A review of neutrino properties

Neutrino oscillations - a historical overview and its projection

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Abstract. In a first part neutrino properties are presented from the beginnings through the letter of Wolfgang Pauli on 4. December 1930 suggesting a new neutral fermion to the "Radioactive Ladies and Gentlemen" at a Conference in Tübingen, contributing to the first oscillation cycle, based on my review at the meeting "Neutrino Telescopes in Venice" in 2005. cited in [1]. In the remaining part I shall present a selection of neutrino properties featuring the structure of mass and mixing of the light and heavy neutrino flavors, a symmetric, complex 6 by 6 matrix within the unifying SO(10) gauge group, and present prospects for the detection of the leptonflavor violating processes $B_s \rightarrow \mu e$; $B \rightarrow K \mu e$, also such involving b-flavored baryons.

1 The beginnings of neutrinos - Wolfgang Pauli's idea 1930

Figure 1: Wolfgang Pauli (1900-1958) in Vienna 1933.

The spin 1/2 fermion was called 'neutron'. The neutron of today was discovered by James Chadwick in 1932, cited in reference [2].
→ Nobel prize for physics 1935.
The name neutrino was coined later by Oscar D'Agostino, Emilio Segrè, Edoardo Amaldi, Franco Rasetti, Ettore Majorana and Enrico Fermi
→ "I ragazzi di via Panisperna".

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Brief an die Gruppe der “Radioaktiven” 1930

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930

Liebe Radioaktive Damen und Herren!

Wie der Überbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren aus-
einandersetzen wird, bin ich angesichts der falschen Statistik der N- und Li 6-Kerne, sowie des
kontinuierlichen β -Spektrums auf einen verzweifelten Ausweg verfallen, um den Wechselsatz
der Statistik¹ und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch
neutrale Teilchen, die ich Neutronen² nennen will, in den Kernen existieren, welche den Spin
1/2 haben und das Ausschließungsprinzip befolgen und sich von Lichtquanten außerdem noch
dadurch unterscheiden, daß sie nicht mit Lichtgeschwindigkeit laufen. — Das kontinuierliche
 β -Spektrum wäre dann verständlich unter der Annahme, daß beim β -Zerfall mit dem Elektron
jeweils noch ein Neutron emittiert wird, derart, daß die Summe der Energien von Neutron
und Elektron konstant ist.

Nun handelt es sich weiter darum, welche Kräfte auf die Neutronen wirken. Das wahr-
scheinlichste Modell für das Neutron scheint mir aus wellenmechanischen Gründen dieses
zu sein, daß das ruhende Neutron ein magnetischer Dipol von einem gewissen Moment μ
ist. Die Experimente verlangen wohl, daß die ionisierende Wirkung eines solchen Neutrons
nicht größer sein kann als die eines γ -Strahls, und dann darf μ wohl nicht größer sein als
 $e \cdot (10^{-13} \text{ cm})$. Ich traue mich vorläufig aber nicht, etwas über diese Idee zu publizieren, und
wende mich erst vertrauensvoll an Euch, liebe Radioaktive, mit der Frage, wie es um den
experimentellen Nachweis eines solchen Neutrons stünde, wenn dieses ein ebensolches oder
etwa 10mal größeres Durchdringungsvermögen besitzen würde wie ein γ -Strahl. ...

Also, liebe Radioaktive, prüfet, und richtet. — Leider kann ich nicht persönlich in Tübingen
erscheinen, da ich infolge eines in der Nacht vom 6. zum 7. Dez. in Zürich stattfindenden
Balles hier unabkömmlich bin. ... Euer untätigster Diener

W. Pauli

¹Heute Pauli'sches Ausschließungsprinzip

²Heute Neutrinos

Topics

1 Base fermions and scalars in SO10

neutrinos are unlike charged fermions - Ettore Majorana

2 Neutrino 'mass from mixing' in vacuo and matter

3 Some perspectives

neutrinos oscillate like neutral Kaons (yes, but how ?) - Bruno Pontecorvo

Key questions → why 3 ? why SO10 ?

Charged fermions are not like neutrinos [3]

We shall consider - 'pour fixer les idées' - 3 fermion families in the (left-) chiral basis,
forming a substrate for the local gauge group

$$SL(2,C) \text{ [or } SO(1,3)] \times SO(10)$$

$$\left(\begin{array}{ccc|cc} u^1 & u^2 & u^3 & \nu & N \\ d^1 & d^2 & d^3 & l^- & l^+ \end{array} \right) \begin{array}{c} \overset{\gamma}{\bullet} \\ \text{F} \end{array}$$

$$\overset{\gamma}{\bullet} \text{F}$$
$$\overset{\gamma}{\bullet} = 1, 2$$
$$\text{F} = e, \mu, \tau$$

$$\rightarrow \text{Fig. 1}$$

[3] Ettore Majorana, 'Teoria simmetrica dell elettrone e positrone ', Nuovo Cimento 14 (1937) 171.

I shall cite two sentences from the reference cited in [3] :

"Per quanto riguarda gli elettroni e i positroni, da essa (via) si può veramente attendere soltanto un progresso formale ...

Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi."

("As far as electrons and positrons are concerned from this (path) one may expect only a formal progress ... We will see in fact that it is perfectly possible to construct, in the most general manner, a theory of neutral elementary particles without negative states.")

... upon normal ordering [PM] .

But the real content of the paper by E. M. (1937) is in the formulae, exhibiting the 'oscillator decomposition' of spin 1/2 fermions as seen and counted by gravity, 1 by 1 and doubled through the 'external' SO2 symmetry associated with electric charge, cited in reference [4] .

