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Gluons, Heavy and Light Quarks in the QCD Vacuum

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Abstract. We are discussing the properties of the QCD vacuum which might be important especially for the understanding of hadrons with small quark core size $\sim 0.3\,fm$. We assume that at these distances the QCD vacuum can be described by the Instanton Liquid Model (ILM). At larger distances, where confinement is important, ILM should be extended to Dyons Liquid Model (DLM). The ILM has only two free parameters, average instanton size $\rho \approx 0.3\,fm$ and average inter-instanton distance $R \approx 1\,fm$, and can successfully describe the key features of light hadron physics. One of the important conceptual results was prediction of the momentum dependent dynamical quark mass $M \sim (packing\ fraction)^{1/2}\,\rho^{-1} \approx 360\,MeV$, later confirmed numerically by evaluations in the lattice. The estimates show that gluon-instanton interaction strength is also big and is controlled by the value of dynamical gluon mass $M_g \approx M$. Heavy quarks interact with instantons much weaker. The heavy quark-instanton interaction strength is given by $\Delta m_Q \sim packing\ fraction\ \rho^{-1} \approx 70\,MeV$. Nevertheless, the direct instanton contribution to the colorless heavy-heavy quarks potential is sizable and must be taken into account. At small distances, where one-gluon exchange contribution to this potential is dominated, we have to take into account dynamical gluon mass M_g. Also, instantons are generating light-heavy quarks interactions and allow to describe the nonperturbative effects in heavy-light quarks systems.

1 Introduction

QCD instanton is a topologically nontrivial classical solution of Yang-Mills (YM) equations for gauge fields in Euclidean space, which is a tunneling path between Chern-Simons (CS) states [1]. Then, within quantum mechanics QCD vacuum can be considered as the lowest energy quantum state of the one-dimensional crystal along the collective CS coordinate [2].

Without any doubt instantons represent a very important topologically nontrivial component of the QCD vacuum. In ILM the external classical gluon field is given by

$$A_\mu = \sum_I A_\mu^I(\gamma_I), \tag{1}$$

where $A_\mu^I(\gamma_I)$ is a generic notation for the QCD (anti)instanton in the singular gauge, described by its collective coordinates γ_I (the position in Euclid 4D space z_I, the size ρ_I and the $SU(N_c)$ color orientation U_I, $4N_c$ variables altogether). The main parameters of the QCD instanton vacuum (

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Instanton Liquid Model(ILM)) are the average instanton size ρ and inter-instanton distance R (see reviews [3, 4]). Their values were phenomenologically estimated as

$$\rho = 1/3\, fm,\ R = 1\, fm \tag{2}$$

and confirmed by theoretical variational calculations [3, 4] and recent lattice simulations of the QCD vacuum [6].

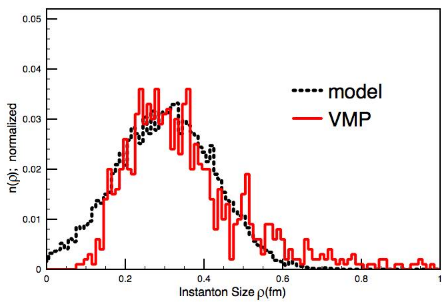

Figure 1. Instanton size distribution function $n(\rho)$ – lattice vs ILM [8].

The instanton size distribution $n(\rho)$ has been studied independently by lattice simulations (see Fig.1). As we can see, for large-size instantons the density $n(\rho)$ is suppressed, which justifies the use of ansatz (1). However, the the large size tail of distribution function $n(\rho)$ becomes important in the confinement regime of QCD. In this regime (as well as for temperature $T > 0$), instead of individual instanton sum (1) we have to replace BRST instantons by KvBLL dyon-instantons [9] described in terms of dyons. So, we get an extension of ILM – Liquid Dyon Model (LDM)[10, 11], which is able to reproduce confinement–deconfinement. Small size instantons can be still described in terms of their collective coordinates. The average size of instantons in LDM is $\bar{\rho} \approx 0.5$ fm [10, 11], while in ILM $\bar{\rho} \approx 0.3$ fm.

The instanton vacuum background (1) leads to nonzero QCD vacuum energy density $\epsilon \approx -500\, MeV/fm^3$ [4] and spontaneous breakdown of chiral symmetry [5] which plays a pivotal and significant role in describing the lightest hadrons and their interactions.

