

# Excited vector mesons: phenomenology and predictions for a yet unknown vector $s\bar{s}$ state with a mass of about 1.93 GeV

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**Abstract.** The firm understanding of standard quark-antiquark states (including excited states) is necessary to search for non-conventional mesons with the same quantum numbers. In this work, we study the phenomenology of two nonets of excited vector mesons, which predominantly correspond to radially excited vector mesons with quantum numbers  $n^{2S+1}L_J = 2^3S_1$  and to orbitally excited vector mesons with quantum numbers  $n^{2S+1}L_J = 1^3D_1$ . We evaluate the decays of these mesons into two pseudoscalar mesons and into a pseudoscalar and a ground-state vector meson by making use of a relativistic quantum field theoretical model based on flavor symmetry. Moreover, we also study the radiative decays into a photon and a pseudoscalar meson by using vector meson dominance. We compare our results to the PDG and comment on open issues concerning the corresponding measured resonances. Within our approach, we are also able to make predictions for a not-yet discovered  $s\bar{s}$  state in the  $n^{2S+1}L_J = 1^3D_1$  nonet, which has a mass of about 1.93 GeV. This resonance can be searched in the upcoming GlueX and CLAS12 experiments which take place at the Jefferson Lab.

## 1 Introduction

In the last decades many theoretical calculations have appeared and numerous experimental observations have been made to explain the structure of matter. This research led to the postulate and verification of quarks, which interact through gluons and build up “white” hadrons. These strong interactions are described by Quantum Chromodynamics (QCD). We consider here bosonic hadrons, the so-called mesons; a significant number of mesons consists of a quark-antiquark ( $q\bar{q}$ ) pair. These are the “conventional mesons”, for more details see the review on the quark model in Ref. [1]. Besides them, there is also a group of non-conventional mesons such as four-quark or gluonic states. Nowadays new evidence of their existence is emerging, see Refs. [2, 3].

Ground-state vector mesons with the principal quantum number  $n = 1$ , spacial angular momentum  $L = 0$  and spin  $S = 1$  are very well known. On the contrary, the understanding of the nature of excited states is not yet so advanced. However, a good description of the excited states is necessary

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to prove the validity of the conventional quark–antiquark assignment and to understand the nature of non–conventional states beyond this standard picture.

In this work we study the phenomenology of two types of excited, quark–antiquark vector mesons (build up from the light  $u$ ,  $d$  and  $s$  quarks) in the low–energy regime, for which there are numerous experimental observations. The first group is identified with the states  $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$  characterized by the principal number  $n = 2$ , spacial angular momentum  $L = 0$  and spin  $S = 1$  (in the old spectroscopy notation  $n^{2S+1}L_J = 2^3S_1$ ). They are (predominantly) radially excited vector mesons. The second group is identified with the states  $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$  characterized by the principal number  $n = 1$ , spacial angular momentum  $L = 2$  and spin  $S = 1$  (which corresponds to  $n^{2S+1}L_J = 1^3D_1$ ). They are (predominantly) orbitally excited vector mesons. The last missing state in this nonet,  $\phi(???)$ , needs still experimental confirmation. By using our approach we make predictions for this putative state.

This paper, based on the publication [4], is organized as it follows: in Sec. 2 we describe the Quantum Field Theoretical (QFT) relativistic Lagrangian of our model; in Sec. 3 we present the results for two types of decays, strong and radiative ones, and we show the predictions for a putative  $s\bar{s}$  state in the nonet of orbitally excited vector mesons. Then, in Sec. 4 we present our conclusions.