The left chiral notation shall be

$$\begin{aligned} (f_k)_F^{\dot{\gamma}} ; \dot{\gamma} = 1, 2 & : \text{spin projection} \\ F = I, II, III & : \text{family label} \\ k = 1, \dots, 16 & : \text{SO10 label} \end{aligned} \quad (1)$$

Lets call the above extension of the standard model the 'minimal nu-extended SM', cited in references [5], [8], [9],
 in the the general vectorlike situation.

$$\begin{aligned} & \left(\begin{array}{cccc|cccc} \bullet & \bullet & \bullet & \nu & \mathcal{N} & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \ell & \widehat{\ell} & \bullet & \bullet & \bullet \end{array} \right)_{F=e, \mu, \tau}^{\dot{\gamma}} \\ & \downarrow \\ & \left(\begin{array}{cc} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{array} \right)_{F=e, \mu, \tau}^{\dot{\gamma}} \end{aligned} \quad (2)$$

The right-chiral base fields are then associated to 1 for 1

$$\begin{aligned} (f_k^*)_{F\alpha} &= \varepsilon_{\alpha\gamma} \left[(f_k)_F^{\dot{\gamma}} \right]^* \\ (\varepsilon = i\sigma_2)_{\alpha\gamma} &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \end{aligned} \quad (3)$$

The matrix ε is the symplectic ($Sp(1)$) unit, as implicit in Ettore Majorana's original paper cited in ref. [3].

The local gauge theory is based on the gauge (sub-) group

$$SL(2, C) \times SU3_c \times SU2_L \times U1_Y \quad (4)$$

... why ? why 'tilt to the left' ? we sidestep a historical overview here !

1.1 Yukawa interactions and mass terms

The doublet(s) of scalars are related to the 'tilt to the left'.

$$\left(\begin{array}{cc} \nu & \mathcal{N} \\ \ell & \widehat{\ell} \end{array} \right)_F \leftrightarrow \left(\begin{array}{cc} \varphi^0 & \Phi^+ \\ \varphi^- & \Phi^0 \end{array} \right) = z \quad (5)$$

The green entries in eq. (4) denote singlets under $SU2_L$.

The quantity z is associated with the quaternionic or octonionic structure inherent to the $(2, 2)$ representation of $SU_{2L} \otimes SU_{2R}$ (beyond the electroweak gauge group) as described in ref. [6].

The Yukawa couplings are of the form (notwithstanding the quaternionic or octonionic structure of scalar doublets)

$$\mathcal{H}_Y = \left[(\varphi^0)^*, (\varphi^-)^* \right] \lambda_{F'F} \times \left\{ \varepsilon_{\dot{\gamma}\dot{\delta}} \mathcal{N}_{F'}^{\dot{\delta}} \begin{bmatrix} \nu^{\dot{\gamma}} \\ \ell^{\dot{\gamma}} \end{bmatrix}_F \right\} + h.c. \quad (6)$$

$$\mathcal{N}_{\dot{\gamma}F'} = \varepsilon_{\dot{\gamma}\dot{\delta}} \mathcal{N}_{F'}^{\dot{\delta}} ; \quad \varepsilon_{\dot{\gamma}\dot{\delta}} = \overline{\varepsilon_{\dot{\gamma}\dot{\delta}}} = \varepsilon_{\gamma\delta}$$

The only allowed Yukawa couplings by $SU_{2L} \otimes U_{1Y}$ invariance are those in eq. (6), with arbitrary complex couplings $\lambda_{F'F}$.

Spontaneous breaking of $SU_{2L} \otimes U_{1Y}$ through the vacuum expected value(s)

$$\begin{aligned} \langle \Omega | \begin{pmatrix} \varphi^0 & \Phi^+ \\ \varphi^- & \Phi^0 \end{pmatrix} (x) | \Omega \rangle &= \\ = \langle z(x) \rangle &= \begin{pmatrix} v_{ch} (v_{ch}^u) & 0 \\ 0 & v_{ch} (v_{ch}^d) \end{pmatrix} \\ v_{ch} &= \frac{1}{\sqrt{2}} \left(\sqrt{2} G_F \right)^{-1/2} = 174.1 \text{ GeV} \end{aligned} \quad (7)$$

independent of the space-time point x . The implied parallelizable nature of $\langle z(x) \rangle$ is by far not trivial and relates in a wider context including triplet scalar representations to potential (nonabelian) monopoles and dyons. (no h.o.),

induces a neutrino mass term through the Yukawa couplings $\lambda_{F'F}$ in eq. (6)

$$\begin{aligned} {}_{F'} \mathcal{N} \nu_F &= \mathcal{N}_{\dot{\gamma}F'} \nu_F^{\dot{\gamma}} = \nu_{\dot{\gamma}F} \mathcal{N}_{F'}^{\dot{\gamma}} \\ \mu_{F'F} &= v_{ch} \lambda_{F'F} \\ \rightarrow \mathcal{H}_\mu &= {}_{F'} \mathcal{N} \mu_{F'F} \nu_F + h.c. = \nu^T \mu^T \mathcal{N} + h.c. \end{aligned} \quad (8)$$

The matrix μ defined in eq. (8) is an arbitrary complex 3×3 matrix, analogous to the similarly induced mass matrices of charged leptons and quarks. In the setting of primary SO10 breakdown, a general (not symmetric) Yukawa coupling $\lambda_{F'F}$ implies the existence in the scalar sector of at least two irreducible representations $(16) \oplus (120)$ key question $\rightarrow$ a 'drift' towards unnatural complexity? It becomes even worse including the heavy neutrino mass terms: 256 (complex) scalars.

