In this paper we are going to study temperature dependence of shear and bulk viscosities of  $SU(3)$ -gluodynamics in the vicinity of confinement/deconfinement phase transition.

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## 2 Details of calculation

In order to calculate the quark-gluon plasma transport coefficients one first needs to measure various correlation functions of energy-momentum tensor components. Bulk viscosity is related to the trace anomaly correlation function:

$$C_\zeta(x_0) = T^{-5} \int d^3\vec{x} \langle \theta(0)\theta(x_0, \vec{x}) \rangle, \quad (1)$$

Correlation function then can be expressed in terms of spectral function  $\rho(\omega)$ :

$$C(x_0) = T^{-5} \int_0^{+\infty} d\omega \rho(\omega) \frac{\cosh \omega \left( \frac{\beta}{2} - x_0 \right)}{\sinh \frac{\omega\beta}{2}}. \quad (2)$$

Having determined the spectral function, one applies the Kubo formula, relating low-frequency behavior of  $\rho(\omega)$  with hydrodynamical transport coefficients:

$$\zeta = \frac{\pi}{9} \lim_{\omega \rightarrow 0} \frac{\rho_\zeta(\omega)}{\omega}. \quad (3)$$

In this way, lattice calculation of bulk viscosity can be divided into two parts. The first part is the measurement of the correlation function  $C(x_0)$  with sufficient accuracy. This part of the calculation requires large computational resources but for the gluodynamics the accuracy of the correlator can be dramatically improved with the help of the two-level algorithm [3]. The second part is the determination of the spectral function  $\rho(\omega)$  from the correlation function  $C(x_0)$ . The last part of the calculation is probably the most complicated, since one should determine continuous spectral function  $\rho(\omega)$  from the integral equation for the set of  $\mathcal{O}(10)$  values of  $C(x_0)$  measured within lattice simulation. Below we will need the the properties of the spectral function. First we recall very general properties: the positivity of the spectral function  $\rho(\omega) > 0$  for positive frequencies  $\omega > 0$  and oddness. At large frequencies one expects that the spectral function approaches the spectral function calculated at weak coupling due to asymptotic freedom of the system. For this reason it is also important to write the expression for the spectral function at the leading-order approximation in strong coupling constant. For the bulk viscosity it reads [4]:

$$\rho_\zeta(\omega) = d_A \left( \frac{11\alpha_s N}{3(4\pi)^2} \right)^2 \frac{\omega^4}{\tanh(\omega\beta/4)}, \quad (4)$$

where  $d_A = N_c^2 - 1 = 8$  for  $SU(3)$ -gluodynamics. It should be noted here that at large  $\omega$  the leading behavior of the spectral function scales as  $\omega^4$  (multiplied by the squared coupling constant  $\alpha_s^2$ ). Although for the shear function the leading order behaviour is given by the same dependence  $\sim \omega^4$ , the numerical factor for the bulk viscosity in Eq. 4 is much smaller. As the result, the contribution of the ultraviolet part into the correlator of the stress-energy tensor is much smaller, what makes it easier to invert the Kubo relation and extract the value of the bulk viscosity as comparing to the shear channel.

We used the continuum expression for the stress-energy trace anomaly with the clover discretization scheme for the tensor  $F_{\mu\nu}$ :

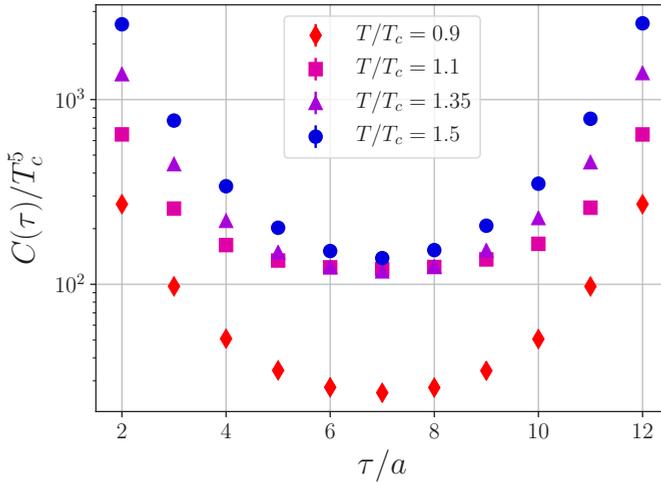
$$F_{\mu\nu}^{(clow)}(x) = \frac{1}{4iga^2} (V_{\mu,\nu}(x) + V_{\nu,-\mu}(x) + V_{-\mu,-\nu}(x) + V_{-\nu,\mu}(x)),$$