## 2 The theoretical model

In order to study the decays of excited vector mesons we utilize an effective QFT relativistic model. In the past, such models have been successfully applied to tensor mesons [5], pseudotensor mesons [6], as well as to pseudovector [7] and (nontrivial) scalar mesons [8, 9]. The Lagrangian of our model fulfills the following conditions: it is invariant under flavour transformation  $U(3)_V$ , parity  $P$ , and charge conjugation  $C$  and it contains four mesonic nonets in form of  $3 \times 3$  matrices with the elements corresponding to  $q\bar{q}$  states, see below. Its explicit form reads:

$$\mathcal{L} = ig_{EPP}Tr([\partial^\mu P, V_{E,\mu}]P) + ig_{DPP}Tr([\partial^\mu P, V_{D,\mu}]P) + g_{EVP}Tr(\tilde{V}_E^{\mu\nu}\{V_{\mu\nu}, P\}) + g_{DVP}Tr(\tilde{V}_D^{\mu\nu}\{V_{\mu\nu}, P\}). \quad (1)$$

The first two terms describe decays of the type  $V_R \rightarrow PP$  and the last two terms describe decays of the type  $V_R \rightarrow VP$  with  $R = E, D$ . Moreover,  $[\cdot, \cdot]$  and  $\{\cdot, \cdot\}$  are the usual commutator and anticommutator, respectively,  $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$  and  $\tilde{V}_R^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(\partial_\alpha V_{R,\beta} - \partial_\beta V_{R,\alpha})$  (the latter is the dual field defined in the standard way). In Eq. (1),  $P$  stands for the matrix of pseudoscalar mesons and it is identified with the states  $\{\pi, K, \eta(547), \eta'(958)\}$ ,  $V_\mu$  describes the matrix of ground–state vector mesons with associated resonances  $\{\rho(770), K^*(892), \omega(782), \phi(1020)\}$ ,  $V_{E,\mu}$  stands for the nonet of radially excited vector mesons  $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$  and finally,  $V_{D,\mu}$  stands for the nonet of orbitally excited vector mesons  $\{\rho(1700), K^*(1680), \omega(1650), \phi(???)\}$ . Moreover, in our relativistic Lagrangian we have four coupling constants,  $g_{EPP}, g_{DPP}, g_{EVP}, g_{DVP}$ , which have been determined in Ref. [4] by using well-known experimental data listed in the Particle Data Group (PDG) [10]. [Note, in order to determine the errors for the coupling constants, the uncertainties on the masses of resonances were not taken into account. For details on how to determine the coupling constants together with the errors, see Ref. [4]]. The numerical values of the coupling constants are reported in Tab 1.

**Table 1.** Results for coupling constants determined by our model.

coupling constant	value
$g_{EPP}$	$3.66 \pm 0.4$
$g_{EVP}$	$18.4 \pm 3.8$
$g_{DPP}$	$7.15 \pm 0.94$
$g_{DVP}$	$16.5 \pm 3.5$

By using a standard QFT calculation, we obtain simple formulas for two strong decay channels ( $V_R \rightarrow PP, V_R \rightarrow VP$ ) and one radiative decay channel ( $V_R \rightarrow \gamma P$ ). The tree-level decay widths of a resonance  $R = \{E, D\}$  read:

$$\Gamma_{R \rightarrow PP} = s_{RPP} \frac{|\vec{k}|^3}{6\pi m_R^2} \left( \frac{g_{RPP}}{2} \lambda_{RPP} \right)^2, \quad \Gamma_{R \rightarrow VP} = s_{RVP} \frac{|\vec{k}|^3}{12\pi} \left( \frac{g_{RVP}}{2} \lambda_{RVP} \right)^2, \quad (2)$$

$$\Gamma_{R \rightarrow \gamma P} = \frac{|\vec{k}|^3}{12\pi} \left( \frac{g_{RVP}}{2} \frac{e_0}{g_\rho} \lambda_{R\gamma P} \right)^2, \quad (3)$$

where  $\vec{k}$  is the value of the three-momentum of one emitted particle from the decaying resonance with mass  $m_R$ , while the masses of the emitted particles are  $m_a$  and  $m_b$ , respectively. The coefficients  $s_{RPP}$  and  $s_{RVP}$  are symmetry factors determined by isospin. Moreover,  $\lambda_{RPP}$ ,  $\lambda_{RVP}$  and  $\lambda_{R\gamma P}$  are coefficients arising from expanding the traces in our Lagrangian (see tables 2-4).