2 a) 'Mass from mixing' in vacuo
 or 'Seesaw' of type 1

neutrinos oscillate like neutral Kaons (yes, but how?) - Bruno Pontecorvo

We will come back to the clearly original idea in 1957 of Bruno Pontecorvo, cited in reference [7].
 but let me first complete the 'flow of thought' embedding neutrino masses in SO10.

The special feature, pertinent to (electrically neutral) neutrinos is, that the ν -extending degrees of freedom $\mathcal{N}$ are singlets under the whole SM gauge group $G_{SM} = SU_{3c} \otimes SU_{2L} \otimes U_{1Y}$, in fact

remain singlets under the larger gauge group $SU5 \supset G_{SM}$. This allows an arbitrary (Majorana-) mass term, involving the bilinears formed from two N -s.

In the present setup (minimal ν -extended SM), as cited in ref. [5], the full neutrino mass term is thus of the form

$$\mathcal{H}_M = \frac{1}{2} [\nu N] \mathcal{M} \begin{bmatrix} \nu \\ N \end{bmatrix} + h.c. \quad (9)$$

$$\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix}; \quad \mathcal{M} = \mathcal{M}^T \rightarrow M = M^T$$

Again within primary SO10 breakdown the full $\mathcal{M}$ extends the scalar sector to the representations $(16) \oplus (120) \oplus (126)$

It is from here where the discussion – to the best of my knowledge – of the origin and magnitude of the light neutrino masses (re-) started in 1974 as documented below.

For 'our world, tilted to the left' see ref. [10].

Correct derivations were subsequently documented in references [11], [12], [13], [14].

We resume the discussion of the mass term in eq. (9).

Especially the 0 entry needs explanation. It is an exclusive property of the minimal ν -extension assumed here.

Since the 'active' flavors ν_F all carry $I_{3w} = \frac{1}{2}$ terms of the form

$$\frac{1}{2} \nu_F \chi_{F'F} \nu_F = \frac{1}{2} \nu^T \chi \nu; \quad \chi = \chi^T \quad (10)$$

cannot arise as Lagrangean masses, except induced by an I_w -triplet of scalars, developing a vacuum expected value, independent from the doublet(s), key questions $\rightarrow$ quo vadis? is this a valid explanation of the 'tilt to the left'? no, at least insufficient!

From the Zauberlehrling by Goethe, cited in reference [15].

2 Neutrino oscillations - historical overview

The idea that light neutrinos have mass and oscillate goes back to Bruno Pontecorvo, but starting with (para-) muonium - antimuonium oscillations [7]

- like $K^0 \leftrightarrow \bar{K}^0$, [17], introducing τ .

Assuming CP conservation there are two equal mixtures of $\mu^- e^+$ and $\mu^+ e^-$ with opposite CP values $\pm$ (at rest and using a semiclassical description of quantum states)

$$|(e\mu)_{\pm}; \tau=0\rangle = \frac{1}{\sqrt{2}} (|e^-\mu^+\rangle \mp |e^+\mu^-\rangle)_{\tau=0} \quad (11)$$

$$|(e^+\mu^-)\rangle = \widehat{C} |e^-\mu^+\rangle$$

For the leptonium case the rest system is a good approximation.

The evolution of the $CP \pm$ states is then characterized by

$$\begin{aligned}\widehat{m}_\alpha &= m_\alpha - \frac{i}{2} \Gamma_\alpha ; \quad \alpha = \pm \text{ with} \\ | (e^- \mu^+) \rangle &= |1\rangle \rightarrow_\tau \\ \frac{1}{\sqrt{2}} \left(| + ; \tau = 0 \rangle e^{-i \widehat{m}_+ \tau} + | - ; \tau = 0 \rangle e^{-i \widehat{m}_- \tau} \right) \\ | (e^+ \mu^-) \rangle &= |2\rangle \rightarrow_\tau \\ \frac{1}{\sqrt{2}} \left(- | + ; \tau = 0 \rangle e^{-i \widehat{m}_+ \tau} + | - ; \tau = 0 \rangle e^{-i \widehat{m}_- \tau} \right)\end{aligned}\quad (12)$$

This reconstructs to

$$\begin{aligned}&\rightarrow \\ |1\rangle &\rightarrow_\tau E_+(\tau) |1\rangle - E_-(\tau) |2\rangle \\ |2\rangle &\rightarrow_\tau -E_-(\tau) |1\rangle + E_+(\tau) |2\rangle \\ E_\pm(\tau) &= \frac{1}{2} \left(e^{-i \widehat{m}_+ \tau} \pm e^{-i \widehat{m}_- \tau} \right)\end{aligned}\quad (13)$$

and leads to the transition relative probabilities indeed identical to the $K^0 \rightarrow |1\rangle ; \bar{K}^0 \rightarrow |2\rangle$ system.

$$\begin{aligned}d p_{1 \leftarrow 1} &= d p_{2 \leftarrow 2} = |E_+(\tau)|^2 d\tau \\ d p_{2 \leftarrow 1} &= d p_{1 \leftarrow 2} = |E_-(\tau)|^2 d\tau \\ |E_\pm(\tau)|^2 &= \\ &= \frac{1}{2} e^{-\frac{1}{2}(\Gamma_+ + \Gamma_-)\tau} \left(\cosh \frac{1}{2} \Delta \Gamma \tau \pm \cos \Delta m \tau \right) \rightarrow\end{aligned}\quad (14)$$

The term $\cos \Delta m \tau$ in eq. (14) indeed signals $(\widehat{e\mu}) \leftrightarrow (\widehat{\bar{e}\mu})$ oscillations, with

$$\Delta m = m_+ - m_- ; \quad \Delta \Gamma = \Gamma_+ - \Gamma_-$$

$$\Delta m = O \left[\left(\frac{\alpha m_e m_{\nu_e \nu_\mu}}{v^2} \right)^2 m_\mu \right] \sim 4 \cdot 10^{-41} \text{ MeV} \quad (15)$$

$$\tau_{osc} = (2\pi) / \Delta m \sim 10^{20} \text{ sec} = 3.3 \cdot 10^{12} \text{ y}$$

"Erstens kommt es anders, zweitens als man denkt." Not only this is clearly unobservable, but eq. (14) ignores CP violation ; (no h.o.) . Details of neutrino mass and mixing, which induces Δm in eqs. (14-15) . "First it happens differently, second as one thinks."