$$V_{\mu,\nu}(x) = \frac{1}{2} (U_{\mu,\nu}(x) - U_{\nu,\mu}(x)). \quad (5)$$

To calculate bulk viscosity one should measure the correlation function. To carry out this measurement we use the two-level algorithm. This algorithm significantly improves the speed of the calculations. It has become conventional to present the value of shear viscosity as a the ratio viscosity-to-entropy density  $\zeta/s$ . For homogeneous systems the entropy density  $s$  can be expressed as  $s = \frac{\epsilon+p}{T}$ , where  $\epsilon$  is the energy density and  $p$  is the pressure.

### 3 Numerical data

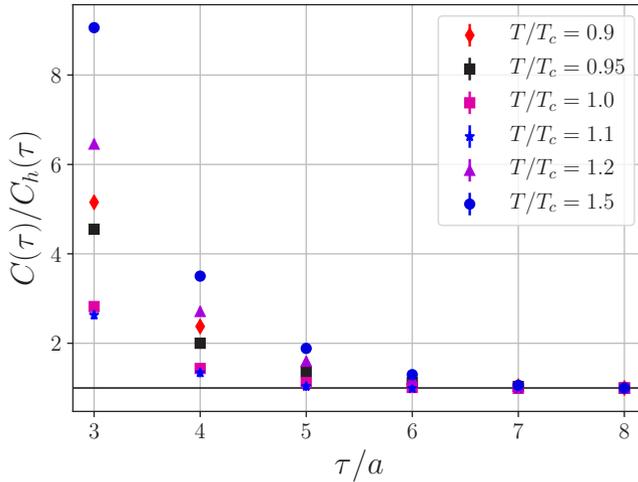
We measured the correlation functions  $C(x_0)$  on the lattice  $16 \times 32^3$  at the temperatures  $T/T_c = 0.90, 0.925, 0.95, 1.0, 1.10, 1.2, 1.35, 1.5$ . Application of two-level algorithm allowed us to get uncertainties not larger than  $\sim 3\%$  at the distance  $\beta/2$  for all temperatures under consideration. For the other points the accuracy is even better. In Fig. 1 we plot the correlation functions for various temperatures. Since we performed the simulation at the fixed lattice size and did not study the finite volume effects, the errors might be underestimated. We plan to address the question of finite volume effects in the future.



**Figure 1.** The correlation functions  $C(x_0)$  for the temperatures  $T/T_c = 0.9, 1.1, 1.35, 1.5$ .

#### 3.1 Bulk viscosity from the middle point estimation

As was discussed before, the contribution of the ultraviolet part of the spectral function to the correlator is suppressed, thus to describe the correlator one should accurately take into account other parts of the spectral density, at small and medium frequencies. If one tries to construct a model function and fit the data, then the number of fitting parameters grows and the fitting procedure becomes complicated and unstable.



**Figure 2.** Ratio of total correlator  $C(x_0)$  to the hydrodynamic part  $C_h(x_0)$  for various temperatures.

However, as our analysis shows, one may use the fact that at large distances  $x_0 \sim \beta/2$  the correlation function is mostly governed by the hydrodynamical contribution, while for small time separations  $x_0$ , on contrary, perturbative part dominates.

In Fig. 2 we plot the ratio of total correlation function to its hydrodynamic part for various temperatures. To do so, we assume that the central point  $x_0 = \beta/2$  is governed by hydrodynamic alone, thus we extract  $\zeta$ , using