### 3 Results

This section presents the results for the decays of nonets of radially and orbitally excited vector mesons. In Sec. 3.1 we concentrate on strong decays of these resonances, while in Sec. 3.2 we show the results for radial decays. In both cases the results are presented in summarizing tables. In total, we determined the decay widths for 64 different decay channels. Then, in Sec. 3.3 we focus on the last missing state of the  $1^3D_1$  nonet, which we called  $\phi(1930)$ .

#### 3.1 Strong decays of excited vector mesons

We calculate the strong decays of excited vector resonances into two pseudoscalar mesons (PP) and into a ground-state vector mesons and a pseudoscalar mesons (VP). In Tab. 2 we present the results for the nonet of radially excited vector mesons,  $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ , and in Tab. 3 for orbitally excited vector mesons,  $\{\rho(1700), K^*(1680), \omega(1650), \phi(???) = \phi(1930)\}$ .

The tables are organized as follows. For each single decay channel we assign the corresponding symmetry factor  $s$  and  $\lambda$  which we obtain from the Lagrangian of the model (1). Then, by using Eq. (2) we calculate the decay widths. As a next step we compared our results with data listed in the PDG [10]. Unfortunately, there is no experimental results for every decay channel. In both tables 2 and 3 we use “+” for a decay process which has been experimentally observed and “-” for those which has not been seen in experiments. The symbol “\*” means that the resonance has not yet been discovered.

The obtained theoretical results are consistent with the experiments. Especially, this is visible in the case of large decays. Yet, there are some inconsistencies between our model and experimental data from PDG: the theoretical decay width  $K^*(1410) \rightarrow K^*(892)$  is too small in comparison to the

**Table 2.** Symmetry factors, amplitude’s coefficients, and decay widths of strong decays of (predominantly) radially excited vector mesons.

<b>Radially excited vector mesons</b>				
Decay channel	Symmetry factor (s)	Amplitude ( $\lambda$ )	Decay width [MeV]	
			Theory	Exp.
$V_E \rightarrow PP$				
$\rho(1450) \rightarrow \bar{K}K$	2	$\frac{1}{2}$	$6.6 \pm 1.4$	$< 6.7 \pm 1.0$
$\rho(1450) \rightarrow \pi\pi$	1	1	$30.8 \pm 6.7$	$\sim 27 \pm 4$
$K^*(1410) \rightarrow K\pi$	3	$\frac{1}{2}$	$15.3 \pm 3.3$	$15.3 \pm 3.3$
$K^*(1410) \rightarrow K\eta$	1	$\frac{1}{2}(\cos \theta_p - \sqrt{2} \sin \theta_p)$	$6.9 \pm 1.5$	-
$K^*(1410) \rightarrow K\eta'$	1	$\frac{1}{2}(\sqrt{2} \cos \theta_p + \sin \theta_p)$	$\approx 0$	-
$\omega(1420) \rightarrow \bar{K}K$	2	$\frac{1}{2}$	$5.9 \pm 1.3$	-
$\phi(1680) \rightarrow \bar{K}K$	2	$\frac{1}{\sqrt{2}}$	$19.8 \pm 4.3$	+
$V_E \rightarrow VP$				
$\rho(1450) \rightarrow \omega\pi$	1	$\frac{1}{2}$	$74.7 \pm 31.0$	$\sim 84 \pm 13$
$\rho(1450) \rightarrow K^*(892)K$	4	$\frac{1}{4}$	$6.7 \pm 2.8$	+
$\rho(1450) \rightarrow \rho(770)\eta$	1	$\frac{1}{2} \cos \theta_p$	$9.3 \pm 3.9$	$< 16.0 \pm 2.4$
$\rho(1450) \rightarrow \rho(770)\eta'$	1	$\frac{1}{2} \sin \theta_p$	$\approx 0$	-
$K^*(1410) \rightarrow K\rho$	3	$\frac{1}{4}$	$12.0 \pm 5.0$	$< 16.2 \pm 1.5$
$K^*(1410) \rightarrow K\phi$	1	$\frac{1}{2\sqrt{2}}$	$\approx 0$	-
$K^*(1410) \rightarrow K\omega$	1	$\frac{1}{4}$	$3.7 \pm 1.5$	-
$K^*(1410) \rightarrow K^*(892)\pi$	3	$\frac{1}{4}$	$28.8 \pm 12.0$	$> 93 \pm 8$
$K^*(1410) \rightarrow K^*(892)\eta$	1	$\frac{1}{4}(\cos \theta_p + \sqrt{2} \sin \theta_p)$	$\approx 0$	-
$K^*(1410) \rightarrow K^*(892)\eta'$	2	$\frac{1}{4}(\sqrt{2} \cos \theta_p - \sin \theta_p)$	$\approx 0$	-
$\omega(1420) \rightarrow \rho\pi$	3	$\frac{1}{2}$	$196 \pm 81$	dominant
$\omega(1420) \rightarrow K^*(892)K$	4	$\frac{1}{4}$	$2.3 \pm 1.0$	-
$\omega(1420) \rightarrow \omega(782)\eta$	1	$\frac{1}{2} \cos \theta_p$	$4.9 \pm 2.0$	-
$\omega(1420) \rightarrow \omega(782)\eta'$	1	$\frac{1}{2} \sin \theta_p$	$\approx 0$	-
$\phi(1680) \rightarrow K\bar{K}^*$	4	$\frac{1}{2\sqrt{2}}$	$110 \pm 46$	dominant
$\phi(1680) \rightarrow \phi(1020)\eta$	1	$\frac{1}{\sqrt{2}} \sin \theta_p$	$12.2 \pm 5.1$	+
$\phi(1680) \rightarrow \phi(1020)\eta'$	1	$\frac{1}{\sqrt{2}} \cos \theta_p$	$\approx 0$	-