From mesonium to neutrino's [16] .

What is to be remembered from reference [18] is the idea of neutrino oscillations, expressed in the *corrected* sentence :

"The effects due to *neutrino flavor transformations* may not be observable in the laboratory, owing to the large R, but they will take place on an astronomical scale."

The $(V - A) \times (V - A)$ form of the Fermi interaction which subsequently clarified the structure of neutrino emission and absorption, was documented almost contemporaneously [18] .

$\Delta m \tau$ from rest system to beam system In notes to Jack Steinberger, lectures on "Elementary particle physics", ETHZ, Zurich WS 1966/67, not documented (again zigzag in time) .

There is time dilatation from rest system to beam system, and also we express time in the beam system by distance ($c = 1$)

$$\tau \rightarrow \frac{d}{\gamma \beta} ; \gamma^{-1} = \sqrt{1 - v^2} ; \beta = v \{ \text{key question} \rightarrow \text{which } v? \} \rightarrow \quad (16)$$

Then we replace Δm , for 12 beam oscillations

$$\Delta m = \frac{\Delta m^2}{2 \langle m \rangle} ; \quad \langle m \rangle = \frac{1}{2} (m_1 + m_2) \quad (17)$$

Thus we obtain, for any 12 oscillation phenomenon

$$\Delta m \tau = \frac{\Delta m^2}{2 \langle m \rangle \beta \gamma} d ; \quad \langle m \rangle \beta \gamma = \langle p \rangle \quad (18)$$

It is apparently clear that $\langle m \rangle \beta \gamma = \langle p \rangle$ represents the average beam momentum, yet this is not really so. Lets postpone the questions (which $\langle p \rangle$? - which d ?) . From eq. (18) it follows

$$\Delta m \tau = \frac{\Delta m^2}{2 \langle p \rangle} d \rightarrow \cos \left(\frac{\Delta m^2}{2 \langle p \rangle} d \right) \quad (19)$$

The oscillation amplitude in vacuo (eq. 19) is well known, yet it contains 'subtleties' .

$$\Delta m^2 d / (2 \langle p \rangle) : \text{what means what ?}$$

The *semiclassical* intuition from beam dynamics and optical interference is obvious. A well collimated and within $\Delta | \vec{p} | / | \langle \vec{p} \rangle |$ – or any similar definition of beam momentum spread – 'monochromatic' beam is considered as a classical line, lets say along the positive z-axis, defining the mean direction from a definite production point ($\vec{x} = 0$) towards a detector, at distance d . But the associated operators for a single beam quantum

$$\widehat{p}_z, \widehat{z} \rightarrow \Delta \widehat{p}_z \Delta \widehat{z} \geq \frac{1}{2} \quad (20)$$

are subject to the uncertainty principle (using units $\hbar = 1$) . The same is true for energy and time. Yet we are dealing in oscillations – with single quantum interference – and thus the spread from one beam quantum to the next is only yielding a 'good guess' of the actual expectation values, e.g. appearing in eq. (20) . The quantity $\langle p \rangle$ in the expression for the phase

$$\Delta m^2 d / (2 \langle p \rangle) \quad (21)$$

essentially presupposes the single quantum production wave function, e.g. in 3 momentum space in a given fixed frame, propagating from a production time t_P to a specific detection space-time point x_D and characterized accounting for all quantum mechanical uncertainties by the distance d . In this framework $\langle p \rangle$ stands for the so evaluated single quantum expectation value This was the content of my notes to the lectures of Jack Steinberger in 1966: in reference [19] h.o.

This was implicit in (e.g.) references [8] and [9] and became obvious in discussing matter effects, specifically for neutrino oscillations (e.g. in the sun) .

Coherence and decoherence in neutrino oscillations (h.o.)

The ensuing is an *incomplete* attempt of a historical overview, going zigzag in time, starting with reference [20] in itself a h.o. .

Just mention is due to two papers : Shalom Eliezer and Arthur Swift, cited in reference [21], submitted 28. July 1975, and Samoil Bilenky and Bruno Pontecorvo, in reference [22], submitted 1. January 1976 , where the phase argument $\Delta m^2 d / (2 \langle p \rangle)$ appears correctly.

Also in 1976 a contribution by Shmuel Nussinov in reference [23], appeared .

Matter effects - MSW for neutrinos Two papers, by Mikheyev and Smirnov and by Wolfenstein are to be singled out : cited in reference [25] and [24] .

The general remark hereto is

Every conceivable coherent or incoherent phenomenon involving photons, is bound to happen (and more) with neutrinos: refraction, double refraction, Čerenkov radiation, $\dots$, see reference [26] .