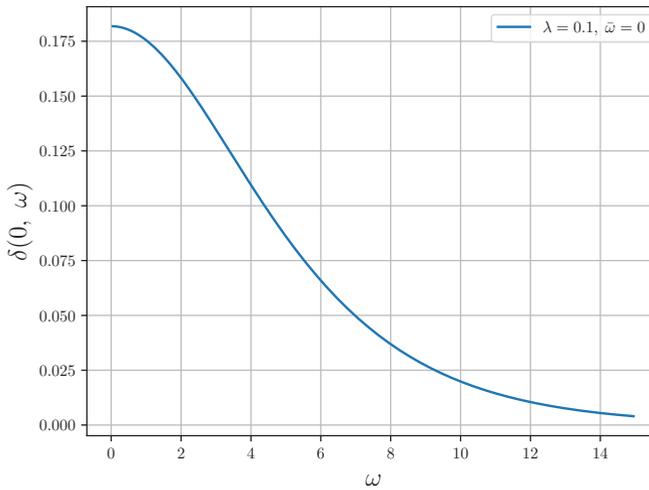
$$C(\beta/2) = \frac{9\zeta}{\pi} \int_0^{+\infty} d\omega \frac{\omega}{\sinh(\omega\beta/2)}.$$

Knowing  $\zeta$ , for every  $x_0$  we calculate  $C_h(x_0)$  as if there was only hydrodynamic contribution and divide the real correlator by  $C_h(x_0)$ . One may notice that, the closer is the temperature to the phase transition, the better the middle point is described by the hydrodynamics alone. Still, for all the temperatures the deviation of correlation function from the middle point, as one moves one time-step back, is negligible, which justifies the middle-point approach. The parameter  $\omega_0$  is varied within 1.5 GeV and 3 GeV, representing the uncertainty in its determination.

### 3.2 Bulk viscosity from the Backus-Gilbert method

We also determined the ratio  $\zeta/s$  using the Backus-Gilbert(BG) method [7, 8]<sup>1</sup>. Within the very similar setup we determined the shear viscosity of  $SU(3)$ -gluodynamics using the Backus-Gilbert method [11]. This approach has considerable advantage over the method based on the fitting procedure or the middle point: one does not need to know the parametrical form of the spectral function to carry out the calculation.

<sup>1</sup>In QCD this approach was recently applied in papers [5, 6]. Backus-Gilbert method was also recently applied in the lattice calculation of graphene conductivity [9].



**Figure 3.** Typical resolution function at  $\lambda = 0.1$  centered at  $\bar{\omega} = 0$ .

We draw the resolution function centered at 0. In Fig. 3 one sees that the width of the resolution function is nearly  $5T$ . As discussed in [11], even in this case, the ultraviolet convoluted with the resolution function, can give a large contribution to the extracted value. To address this issue, we subtracted the ultraviolet contribution before applying Backus-Gilbert method. The ultraviolet part is assumed to have the form:

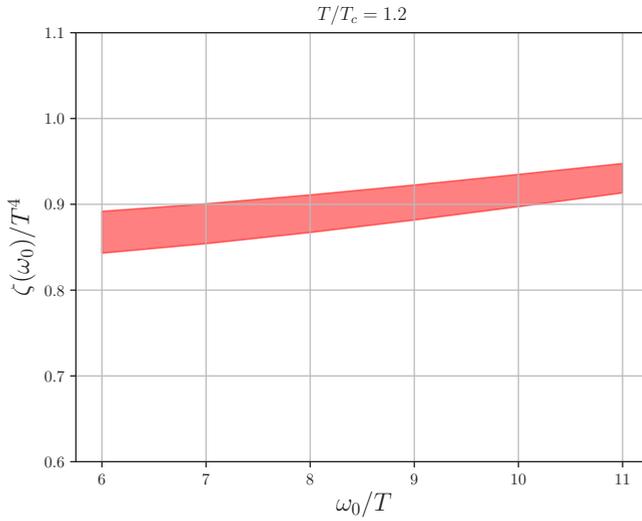
$$\rho(\omega) = A\theta(\omega - \omega_0)\rho_{\text{lat}}(\omega). \tag{6}$$

The parameter  $A$  is determined from the assumption that the smallest points  $x_0 = 2, 3$  of the correlation function are described completely by the ultraviolet tail. The parameter  $\omega_0$  is varied within range from 1.5 GeV to 3.0 GeV, representing the uncertainty in the position of transition from hydrodynamics to the ultraviolet. Varying this parameter, one changes the estimated value of  $\zeta$ , as shown in Fig. 4. We see, that the final value of  $\zeta$  depends on the parameter  $\omega_0$  very mildly. It means, the ultraviolet contribution is indeed small and Backus-Gilbert method gives robust results. Since the precise value of  $\omega_0$  is not known we include this dependence on  $\omega_0$  in the uncertainty of our final results.

