

Semileptonic decays and $|V_{xb}|$ determinations

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Abstract. We briefly summarize up-to-date results on the determination of the parameters of the Cabibbo-Kobayashi-Maskawa matrix $|V_{cb}|$ and $|V_{ub}|$, which play an important role in the unitarity triangle and in testing the Standard Model, and recent results on semileptonic B meson decays involving a τ lepton.

1 Introduction

We briefly review recent results on the semi-leptonic B decays and on the determination of the parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $|V_{cb}|$ and $|V_{ub}|$, which play an important role in the unitarity triangle and in testing the Standard Model (SM). For instance, the parameter ϵ_K depends on $|V_{cb}|^4$, while the ratio $|V_{ub}/V_{cb}|$ directly constrains one side of the unitarity triangle. The SM does not predict the values of the CKM matrix elements and the most precise measurements of $|V_{cb}|$ and $|V_{ub}|$ come from semi-leptonic decays, that being tree level at the lowest order in the SM are generally considered unaffected by new physics. The inclusive and exclusive semi-leptonic searches rely on different theoretical calculations and on different experimental techniques which have, to a large extent, uncorrelated statistical and systematic uncertainties. This independence makes the agreement between determinations of $|V_{cb}|$ and $|V_{ub}|$ values from inclusive and exclusive decays a useful test of our understanding of experimental data extraction and underlying theory (see e.g. [1–5] and references therein). We discuss up-to-date tensions between the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ within the SM and recent results on semileptonic B meson decays involving a τ lepton.

2 Exclusive $|V_{cb}|$ determination

For negligible lepton masses ($\ell = e, \mu$), the differential ratios for the semi-leptonic CKM favoured decays $B \rightarrow D^{(*)} \ell \nu$ can be written as

$$\begin{aligned} \frac{d\Gamma}{d\omega}(B \rightarrow D^* \ell \nu) &\propto G_F^2 (\omega^2 - 1)^{\frac{1}{2}} |V_{cb}|^2 \mathcal{F}(\omega)^2 \\ \frac{d\Gamma}{d\omega}(B \rightarrow D \ell \nu) &\propto G_F^2 (\omega^2 - 1)^{\frac{3}{2}} |V_{cb}|^2 \mathcal{G}(\omega)^2 \end{aligned} \quad (1)$$

The recoil parameter $\omega = p_B \cdot p_{D^{(*)}} / m_B m_{D^{(*)}}$ corresponds to the energy transferred to the leptonic pair. For the exact expression of the differentials in Eq. (1) we refer to the current literature. Here we care

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to emphasize the dependence on a single form factor, $\mathcal{F}(\omega)$ for $B \rightarrow D^* \ell \nu$ and $\mathcal{G}(\omega)$ for $B \rightarrow D \ell \nu$, and the phase space vanishing at the no-recoil point $\omega = 1$ in both cases.

In the heavy quark limit both form factors are related to a single Isgur-Wise function, $\mathcal{F}(\omega) = \mathcal{G}(\omega) = \xi(\omega)$, which is normalized to unity at zero recoil, that is $\xi(\omega = 1) = 1$. There are non-perturbative corrections to this prediction, expressed at the zero-recoil point by the heavy quark symmetry under the form of powers of Λ_{QCD}/m , where $m = m_c$ and m_b . Other corrections are perturbatively calculable radiative corrections from hard gluons and photons.

In order to extract the CKM factors, we need not only to compute the form factors, but also to measure experimental decay rates, which vanish at zero-recoil. Therefore, experimental points are extrapolated to zero recoil, using a parametrization of the dependence on ω of the form factor.

Recent determinations adopt a parametrization where ω is mapped onto a complex variable z via the conformal transformation $z = (\sqrt{\omega + 1} - \sqrt{2})/(\sqrt{\omega + 1} + \sqrt{2})$. The form factors may be written in form of an expansion in z , which converges rapidly in the kinematical region of heavy hadron decays. The coefficients of the expansions are subject to unitarity bounds based on analyticity. Common examples are the CLN (Caprini-Lellouch-Neubert) [6], the BGL (Boyd-Grinstein-Lebed) [7] and the BCL (Bourrely-Caprini-Lellouch) [8] parameterizations. They are all constructed to satisfy the unitarity bounds, but the CLN approach differs mostly in its reliance on next-to-leading order HQET relations between the form factors. Recently, the reliability of the CLN approach has been questioned in both $B \rightarrow D \ell \nu$ [9] and $B \rightarrow D^* \ell \nu$ [10, 11] channels.

The experiments, by measuring the differential decay rates with a variety of methods, provide inputs for several fits, that, among other parameters, aim at estimating the CKM values. A combined fit of the $B \rightarrow D^{(*)} \ell \nu$ differential rates and angular distributions, consistently including the HQET relations to $O(\Lambda_{QDE}/m_{c,b}, \alpha_s)$, has recently been performed. Under various fit scenarios, that use or omit lattice QCD and QCD sum rule predictions, they constrain the leading and subleading Isgur-Wise functions [12].