The forward scattering amplitude and refractive index relation is (a semiclassical one)
 $\rightarrow$ plane wave distortion in the z-direction

$$\begin{aligned} f_0 &\equiv f_{forward}^{lab} = (8\pi m_{target})^{-1} T_{forward} \\ Im f_0 &= (k_{lab} / (4\pi)) \sigma_{tot} ; k_{lab} \rightarrow k \\ e^{ikz} &\rightarrow e^{in kz} = e^{ikz} e^{i(2\pi/k) \varrho_N f_0 z} \\ &= e^{ikz} e^{i[(2\pi/k^2) \varrho_N f_0] kz} \end{aligned} \quad (22)$$

$$n = 1 + (2\pi/k^2) \varrho_N f_0 ; \langle v \rangle_{mat.} \sim 1 / (Re n) \stackrel{<}{>} ? 1$$

ϱ_N = mean number density of (target-) matter

key question $\rightarrow$ which is the fully quantum mechanical description ?

T = invariantly normalized (elastic-) scattering amplitude

for neutrinos indeed, tribute – to many who 'really did it' – and to a pioneer of solar physics and beyond (no h.o.) , cited in reference [27] . at low energy :

$$\mathcal{H}_\nu \sim 2\sqrt{2} G_F \left(\begin{aligned} &\bar{\nu}_\alpha \gamma_L^\mu \nu_\beta \bar{\ell}_\beta \gamma_{\mu L} \ell_\alpha \\ &+ \bar{\nu}_\alpha \gamma_L^\mu \nu_\alpha j_{\mu n}(\ell, q) \varrho \\ &+ \frac{1}{4} \bar{\nu}_\alpha \gamma_L^\mu \nu_\alpha \bar{\nu}_\beta \gamma_L^\mu \nu_\beta \varrho \end{aligned} \right) \quad (23)$$

$\alpha, \beta = I, II, III$ for family ; ϱ : e.w. neutral current parameter

The second and third of the latter induces – tiny – matter distortions on relic neutrinos , maybe it is worth while to work them out ? terms on the r.h.s. of eq. (23) – in matter consisting of hadrons and electrons – do not distinguish

between neutrino flavors. So the relative distortion of ν_e –
 by electrons at rest – is

$$\begin{aligned} \Delta_e \mathcal{H}_\nu &\rightarrow \sqrt{2} G_F \nu_e^{\beta*} \nu_e^{\beta} \langle e^* e \rangle_e \\ &= \left(\sqrt{2} G_F \varrho_{n_e} \right) \nu_e^{\beta*} \nu_e^{\beta} \end{aligned} \quad (24)$$

$$K_e = \sqrt{2} G_F \varrho_{n_e} ; (K_e \rightarrow -K_e \text{ for } e^- \rightarrow e^+)$$

The spinor field equation in the above semiclassical approximation, in (chiral) basis becomes – suppressing all indices – and allowing for an $\vec{x}$ dependent electron density

$$\begin{aligned}
 (i \partial_t - \kappa) \nu &= \mathcal{M}^\dagger \bar{\nu} \\
 (i \partial_t + \kappa) \bar{\nu} &= \mathcal{M} \nu \\
 \kappa &= K_e P_e - \frac{1}{i} \vec{\sigma} \vec{\nabla} \mathbb{I} ; \nu = \nu_f^{\beta} ; \bar{\nu} = \varepsilon_{\alpha\beta} \left(\nu_f^{\beta} \right)^* \\
 f &= 1, \dots, 6 ; P_e = \delta_{ef} \delta_{ef'} ; \mathbb{I} = \delta_{ff'} ; \mathcal{M} = \mathcal{M}_{ff'} \\
 (\nu, \bar{\nu}) (t, \vec{x}) &: \text{fields} ; * : \text{hermitian conjugation} \\
 K_e &= K_e(\vec{x}) = \sqrt{2} G_F \varrho_{n_e}(\vec{x})
 \end{aligned} \tag{25}$$

Here we substitute fields by wave functions From eq. (26) we obtain the 'squared' form

$$\begin{aligned}
 (E^2 - \kappa^2 - \mathcal{M}^\dagger \mathcal{M}) \nu &= [\kappa, \mathcal{M}^\dagger] \bar{\nu} \\
 (E^2 - \kappa^2 - \mathcal{M} \mathcal{M}^\dagger) \bar{\nu} &= [\mathcal{M}, \kappa] \nu \\
 [\mathcal{M}, \kappa] &= K_e [\mathcal{M}, P_e] ; [\kappa, \mathcal{M}^\dagger] = [\mathcal{M}, \kappa]^\dagger \\
 \text{The purple quantities in eq. (27) give (e.g. in the sun) negligible effects for the light flavors} &\rightarrow 0. \\
 \kappa^2 &= -\Delta - 2 P_e K_e \frac{1}{i} \vec{\sigma} \vec{\nabla} - P_e \vec{\sigma} \left(\frac{1}{i} \vec{\nabla} K_e \right) + P_e K_e^2
 \end{aligned} \tag{26}$$

In eqs. (26-27) the mainly neutrinos have negative helicity, which can be specified precisely if the purple quantities are ignored, whereas the mainly antineutrinos carry positive helicity, in the ultrarelativistic limit $p = |\vec{p}| \gg |m|$.

$$\kappa^2 \rightarrow p^2 \pm 2 p K_e P_e : \begin{cases} + \text{ for neutrinos} \\ - \text{ for antineutrinos} \end{cases} \tag{27}$$

The mass diagonalization yields correspondingly for the mixing in vacuo approximatively

$$\mathcal{M}^\dagger \mathcal{M} \rightarrow \bar{u} m_{diag}^2 \bar{u}^{-1} ; \mathcal{M} \mathcal{M}^\dagger \rightarrow u m_{diag}^2 u^{-1} \tag{28}$$

In eq. (29) the red quantities refer to the three light flavors So for real (i.e. orthogonal) u , neutrinos distorted by electrons react identically to antineutrinos relative to positrons.