For applications of the ILM to the physics of the heavy quarks, we may notice that the typical sizes of quarkonia in the potential model framework [12] are $r_{J/\psi} \sim 0.25 - 0.47\, fm,\ \ r_\Upsilon = 0.14 - 0.2\, fm$ Similar estimate of nucleon quark core size gives $r_N \sim 0.3 - 0.5$ fm [13]. Since small quark core size hadrons are insensitive to the confinement we may safely apply ILM for their description.

2 Light quarks in ILM

Zero modes are the solutions of the Dirac equation $(\hat{p} + g\hat{A}_\pm)\Phi_{\pm,0}(x, \zeta_\pm) = 0$. Their dominance in the single instanton light quark propagator provide summation of the multi-scattering series for the ILM light quark propagator and leads to the low-frequencies part of the light quarks partition function

[3–5, 14]

$$\begin{aligned}
Z[\xi^+,\xi] &= \int D\zeta\,\mathrm{Det}_{low}(\hat{p}+g\hat{A}+im)\exp\left(-\xi^+(\hat{p}+g\hat{A}+im)^{-1}\xi\right) = \qquad (3)\\
&= \int D\zeta\prod_f D\psi_f D\psi_f^\dagger \exp\int\left(\psi_f^\dagger(\hat{p}\;+\;im_f)\psi_f+\psi_f^\dagger\xi_f+\xi_f^+\psi_f\right)\\
&\times\prod_f\left\{\prod_+^{N_+}V_{+,f}[\psi^\dagger,\psi]\prod_-^{N_-}V_{-,f}[\psi^\dagger,\psi]\right\},\\
V_{\pm,f}[\psi^\dagger,\psi] &= i\int dx\left(\psi_f^\dagger(x)\,\hat{p}\Phi_{\pm,0}(x;\zeta_\pm)\right)\int dy\left(\Phi^\dagger_{\pm,0}(y;\zeta_\pm)(\hat{p}\,\psi_f(y)\right). \qquad (4)
\end{aligned}$$

where $\psi^\dagger,\psi$ correspond to constituent quarks, $\Phi_{\pm,0}$are the zero modes in the single instanton and antiinstanton fields respectively.

Small packing fraction $(\rho/R)^4\approx 0.01$ justifies independent averaging over collective coordinates ζ of each instanton and leads to the non-local t'Hooft-like vertex with $2N_f$-quarks legs

$$\overline{V_\pm[\psi^\dagger,\psi]}=\int d\zeta_\pm\prod_f V_{\pm,f}\left[\psi^\dagger,\psi\right]. \qquad (5)$$

Spontaneous Breaking of the Chiral Symmetry. The calculation of $Z[\xi^+,\xi]$ in the saddle-point approximation (leading order in $1/N_c$) leads to the Spontaneous Breaking of the Chiral Symmetry (SBCS). One of the manifestations of the SBCS is the dynamical quark mass $M(q)$ (see Fig.2).

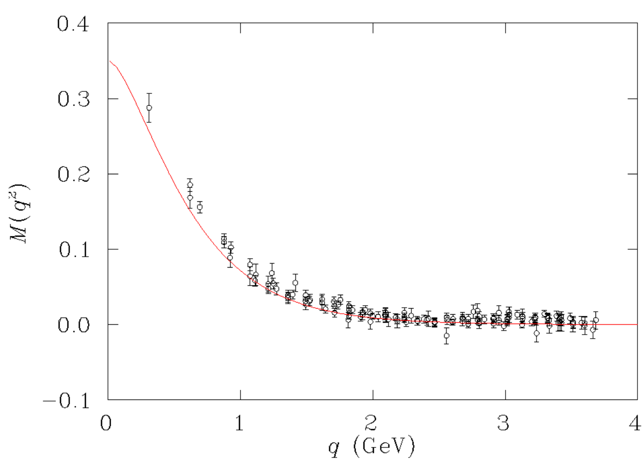

Figure 2. Dynamical light quark mass M(q). ILM at $\rho = 0.33\,fm, R = 1\,fm$ vs lattice [7].

For small momenta $M(0)\approx 360$ MeV and is related to the strength of light quark-instanton interaction. Even in the leading order over $1/N_c$, the ILM successfully reproduces quark condensate, pion and nucleon properties etc (see e.g. [3, 4, 15]). In the next to leading order $1/N_c$ corrections , we have successfully reproduced the Low Energy Constants of ChPT [5].