experiment; on the contrary,  $\rho(1700) \rightarrow \bar{K}K$  is too large when compared to data. These discrepancies are also visible when studying ratios between partial widths, which are discussed in details together with other open issues in Ref. [4].

Our results confirm the validity of the interpretation of both nonets as predominantly quark–antiquark states. Note, however, that some experimental results are quite old and new determination would be very useful.

**Table 3.** Symmetry factors, amplitude's coefficients, and decay widths of strong decays of (predominantly) orbitally excited vector mesons.

<b>Orbitally excited vector mesons</b>				
Decay channel	Symmetry factor (s)	Amplitude ( $\lambda$ )	Decay width [MeV]	
			Theory	Exp.
$V_D \rightarrow PP$				
$\rho(1700) \rightarrow \bar{K}K$	2	$\frac{1}{2}$	$40 \pm 11$	$8.3^{+10}_{-8.3}$
$\rho(1700) \rightarrow \pi\pi$	1	1	$140 \pm 37$	$75 \pm 30$
$K^*(1680) \rightarrow K\pi$	3	$\frac{1}{2}$	$82 \pm 22$	$125 \pm 43$
$K^*(1680) \rightarrow K\eta$	1	$\frac{1}{2}(\cos \theta_p - \sqrt{2} \sin \theta_p)$	$52 \pm 14$	-
$K^*(1680) \rightarrow K\eta'$	1	$\frac{1}{2}(\sqrt{2} \cos \theta_p + \sin \theta_p)$	$0.72 \pm 0.02$	-
$\omega(1650) \rightarrow \bar{K}K$	2	$\frac{1}{2}$	$37 \pm 10$	-
$\phi(1930) \rightarrow \bar{K}K$	2	$\frac{1}{\sqrt{2}}$	$104 \pm 28$	*
$V_D \rightarrow VP$				
$\rho(1700) \rightarrow \omega\pi$	1	$\frac{1}{2}$	$140 \pm 59$	+
$\rho(1700) \rightarrow K^*(892)K$	4	$\frac{1}{4}$	$56 \pm 23$	$83 \pm 66$
$\rho(1700) \rightarrow \rho(770)\eta$	1	$\frac{1}{2} \cos \theta_p$	$41 \pm 17$	$68 \pm 42$
$\rho(1700) \rightarrow \rho(770)\eta'$	1	$\frac{1}{2} \sin \theta_p$	$\approx 0$	-
$K^*(1680) \rightarrow K\rho$	3	$\frac{1}{4}$	$64 \pm 27$	$101 \pm 35$
$K^*(1680) \rightarrow K\phi$	1	$\frac{1}{2\sqrt{2}}$	$13 \pm 6$	-
$K^*(1680) \rightarrow K\omega$	1	$\frac{1}{4}$	$21 \pm 9$	-
$K^*(1680) \rightarrow K^*(892)\pi$	3	$\frac{1}{4}$	$81 \pm 34$	$96 \pm 33$
$K^*(1680) \rightarrow K^*(892)\eta$	1	$\frac{1}{4}(\cos \theta_p + \sqrt{2} \sin \theta_p)$	$0.5 \pm 0.2$	-
$K^*(1680) \rightarrow K^*(892)\eta'$	2	$\frac{1}{4}(\sqrt{2} \cos \theta_p - \sin \theta_p)$	$\approx 0$	-
$\omega(1650) \rightarrow \rho\pi$	3	$\frac{1}{2}$	$370 \pm 156$	$205 \pm 23$
$\omega(1650) \rightarrow K^*(892)K$	4	$\frac{1}{4}$	$42 \pm 18$	-
$\omega(1650) \rightarrow \omega(782)\eta$	1	$\frac{1}{2} \cos \theta_p$	$32 \pm 13$	$56 \pm 30$
$\omega(1650) \rightarrow \omega(782)\eta'$	1	$\frac{1}{2} \sin \theta_p$	$\approx 0$	-
$\phi(1930) \rightarrow K\bar{K}^*$	4	$\frac{1}{2\sqrt{2}}$	$260 \pm 109$	*
$\phi(1930) \rightarrow \phi(1020)\eta$	1	$\frac{1}{\sqrt{2}} \sin \theta_p$	$67 \pm 28$	*
$\phi(1930) \rightarrow \phi(1020)\eta'$	1	$\frac{1}{\sqrt{2}} \cos \theta_p$	$\approx 0$	*