From Hans Bethe, cited in reference [27], with apologies to Mikheyev and Smirnov and many.

→ Further references to the $\odot$ LMA solution can be found in a paper by Alexei Smirnov in reference [28], as well as many references cited therein, (no h.o.).

$$\Delta m_{\odot}^2 = 7.9 \cdot 10^{-5} \text{ ev}^2 ; \tan^2 \vartheta_{\odot} = 0.40 ; \vartheta \sim 32.3^\circ \tag{29}$$

See reference [29].

$$\Delta m_{\odot}^2 = \left(7.9_{-0.6}^{+0.5} \right) \cdot 10^{-5} \text{ ev}^2 ; \tan^2 \vartheta_{\odot} = 0.40_{-0.07}^{+0.09} \text{ Fig.} \rightarrow \tag{30}$$

Derivations are discussed in references [29] and [31].

FIG. 1. The masses of two flavors of neutrinos as a function of density. The curves nearly cross at one point. The electron-antineutrino mass $\bar{\nu}_e$ is also shown.

Figure 1. The 90%, 95%, 99% and 99.73% C.L. allowed regions in the Δm^2_{21} - $\sin^2 \theta_{12}$ plane, obtained in a 3- ν oscillation analysis of the solar neutrino, KL and CHOOZ data [15].

For further experimental references to the $\odot$ LMA solution see reference [28] , see also many references cited therein, (no h.o.) .

$$\begin{aligned} \Delta m^2_{\odot} &= \left(7.9^{+0.5}_{-0.6}\right) \cdot 10^{-5} \text{ eV}^2 \ ; \ \tan^2 \vartheta_{\odot} = 0.40^{+0.09}_{-0.07} \\ \left|\Delta m^2_{23}\right| &= \left(2.1^{+2.1}_{-0.8}\right) \cdot 10^{-3} \text{ eV}^2 \ ; \ \sin^2 2 \vartheta_{23} = 1.0_{-0.15} \\ \sin^2 \vartheta_{13} &\leq 0.05 \quad \text{at } 99.73 \% \text{ C.L.} \end{aligned} \tag{32}$$

This shall conclude my – necessarily partial – historical overview, from the time of the year 2005, cited in reference [1] . What follows must be cut short as well .

reaction : $\bar{\nu}_{e\odot} + p \rightarrow e^- + n$ from ~ 20 reactors $\rightarrow$

$$\begin{aligned} \Delta m^2_{\odot} &= \left(7.9^{+0.5}_{-0.6}\right) \cdot 10^{-5} \text{ eV}^2 \ ; \ \tan^2 \vartheta_{\odot} = 0.40^{+0.09}_{-0.07} \\ \left|\Delta m^2_{23}\right| &= \left(2.1^{+2.1}_{-0.8}\right) \cdot 10^{-3} \text{ eV}^2 \ ; \ \sin^2 2 \vartheta_{23} = 1.0_{-0.15} \\ \sin^2 \vartheta_{13} &\leq 0.05 \quad \text{at } 99.73 \% \text{ C.L.} \end{aligned} \tag{33}$$

2.1 Mass from mixing $\rightarrow$ the subtle things

key questions $\rightarrow$ which is the scale of M ? $O(10^{10})$ GeV $\rightarrow$ is there any evidence for this scale today ? hardly ! $\rightarrow$ and what about susy ?

How is the mass matrix of the form in eq. (9) diagonalized exactly ?

$$\begin{aligned}
 \mathcal{M} &= \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} = U \mathcal{M}_{diag} U^T = K \mathcal{M}_{bl.diag} K^T \\
 \mathcal{M}_{diag} &= \mathcal{M}_{diag} (m_1, m_2, m_3; M_1, M_2, M_3) \\
 \mathcal{M}_{bl.diag} &= U_0 \mathcal{M}_{diag} U_0^T; U_0 = \begin{pmatrix} u_0 & 0 \\ 0 & v_0 \end{pmatrix} \\
 U &= K U_0 = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}
 \end{aligned}
 \tag{34}$$

The *at least* perturbatively appropriate notion of mass in the assumed fundamental presence of gravity, has to be selfconsistently justified. It can then only have perturbative validity through a limited range of energies of multiparticle lepton scattering states.

Leptonic heavy gauge bosons, necessarily shall thus have masses low enough, to allow their decay amplitudes, i.e. their widths, to be calculated as if gravity effects are negligible, i.e. whence Poincaré invariance, again approximately, holds.

We shall come back to this point in (a) subsequent section(s), leading to the formulation of the trace anomaly in ref. [41], whereby a seminal paper by Mikhail Shifman, Arkady Vainshtein and Valentin Zakharov showed me the way, in ref. [43], after quite some time of derivation and checking.

$$U = K U_0 ; K^{-1} M K^{-1 T} = M_{bl.diag.} = \begin{pmatrix} \mathcal{M}_1 & 0 \\ 0 & \mathcal{M}_2 \end{pmatrix}$$

$$U = \begin{pmatrix} (1 + t t^\dagger)^{-1/2} u_0 & (1 + t t^\dagger)^{-1/2} t v_0 \\ -t^\dagger (1 + t t^\dagger)^{-1/2} u_0 & (1 + t t^\dagger)^{-1/2} v_0 \end{pmatrix} \quad (35)$$