3 Heavy quarks in ILM

It is known that for the description of heavy quark systems we can develop a systematic $1/m_Q$expansion. In the leading order the heavy quark propagator is given by $w = \int D\zeta(\theta^{-1} - iA_4)^{-1}$, where $< t_2|\theta|t_1 >= \theta(t_2 - t_1)$ and $< T|w|0 >= \int D\zeta\, P\exp(i\int_L dx_4 A_4)$, $L = (\vec{x}, 0, T)$. The problem of the averaging over instantons was solved in [16] using the framework suggested in [17] , which leads to $w^{-1} = \theta^{-1} + \sum_i \int d\zeta_i \left(w - a_i^{-1}\right)^{-1}$.

In view of the low density of the instanton liquid, it makes sense ot develop a systematic expansion over the dimensionless effective parameter–the packing fraction $\rho^4 N/V = \rho^4/R^4 \approx 0.01$. To the lowest order in the instanton density the heavy quark propagator is given by

$$w^{-1} = \theta^{-1} - \frac{N}{2}\mathrm{tr}_c \sum_{\pm} \theta^{-1}(w_\pm - \theta)\theta^{-1} + O(N^2/V^2), \tag{6}$$

where we introduced shorthand notation for the single (anti)instanton heavy quark propagator $w_\pm = \int d\zeta_\pm \left(\theta^{-1} - iA_{\pm,4}\right)^{-1}$. Instanton medium contribution to the heavy quark mass is given by [16, 18]

$$\Delta m_Q = 16\pi i_0(0)(\rho^4/R^4)\rho^{-1}/N_c, \quad i_0(0) = 0.55, \tag{7}$$

where form-factor $i_0(x)$ was defined at [16]. We estimate that the value of Δm_Q should be within $\Delta m_Q \approx 70\,\mathrm{MeV}$ (for the set of parameters (2)) and $\Delta m_Q \approx 140\,\mathrm{MeV}$ (for $\rho = 0.36\,\mathrm{fm}, R = 0.89\,\mathrm{fm}$). Δm_Q is related to the strength of a heavy quark-instanton interaction.

Colorless state heavy quark-antiquark potential in ILM. Static central and spin-dependent parts of the heavy quark-antiquark potential can be obtained from ILM averaged Wilson loop over rectangular contour $\vec{r} \times T$ with $T \to \infty$ [19]. The application of the framework [16, 21] gives the potential

$$V(\vec{r}) = V_C(r) + V_{SS}(r)(\vec{S}_Q \cdot \vec{S}_{\bar{Q}}) + V_{LS}(r)(\vec{L} \cdot \vec{S}) + V_T(r)\left[3(\vec{S}_Q \cdot \vec{n})(\vec{S}_{\bar{Q}} \cdot \vec{n}) - \vec{S}_Q \cdot \vec{S}_{\bar{Q}}\right], \tag{8}$$

where $\vec{L}$ is the angular moemntum of the rlative motion, $\vec{S}_{Q,\bar{Q}}$ are the spins of the quarks, and

$$V_{SS}(r) = \frac{1}{3m_Q^2}\nabla^2 V_C(r), V_{LS}(r) = \frac{1}{2m_Q^2}\frac{1}{r}\frac{dV_C(r)}{dr}, V_T(r) = \frac{1}{3m_Q^2}\left(\frac{1}{r}\frac{dV_C(r)}{dr} - \frac{d^2V_C(r)}{dr^2}\right). \tag{9}$$

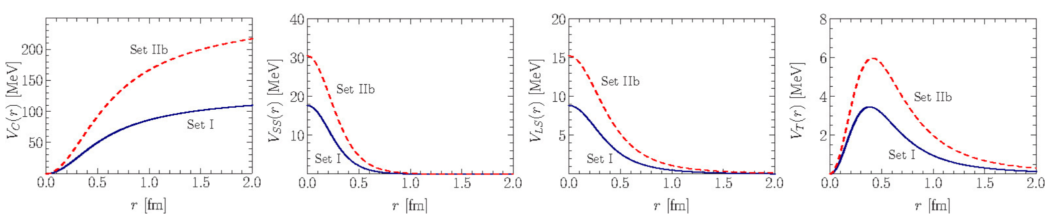

Figure 3. Solid curve ∼ Set I $\rho = 0.33$ fm and $R = 1$ fm [3, 4]. Dashed one ∼ Set II $\rho = 0.36$ fm, $R = 0.89$ fm [5, 6], $m_c = 1275$ MeV.