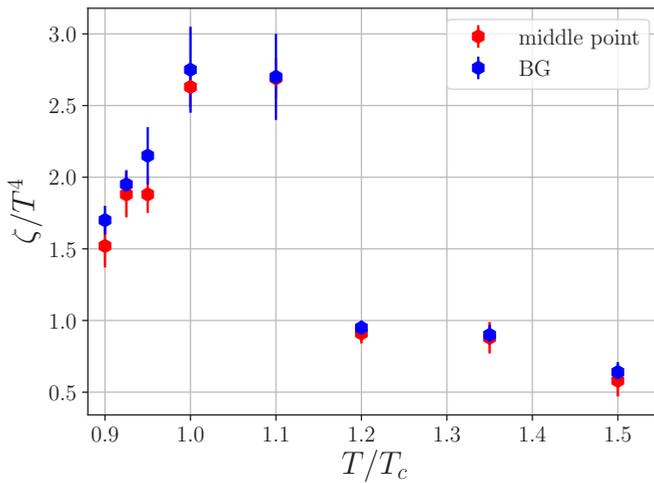
## 4 Results and conclusions

In this paper we measured the values of the bulk viscosity in  $SU(3)$ -gluodynamics for a set of temperatures  $T/T_c \in [0.9, 1, 5]$ . We used two methods for extraction the value of the bulk viscosity from the lattice data for the correlator of the stress-energy tensor: the one, based on the middle point estimation, and the non-parametrical Backus-Gilbert method. The data, obtained within both methods is shown in Fig. 5. Within errors these approaches are consistent and agree with the resent experimental data analysis.

Finally, one may ask the question whether the quark-gluon plasma can be described perturbatively or it is essentially a strongly-interacting system. If the QGP is weakly-interacting then by the per-



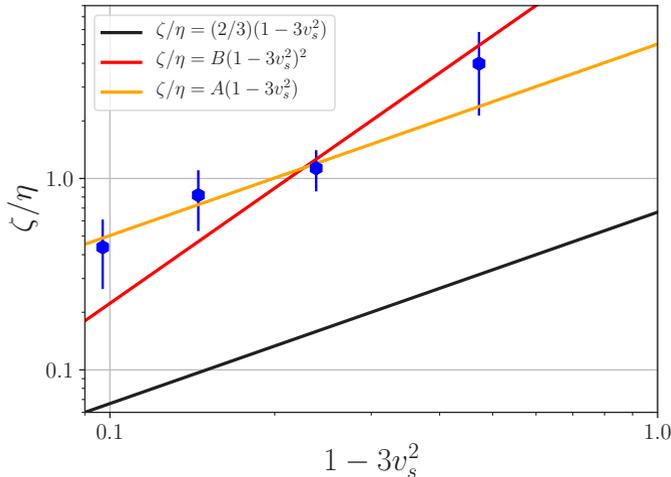
**Figure 4.** Bulk viscosity dependence on the position of transition between hydrodynamic and ultraviolet.



**Figure 5.** The data obtained within both methods. Red points represent the values of  $\zeta/s$  obtained within the middle-point approach, blue points stand for the Backus-Gilbert method result.

turbative computation [10],  $\zeta/\eta \propto (1 - 3v_s^2)^2$ , where  $v_s$  is the velocity of sound in the medium. In the other hand, AdS/CFT calculations that accounts deviations from conformality at strong coupling

gives  $\zeta/\eta \propto (1 - 3v_s^2)$  [12]. The strong coupling analysis also suggests [13], that the ratio  $\zeta/\eta$  should be greater than  $(2/3)(1 - 3v_s^2)$ .



**Figure 6.** Fitting the ratio  $\zeta/\eta$  in both strongly- and weakly-interacting scenarios above the phase transition. The lower bound  $(2/3)(1 - 3v_s^2)$  is also drawn.

Using the data on shear viscosity that we previously obtained in [11], we fit the data above phase transition, as shown in Fig. 6. The fitting procedure favors the strongly-interacting scenario with  $\chi^2/\text{ndof} \sim 1$  contrary to  $\chi^2/\text{ndof} \sim 4$  for weakly-interacting scenario. Thus, our data suggests that the quark-gluon plasma is essentially a strongly-interacting system and cannot be described perturbatively.

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