### 3.2 Radiative decays of excited vector mesons

We consider the radiative decays of excited vector mesons into a photon and pseudoscalar meson ( $\gamma P$ ). We evaluate this type of process by using Vector Meson Dominance (VMD) (see e.g. Ref. [11]). In practice, we replace the vector field strength tensor according to the formula

$$V_{\mu\nu} \rightarrow V_{\mu\nu} + \frac{e_0}{g_\rho} Q F_{\mu\nu}, \quad (4)$$

where  $F_{\mu\nu}$  is the field strength tensor of the photons. Moreover,  $g_p = 5.5 \pm 0.5$  is the coupling constant which determines the interaction,  $e_0 = \sqrt{4\pi\alpha}$  stands for the electric charge of the proton, and  $Q = \text{diag}\{\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\}$  represents the charge matrix of the quarks. In Tab. 4 we report the results for radiative decays of both nonets of excited vector mesons. We use the following notation:

**Table 4.** Amplitude’s coefficients and decay widths of radiative decays of radially ( $V_E$ ) and orbitally ( $V_D$ ) excited vector mesons.

Decay channel	$\lambda$	$V_E$		$V_D$	
		Decay width [MeV]			
		Theory	Exp.	Theory	Exp.
$\rho_R \rightarrow \gamma\pi$	$\frac{1}{6}$	$0.072 \pm 0.042$	–	$0.095 \pm 0.058$	–
$\rho_R \rightarrow \gamma\eta$	$\frac{1}{2} \cos \theta_p$	$0.23 \pm 0.14$	$\sim 0.2 - 1.5$	$0.35 \pm 0.21$	–
$\rho_R \rightarrow \gamma\eta'$	$\frac{1}{2} \sin \theta_p$	$0.056 \pm 0.033$	–	$0.13 \pm 0.08$	–
$K_R^* \rightarrow \gamma K$	$\frac{1}{3}$	$0.18 \pm 0.11$	$< 0.0529$	$0.30 \pm 0.18$	–
$\omega_R \rightarrow \gamma\pi$	$\frac{1}{2}$	$0.60 \pm 0.36$	$1.90 \pm 0.75$	$0.78 \pm 0.47$	–
$\omega_R \rightarrow \gamma\eta$	$\frac{1}{6} \cos \theta_p$	$0.023 \pm 0.014$	–	$0.035 \pm 0.021$	–
$\omega_R \rightarrow \gamma\eta'$	$\frac{1}{6} \cos \theta_p$	$0.0050 \pm 0.0030$	–	$0.012 \pm 0.007$	–
$\phi_R \rightarrow \gamma\eta$	$\frac{1}{3} \sin \theta_p$	$0.14 \pm 0.09$	+	$0.19 \pm 0.12$	*
$\phi_R \rightarrow \gamma\eta'$	$\frac{1}{3} \cos \theta_p$	$0.076 \pm 0.045$	–	$0.13 \pm 0.08$	*