The ILM contributions to the mass shifts of the charmonium states [21] are given in the Table 1:

We can see that the instanton effects are not small ∼ 30−40 % in comparison with the experimental data and strongly depend on instanton liquid parameters.

Table 1. ILM contribution to the charmonium states. $\Delta M_{c\bar{c}} = M_{c\bar{c}} - 2m_c$ in [MeV].

$\Delta M_{c\bar{c}}(J^P)$	Set I	Set II	Exp.
$\Delta M_{\eta_c}(0^-)$	118,81	203,64	$433,6 \pm 0.6$
$\Delta M_{J/\psi}(1^-)$	119,57	205,36	$546,916 \pm 0.11$
$\Delta M_{\chi_{c0}}(0^+)$	142,43	250,86	$864,75 \pm 0.31$

4 Gluons in ILM

Scalar "gluons" in ILM. There operators $\Delta_I^{-1} = (p + A_I)^2$ and $\Delta^{-1} = (p + \sum_i A_i)^2$ do not have zero mdoes, for this reason their inverse operators are well-defined,

$$\Delta = (p^2 + \sum_i(\{p, A_i\} + A_i^2) + \sum_{i\neq j} A_i A_j)^{-1}, \quad \Delta_i = (p^2 + \{p, A_i\} + A_i^2)^{-1}, \quad \Delta_0 = p^{-2}. \tag{10}$$

where $\tilde{\Delta} = (p^2 + \sum_i(\{p, A_i\} + A_i^2))^{-1}$.

The propagator in ILM is $\bar{\Delta} \equiv < \tilde{\Delta} > = \int D\zeta\, \Delta$. Let's start first with evaluation of $\bar{\tilde{\Delta}}$. The extension of Pobylitsa equation to the present case is

$$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} = \sum_i < \{\bar{\tilde{\Delta}} + (\Delta_i^{-1} - \Delta_0^{-1})^{-1}\}^{-1} > \tag{11}$$

Since effective parameter of instanton density expansion in fact is $\rho^4/R^4 \sim (1/3)^4 = 0.012$, we may neglect the higher orders of the expansion and we have at the first order in the instanton density

$$\bar{\tilde{\Delta}}^{-1} - \Delta_0^{-1} = N\Delta_0^{-1}(\bar{\Delta}_I - \Delta_0)\Delta_0^{-1}, \tag{12}$$

It is obvious that with the same accuracy we have $\bar{\Delta} = \bar{\tilde{\Delta}}$.

Scalar "gluon" dynamical mass in ILM. Applying the well-known result for the Δ_I [22], we found

$$M_s(q) = \left[\frac{3\rho^2}{(N_c^2 - 1)R^4} 4\pi^2\right]^{1/2} q\rho K_1(q\rho), \tag{13}$$

which was also obtained in [23]. For the set of parameters (2) we got an estimate $M_s(0) = 256\, MeV$ and the form-factor $q\rho K_1(q\rho)$ given by Fig.4:

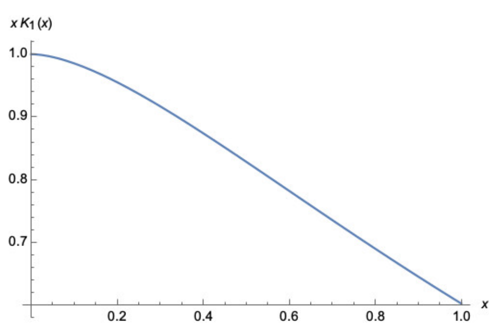

Figure 4. the form-factor $q\rho K_1(q\rho)$

Gluons. Zero-modes problem. The quadratic form of the gluon fluctuations a_μ in the background of a single instanton is $(a_\mu M^I_{\mu\nu} a_\nu)$. The matrix $M^I_{\mu\nu}$ has $4N_c$ zero-modes $M^I_{\mu\nu}\phi^i_\nu = 0$, which correspond to the fluctuations along of the collective coordinates ζ_I. If we define $P^I_{\mu\nu}$ as the zero-modes projection operatorthen, the single instanton gluon propagator $S^{\ I}_{\mu\nu}$ is given by

$$M^I_{\mu\nu} S^{\ I}_{\nu\rho} = \delta_{\mu\nu} - P^I_{\mu\nu} \tag{14}$$

The explicit solution for Eq.(14) was found in [22].