↓ ... →

$$\mathcal{M}_1 = -t \mathcal{M}_2 t^T ; \begin{bmatrix} \mathcal{M}_1 = u_0 m_{diag} u_0^T : \text{light } 3 \\ \mathcal{M}_2 = v_0 M_{diag} v_0^T : \text{heavy } 3 \end{bmatrix}$$

$$\mathcal{M}_1 = -t \mathcal{M}_2 t^T ; \begin{bmatrix} \mathcal{M}_1 = u_0 m_{diag} u_0^T : \text{light } 3 \\ \mathcal{M}_2 = v_0 M_{diag} v_0^T : \text{heavy } 3 \end{bmatrix} \quad (36)$$

In eq. (36) all matrices are 3×3 , t describes light - heavy mixing generating mass from mixing . This is documented in ref. [44] Of course 6 by 6 matrices have been diagonalised before .

u_0 (unitary) accounts for light-light mixing and v_0 (unitary) for heavy-heavy (re)mixing .
 it is 'driven' by μ (in $M \leftarrow$) and determined →

from the quadratic equation

$$t = \mu^T M^{-1} - t \mu \bar{t} M^{-1} ; \text{ to be solved by iteration } \rightarrow$$

$$t_{n+1} = \mu^T M^{-1} - t_n \mu \bar{t}_n M^{-1} ; t_0 = 0$$

$$t_1 = \mu^T M^{-1} , t_2 = t_1 - \mu^T M^{-1} \mu \mu^\dagger \bar{M}^{-1} M^{-1}$$

$$\dots ; \lim_{n \rightarrow \infty} t_n = t \quad (37)$$

This is essentially different from the mixing of *identical* $SU(2)_L \times U(1)_Y$ representations, i.e. charged base fermions. → the PMNS-matrix, cited in [40] ; the authors assumed (in 1962) only light-light mixing .

The subtleties inherent to the (iterative) solution of eq. 37 have recently been correctly tied to the breaking of supersymmetry on a ll scales in ref. [45] . →

Finally lets turn to the mixing matrix u_{11} (eqs. 34-35)

$$u_{11} = (1 + t t^\dagger)^{-1/2} u_0 \sim u_0 - \frac{1}{2} t t^\dagger u_0$$

$$t t^\dagger = O \left(\bar{m} / \bar{M} \right) \sim 10 \text{ meV} / 10^{10} \text{ GeV} = 10^{-21} \quad (38)$$

The estimate in eq. (38) is very uncertain and assumes among other things $m_1 \sim 1 \text{ meV}$, nevertheless it follows, for by now more than ten years, on the same grounds as the smallness of light neutrino masses, that the deviation of u_{11} from u_0 is tiny .

"Much ado about nothing" , William Shakespeare ; in ref. [38] .

For details in the derivation of the approximations displayed in eq. (37) I refer to ref. [46] .
 I should like - in commemoration of my former collaborator Bernhard Scheuner - to show a photograph of Heimenschwand, BE, the community where he was born in the southern Emmental .

3 Mass and Mixing of 3 + 3 light and heavy neutrino flavors

The details of the complex symmetric 6 by 6 matrix $\mathcal{M}$ was displayed in eqs. 9 and 35 - 37 .
 In the framework of three families of minimal neutrino mass extended fermions, within a unified or enveloping charge like gauge group , cited in references [41] , [42] .

$$\mathcal{G}_{env} = \text{SO10} \quad (39)$$

It may be useful, to define the mass in a renormalization group covariant way, in which case it does not describe the physical mass of neutrino's, neither light nor heavy ones. Then it is necessary to use a loop expansion in the SO10 associated couplings, a procedure denoted loop expansion of physical neutrino mass.

For a recent discussion of these issues let me refer to reference [47] . → It must here be 'remembered' that the notion of physical neutrino mass in a unifying theory *including* gravity is not unconditionally defined, except after establishing conditions of (here) leptonic scattering amplitudes, for which local Poincaré covariance is a good approximation. It can not be an exact one, as follows from the spontaneous scale breaking induced by the trace anomaly, as exposed in references [41] and here at ICNFP2014, as cited in reference [42] . → Given the premises of simultaneous

- a) charge like gauging as in SO10
  - b) gauging of orientation as in gravity,
but in $d + 1$ space time dimensions with $\infty > d \geq 3$
  - c) a finite number of defining constants, as required for
substrate fields, compatible with strict locality,
implying causality
- it turns out that the conditions a) , b) ,

c) can not be fulfilled This statement is at present a conjecture, in the process of being checked . .
 → Two consequences shall be singled out below

1 Chargelike gauge fields have to be generated as composite ones from structures in space time.

2 Any realization of supersymmetry, global and/or local, containing a super Yang Mills structure in accord with our SO10 unification hypothesis, is broken on all scales as a consequence of the trace anomaly.

→
In reference [48], Keith Olive considers the unification of the 3 charge like coupling constants, with the field content underlying SO10 unification, but without supersymmetry. First let me quote his introductory remarks "1. Introduction

For well over 30 years, supersymmetry and the minimal supersymmetric standard model (MSSM) have been prime examples of extensions beyond the standard model (SM) of strong and electroweak interactions. Apart from its algebraic beauty, there are many motivations for supersymmetry. These include the theory’s ability to provide gauge coupling unification and address the gauge hierarchy problem. → It is well known that the additional fields predicted in the MSSM, if present at low energy, alter the running of the gauge couplings as shown in Fig. 1 (→ Fig. -2). At one-loop, the running of the gauge couplings is given by

$$Q \frac{d}{Q} \alpha_i = \frac{b_i}{2 \pi} \alpha_i^2 \tag{40}$$

where the coefficients of the SM beta functions are $b_i = 41/10, -19/6, -7$ for the $U1_y, SU2_L$, and $SU3_c$ gauge couplings respectively.