$\{\rho_R, K_R^*, \phi_R, \omega_R\} = \{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$  stands for the nonet of radially excited vector mesons and  $\{\rho_R, K_R^*, \phi_R, \omega_R\} = \{\rho(1700), K^*(1680), \omega(1650), \phi(1930)\}$  for orbitally excited vector mesons. We use the same notation as in the case of strong decays to indicate which decay process was observed and which one was not. Here, experimental data are rather poor, this is why most of the results are predictions.

### 3.3 Predictions for $\phi(1930)$ resonance

Our approach allows us to make predictions for a not-yet experimentally observed  $s\bar{s}$  resonance:  $\phi(???) = \phi(1930)$ . The existence of this missing state was discussed in the framework of the quark model [1] and recently re-elaborated in Ref. [12], based on the similarity between non-strange and strange meson families. In order to perform the calculations we have to know the mass of this putative state, which we fix by making a simple estimate. We notice that the mass difference between other member in both nonets of excited vector mesons is approximately the same and reads 250 MeV (this is due to the same dynamics describing these resonances). We shall apply this relation also to our novel state. By adding this difference to the mass of  $\phi(1680)$ , we get the mass of the state  $\phi(???)$ : 1930 MeV, hence  $\phi(1930)$ . By using this assumption, we calculate the decay widths of this resonance, as it is shown in Tab. 5. These results are predictions.

Our calculation shows that  $\phi(1930)$  has a quite broad width ( $\approx 400 \pm 100$  MeV), which is probably the reason why it is still missing in experiments. It is promising that in the near future two photoproduction-based experiments, CLAS12 [13] and GlueX [14–16], will take place at Jefferson Lab and will be able to investigate that energy region.

**Table 5.** Predicted values of decay widths for the putative state  $\phi(1930)$ .

Resonance $\phi(1930)$	
Decay channel	Decay width [Mev]
$\phi(1930) \rightarrow KK$	$104 \pm 28$
$\phi(1930) \rightarrow K\bar{K}^*$	$260 \pm 109$
$\phi(1930) \rightarrow \phi(1020)\eta$	$67 \pm 28$
$\phi(1930) \rightarrow \phi(1020)\eta'$	$\approx 0$
$\phi(1930) \rightarrow \gamma\eta$	$0.19 \pm 0.12$
$\phi(1930) \rightarrow \gamma\eta'$	$0.13 \pm 0.08$

## 4 Conclusions

In this work we have studied strong and radiative decays of two nonets of excited vector mesons:  $\{\rho(1450), K^*(1410), \omega(1420), \phi(1680)\}$ , which predominantly corresponding to radially excited vector mesons, and  $\{\rho(1700), K^*(1680), \omega(1650), \phi(1930)\}$ , predominantly corresponding to orbitally excited vector mesons. In our approach we used an effective QFT Lagrangian based on flavour symmetry. In total, we evaluated 64 different decay processes which we compared to experimental data from PDG (see results in tables 2-5). The overall agreement of our theory with experimental results shows the validity of the interpretation of these states as quark-antiquark objects. Moreover, we have made predictions for an undiscovered  $s\bar{s}$  resonance belonging to nonet of orbitally excited vector mesons, which we named  $\phi(1930)$ . Our calculations have shown that this missing state is broad. However, we expect new data from CLAS12 and GlueX experiments, that could shed light on its existence.

## Acknowledgements

The authors thank C. Reisinger for cooperation and acknowledge support from the Polish National Science Centre (NCN) through the OPUS project no. 2015/17/B/ST2/01625.

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