Now, in order to extend Pobylitsa Equation, we introduce artificial gluon mass m and define additional quantities $G^I_{m,\rho\nu}$ and $g^I_{m,\mu\nu}$ defined as [24]

$$\left(M^I_{\mu\rho} + m^2\delta_{\mu\rho}\right) g^I_{m,\rho\nu} = \delta_{\mu\nu} - P^I_{\mu\nu}, \ \left(M^I_{\mu\rho} + m^2\delta_{\mu\rho}\right) G^I_{m,\rho\nu} = \delta_{\mu\nu}, \tag{15}$$

$$\lim_{m\to 0} g^I_{m,\mu\nu} = S^{\ I}_{\mu\nu} \tag{16}$$

where [24] $G^I_{m,\rho\nu} = g^I_{m,\rho\nu} + \frac{1}{m^2} P^I_{\rho\nu}$. It is clear that $G^{I^{-1}}_{m,\mu\rho} = \left(M^I_{\mu\rho} + m^2\delta_{\mu\rho}\right)$.

Dynamical gluon mass in ILM [25]. Let's repeat the way to Pobylitsa Equation for ILM "scalar" gluon propagator $\bar{\Delta}$ and neglect by second and higher orders terms in density expansion. We get

$$\bar{G}_{m,\rho\nu} - G^0_{m,\rho\nu} = N\left(\bar{G}^I_{m,\alpha\nu} - G^0{}_{m,\rho\nu}\right). \tag{17}$$

Finally we get at $m \to 0$ limit the dynamical gluon mass

$$M_g^2 \delta_{\rho\nu} = N S^{0^{-1}}_{\ \rho\sigma} (\bar{S}^{\ I}_{\sigma\mu} - S^{\ 0}_{\ \sigma\mu}) S^{0^{-1}}_{\ \mu\nu} \tag{18}$$

From the well-known result for the $S^{\ I}_{\sigma\mu}$ [22] we conclude

$$M_g^2(q) = 2M_s^2(q). \tag{19}$$

For the set of parameters (2) we estimate that $M_g(0) = 362\, MeV$.

5 Heavy-light quarks interactions in ILM

If heavy and light quarks are interacting with the same instanton, they are effectively interacting with each other. Our aim here is to derive light-heavy quarks interaction term induced by this mechanism. We solve this taking into account light quarks determinant in the measure as $D\zeta \Rightarrow D\zeta\,\mathrm{Det}_{low}(\hat{p} + g\hat{A} + im)$. With account of light quarks ILM heavy quark propagator becomes

$$\int \prod_f D\psi_f D\psi^\dagger_f \exp\int \left(\psi^\dagger_f(\hat{p} \ + \ im_f)\psi_f\right) \prod_{\pm} \left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{N_\pm} < T|w[\psi,\psi^\dagger]|0 >, \tag{20}$$

where

$$w[\psi,\psi^\dagger] = \prod_{\pm}\left(\overline{V_\pm[\psi^\dagger,\psi]}\right)^{-N_\pm} \int D\zeta\,(\theta^{-1} - iA_4)^{-1} \prod_f \prod_{\pm}^{N_\pm} V_{\pm,f}[\psi^\dagger,\psi] \tag{21}$$

The solution of the extended Pobilitca Eq. is [26]

$$w^{-1}[\psi,\psi^\dagger] = \theta^{-1} - \frac{N}{2}\sum_{\pm} \frac{1}{\overline{V_\pm[\psi^\dagger,\psi]}} \Delta_{H,\pm}[\psi^\dagger,\psi] + O(N^2/V^2), \tag{22}$$

$$\Delta_{H,\pm}[\psi^\dagger,\psi] = \int d\zeta_\pm \prod_f V_{\pm,f}[\psi^\dagger,\psi]\theta^{-1}(w_\pm - \theta)\theta^{-1}. \tag{23}$$

and it defines the heavy (Q)-light quarks(ψ) interaction term

$$S_{Q\psi} = -\lambda \sum_{\pm} Q^{\dagger}\Delta_{H,\pm}[\psi^{\dagger}, \psi]Q, \tag{24}$$

where the coupling λ is taken from the saddle-point approximation in light quarks partition function $Z[\xi, \xi^{\dagger}]$ [5, 14].