The scale evolution is shown in Fig. -2 below →

Fig. -2 : The left panel shows the SO10 evolution of the coupling constants, while the right panel shows the same but with supersymmetric doubling of the 16-representation of fermions modifying the evolution at a scale of energy within ~ 1 TeV of the electroweak scale . →

I further quote from ref. [48] : "It is also well known that the quartic coupling, λ , of the Higgs potential, (denoting the physical Higgs scalar field by h)

$$V = \frac{1}{4} \lambda h^4 + \frac{1}{2} m^2 h^2 \tag{41}$$

runs negative at a scale $Q \sim 10^{10}$ GeV ... is indeed in the the 1 scalar doublet realization of electroweak global symmetry breaking a nontrivial source of estimate of the mass scale, where new physics must appear."

This scale being of the order of 10^{10} GeV is an independent derivation from the standard model $\rightarrow$ as referred to in eq. 41 above and the estimate, derived in eq. 38 repeated below :

$$\begin{aligned} u_{11} &= \left(1 + t t^\dagger\right)^{-1/2} u_0 \sim u_0 - \frac{1}{2} t t^\dagger u_0 \\ t t^\dagger &= O\left(\frac{\overline{m}}{\overline{M}}\right) \sim 10 \text{ meV} / 10^{10} \text{ GeV} = 10^{-21} \end{aligned} \quad (42)$$

In order to get the numerical relation in the last line of eq. 42, normal hierarchy for the three light masses and 1 meV for the mass of the lightest neutrino flavor have to be assumed. $\rightarrow$

3.1 Rare decay processes associated with B - L breaking , showing deviations from oppositely charged lepton pair decays, with assessed significant deviations in their e - mu ratio from the standard model at the LHC

In reference [49], the four authors, Andreas Crivellin, Dario Müller, Adrian Signer and Yannick Ulrich, consider the constituent transitions

$$b \rightarrow s \ell^+ \ell^- ; \ell^- = e^-, \mu^-, (\tau^-) \quad (43)$$

Upon hadronization the directly leptonnumber violating processes $B_s \rightarrow \mu e$; $B \rightarrow K \mu e$ appear, which are within the reach of LHCb and Belle II. $\rightarrow$

I quote from ref. [49] :

"Taking this approach and including the new LHCb result for $R^{(K)} = (B \rightarrow K \mu^+ \mu^-) / (B \rightarrow K e^+ e^-)$, measuring lepton flavour universality (LFU) violation, the global significance for NP increased above the 5σ level [50] . In addition, the combination of the ratios $R(D^{(HW)}) = (B \rightarrow D^{(HW)} \tau \nu) / (B \rightarrow D^{(HW)} \ell \nu)$ also differs by 3.9σ from its SM prediction. All together this strongly motivates to examine LFU violation in semileptonic B decays in the context of NP. Since $b \rightarrow s \ell^+ \ell^-$ processes are semileptonic, leptoquarks (LQ) provide a natural explanation for these anomalies. They give tree-level contributions to these processes but contribute for example to $\Delta F = 2$ processes only at the loop level, therefore respecting the bounds from other flavour observables. $\rightarrow$ Furthermore, since in $R^{(D^*)}$ an O(10%) effect compared to the tree-level SM is needed, also a NP tree-level effect is required. Here LQ are probably even the most promising solution. In fact ... a model for a simultaneous explanation of $b \rightarrow s \ell^+ \ell^-$ data together with $R^{(D^{(HW)})}$ has been proposed which is compatible with the bounds from $B K^{(HW)} - \nu \bar{\nu}$, electroweak precision data and direct LHC searches ... . Interestingly, LQ also provide a natural solution to the anomaly in the magnetic moment of the muon due to the possible enhancement by m_t / m_μ through an internal chirality flipping .... The dots in the text quoted above and below stand for references quoted in ref. [49], that are not essential for our discussion here. The model independent fit to $R(K)$ and $R(K^*)$ allows for NP contributions to electrons or muons separately, but also to both simultaneously ... ."

$\rightarrow$ "Once the other data on $b \rightarrow s \mu^+ \mu^-$ is included, NP in muons is required but is only optional for electrons. However, the best fit value suggests a simultaneous NP contribution to electrons as well .... It is well known that once LQ couple to muons and electrons simultaneously, they give rise to lepton flavour violating B decays and to $\mu \rightarrow e \gamma$ ... ."

$\rightarrow$ "Once the other data on $b \rightarrow s \mu^+ \mu^-$ is included, NP in muons is required but is only optional for electrons. However, the best fit value suggests a simultaneous NP contribution to electrons as well .... It is well known that once LQ couple to muons and electrons simultaneously, they give rise to lepton flavour violating B decays and to $\mu \rightarrow e \gamma$ ... ."

Fig. 1 ($\rightarrow$ Fig. -3)

$\rightarrow$

FIG. 1: Feynman diagram generating $\mu \rightarrow e \gamma$ in models with leptoquarks.

"Both $\mu \rightarrow e \gamma$ and lepton flavour violating B decays with μe final states are experimentally very interesting and precise upper limits for these processes already exist. For $\mu \rightarrow e \gamma$ the current experimental bound, obtained by the MEG collaboration ... , is

$$Br [\mu \rightarrow e \gamma] \leq 4.2 \cdot 10^{-13} \quad (44)$$

and MEG II ... at the Paul Scherrer Institute (PSI) will significantly improve on this bound in the future." Here let me recall my estimate in ref. [10] ($\mu \rightarrow e \gamma$ at a rate of 1 out of 10^9 muon decays ?)

$$Br [\mu \rightarrow e \gamma] \sim 3.9 \cdot 10^{-56} \quad (45)$$

Between the estimates in eqs. 44 and 45 there are 43 orders of magnitude .

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