Heavy–light quarks interactions ($N_f = 1$) **[26].** In this case the heavy-light quarks interaction term is given by

$$S_{Q\psi} = i \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^3q}{(2\pi)^3}(2\pi)^4\delta^3(\vec{k}_2 + \vec{k}_1 - \vec{q})\delta(k_{2,4} - k_{1,4})(M(k_1)M(k_2))^{1/2}\Delta m_Q R^4 \tag{25}$$

$$\frac{i_0(q\rho)}{i_0(0)}\left[\frac{2N_c^2 - N_c}{2N_c^2 - 2}\psi^+(k_1)\psi(k_2)Q^+Q + \frac{N_c^2 - 2N_c}{2N_c^2 - 2}(\psi^+(k_1)QQ^+\psi(k_2) + \psi^+(k_1)\gamma_5 QQ^+\gamma_5\psi(k_2))\right]$$

The first term is the heavy quark–light meson interaction term, while the second and the third terms correspond to Qq mesons degenerated on parity. Similar expression was derived in [18].

Light quarks contribution to the $Q\bar{Q}$ potential ($N_f = 1$) **[26].** With account of the light quarks averaged previously defined Wilson loop is given by

$$\int \prod_f D\psi_f D\psi_f^{\dagger} \exp \int \left(\psi_f^{\dagger}(\hat{p} + im_f)\psi_f\right) \prod_{\pm} \left(\overline{V_{\pm}[\psi^{\dagger}, \psi]}\right)^{N_{\pm}} \mathrm{tr} < T|W[\psi, \psi^{\dagger}]|0 >, \tag{26}$$

$$< T|W[\psi, \psi^{\dagger}]|0 >= \prod_{\pm} \left(\overline{V_{\pm}[\psi^{\dagger}, \psi]}\right)^{-N_{\pm}} \int D\zeta \prod_f \prod_{\pm}^{N_{\pm}} V_{\pm,f}[\psi^{\dagger}, \psi]$$
$$\times P \exp(i \int_{L_1} dx_4 A_4) P \exp(i \int_{L_2} dx_4 A_4) \tag{27}$$

In the first order on instanton density the solution of extended Pobilitca Eq. is given by

$$W^{-1}[\psi, \psi^{\dagger}] = w_1^{-1}[\psi, \psi^{\dagger}](\times)w_2^{-1,T}[\psi, \psi^{\dagger}] - \frac{N}{2}\sum_{\pm}\left(\overline{V_{\pm}[\psi^{\dagger}, \psi]}\right)^{-1}$$
$$\times \int d\zeta_{\pm} \prod_f V_{\pm,f}[\psi_f^{\dagger}, \psi_f]\left(\theta^{-1}\left(w_{\pm}^{(1)} - \theta\right)\theta^{-1}\right)(\times)\left(\theta^{-1}\left(w_{\pm}^{(2)} - \theta\right)\theta^{-1}\right)^T + O(\frac{N^2}{V^2}) \tag{28}$$

where the superscript T means the transposition and $(\times)$ stands for the tensor product.

It is obvious that first term in (28) describes light quarks exchange between heavy quarks, which leads to the potential $V_{lq}(r/\rho)$ [26], see Fig.5.

$N_f = 2$**. Heavy quark light mesons interaction term [26].** Heavy quark light quarks interaction term (24) has an essential part which is co-product of colorless heavy and light quark factors. From this one and at the saddle points we have the effective action for the mesons and colorless heavy quark $Q^{\dagger}Q$ bilinear as

$$S[\sigma', \vec{\phi}', \eta', \vec{\sigma}', Q^{\dagger}Q] = -\mathrm{Tr}\ln\frac{\hat{p} + i(m + M(p))}{\hat{p} + im} + N/2 + \frac{1}{2}\int d^4x\left(\sigma'^2 + \vec{\phi}'^2 + \vec{\sigma}'^2 + \eta'^2\right)$$
$$-\mathrm{Tr}\ln\left[1 + \frac{1}{\hat{p} + i(m + M(p))}\frac{iM}{\sigma_0}F\left(\sigma' + i\gamma_5\vec{\tau}\vec{\phi}' + i\vec{\tau}\vec{\sigma}' + \gamma_5\eta'\right)F\right] \tag{29}$$
$$+\mathrm{Tr}\frac{1}{\hat{p} + i(m + M(p)) + \frac{iM}{\sigma_0}F\left(\sigma' + i\gamma_5\vec{\tau}\vec{\phi}' + i\vec{\tau}\vec{\sigma}' + \gamma_5\eta'\right)F}$$
$$\times i\left(M(p) + \frac{M}{\sigma_0}F\left(\sigma' + i\gamma_5\vec{\tau}\vec{\phi}' + i\vec{\tau}\vec{\sigma}' + \gamma_5\eta'\right)F\right)\left(\frac{i}{2}\Delta m_Q R^4 \int e^{-ipx}\frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{i_0(p\rho)}{i_0(0)}Q^{\dagger}Q\right).$$

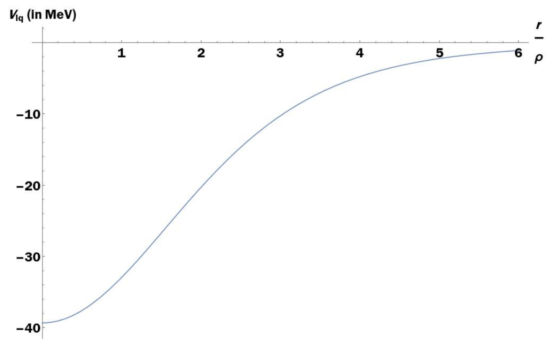

Figure 5. Heavy quark–antiquark potential $V_{lq}(r/\rho)$ (in MeV), generated by light quarks, at Set II $\rho = 0.36$ fm, $R = 0.89$ fm.

The first and the second lines describe mesons and their interactions, while the third and the forth one describe the renormalization of the heavy quark mass and heavy quark-light quark mesons interactions terms. From the Eq. (29) we have the heavy quark-pion interaction term:

$$S_{Q\pi} = i\Delta m_Q R^4 \frac{F^2_{\pi Q}}{4} \int d^4x\, \mathrm{tr}_f \partial_\mu U(x)\partial_\mu U^\dagger(x) \int e^{-ipx} \frac{d^4p_1}{(2\pi)^4}\frac{d^4p_2}{(2\pi)^4}\frac{i_0(p\rho)}{i_0(0)} Q^\dagger(p_2)Q(p_1) \tag{30}$$

where the matrix pion field $U \approx (\sigma + i\vec{\tau}\vec{\phi}')/\sigma_0$, $F^2_{\pi Q} \approx 0.7F^2_\pi$ and $p = p_1 - p_2$.

The similar approach for the calculations of $Q\bar{Q}$ correlators (Wilson loop) with account of light quarks will provide the interaction term of the pair of heavy quarks with pions $S_{QQ\pi}$. Both of these terms $S_{QQ\pi}$ and $S_{Q\pi}$ are responsible for the two-pions transitions in heavy quarkoniums.

6 Discussion and future work

- Instantons with sizes $\rho \sim$ hadron quark core sizes r give most essential contribution to their properties. In the case of lowest on energy hadron states the ILM is applicable.
- The strength of a heavy quark-instanton interaction is defined by

  $\Delta m_Q \sim packing\ fraction\ \rho^{-1} \sim 70\, MeV$ (at $\rho = 0.33\, fm, R = 1\, fm$)

  is small, while the strength of a light quark-instanton and gluon-instanton interactions are much more larger and given by the dynamical quark and dynamical gluon masses

  $M_g \sim M \sim (packing\ fraction)^{1/2}\ \rho^{-1} \approx 360\, MeV$ (at the same ρ, R).
- Instantons naturally generate also heavy-light quarks interaction, which might be important for the heavy quarkonium and heavy-light quarks systems properties. It can be responsible for the traces of SBCS in heavy quarks physics.

Future work.

- Extend the calculations of heavy-heavy quarks potential with ILM modified gluons.
- Take into account light quarks in the observables of heavy quark physics:
  - $Q\bar{Q}$ pions transitions;

– heavy-light mesons properties.

- Consider ILM generated gluon-light quarks interactions and related problems of exotic hadrons.

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