2.1 $B \rightarrow D^* \ell \nu$ channel

Until now, the FNAL/MILC collaboration has been the only one performing the non perturbative determination of the form factor $\mathcal{F}(1)$, at zero recoil, for the $B \rightarrow D^* \ell \nu$ channel in the lattice unquenched $N_f = 2 + 1$ approximation, and their latest estimate gives the value [13]

$$\mathcal{F}(1) = 0.906 \pm 0.004 \pm 0.012 \quad (2)$$

The first error is statistical and the second one is the sum in quadrature of all systematic errors. The lattice QCD theoretical error is now commensurate with the experimental error (they contribute respectively for about 1.4% and 1.3%), while the QED error contributes for about 0.5%. Large discretization error could be in principle be reduced by going to finer lattice spacings or larger lattice sizes. The total uncertainty is around the (1-2)% level.

There are two recent $|V_{cb}|$ determinations from the Heavy Flavour and Lattice Averaging Groups, HFLAV and FLAG respectively, that use the form factor (2); we report them in Table 1. Using the CLN parametrization, the 2016 HFLAV average [14] gives

$$|V_{cb}| = (39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}) \times 10^{-3} \quad (3)$$

where the first uncertainty is experimental and the second error is theoretical (lattice QCD calculation and electro-weak correction). The 2016 FLAG $N_f = 2 + 1$ $|V_{cb}|$ average value yields [15]

$$|V_{cb}| = (39.27 \pm 0.49_{\text{exp}} \pm 0.56_{\text{latt}}) \times 10^{-3} \quad (4)$$

This average employs the 2014 HFLAV experimental average [16] $\mathcal{F}(1)\eta_{EW}|V_{cb}| = (35.81 \pm 0.45) \times 10^{-3}$ and the value $\eta_{EW} = 1.00662$.

The HPQCD collaboration has presented preliminary results for the $B \rightarrow D^*$ form factor at zero recoil, based on relativistic HISQ charm quark and NRQCD bottom quark, giving the estimate $|V_{ub}| = (41.5 \pm 1.7) \times 10^{-3}$ [17].

Many experiments have measured the differential decay rate as a function of ω , but only recently, and for the first time, the unfolded fully-differential decay rate and associated covariance matrix have been published, by the Belle collaboration [18]. Using the CLN parametrization and the lattice form factor value, they extract the value [18]

$$|V_{cb}| = (37.04 \pm 1.3) \times 10^{-3} \quad (5)$$

Using Belle data, it has been shown that when switching from the CLN to the BGL form the determination of $|V_{cb}|$ shifts beyond the quoted experimental precision [10, 11, 19]. These analyses are consistent with each other and give in the BGL framework, along with the lattice value given for the zero recoil form factor, the values [11]

$$|V_{cb}| = (41.9^{+2.0}_{-1.9}) \times 10^{-3} \quad (6)$$

and [10]

$$|V_{cb}| = (41.7^{+2.0}_{-2.1}) \times 10^{-3} \quad (7)$$

The central value is higher than the corresponding value in CLN parametrization. However, it has also been argued that fits that yield the higher values of $|V_{cb}|$ suggest large violations of heavy quark symmetry and tension with lattice predictions of the form factor ratios [20].

Moving to estimates of the form factor via zero recoil sum rules, we have [21, 22]

$$\mathcal{F}(1) = 0.86 \pm 0.01 \pm 0.02 \quad (8)$$

where the second uncertainty accounts for the excited states. This value is in good agreement with the lattice value in Eq. (2), but slightly lower in the central value. That implies a relatively higher value of $|V_{cb}|$, that is

$$|V_{cb}| = (41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}) \times 10^{-3} \quad (9)$$

where the HFAG averages [23] have been used. The theoretical error is more than twice the error in the lattice determination (3).

2.2 The $B \rightarrow D\ell\nu$ channel

For $B \rightarrow D\ell\nu$ decay, the FNAL/MILC collaboration has calculated in 2015 the form factors in the unquenched lattice-QCD approximation [24] for a range of recoil momenta. By parameterizing their dependence on momentum transfer using the BGL z-expansion, they determine $|V_{cb}|$ from the relative normalization over the entire range of recoil momenta, which reads [24]

$$|V_{cb}| = (39.6 \pm 1.7_{\text{exp+QCD}} \pm 0.2_{\text{QED}}) \times 10^{-3} \quad (10)$$

The average value is almost the same than the one inferred from $B \rightarrow D^*\ell\nu$ decay by the same collaboration, see Eq. (3) and Table 1.

Results on $B \rightarrow D\ell\nu$ form factors at non-zero recoil have also been given the same year by the HPQCD Collaboration [25]. Their results are based on the non-relativistic QCD (NRQCD) action

for bottom and the Highly Improved Staggered Quark (HISQ) action for charm quarks, together with $N_f = 2 + 1$ MILC gauge configuration. A joint fit to lattice and 2009 BaBar experimental data [26] allows the extraction of the CKM matrix element $|V_{cb}|$, using the CLN parametrization. It gives [25]

$$|V_{cb}| = (40.2 \pm 1.7_{\text{latt+stat}} \pm 1.3_{\text{syst}}) \times 10^{-3} \quad (11)$$

The first error consists of the lattice simulation errors and the experimental statistical error and the second error is the experimental systematic error. The dominant error is the discretization error, followed by higher order current matching uncertainties. The former error can be reduced by adding simulation data from further ensembles with finer lattice spacings.

In 2015 the decay $B \rightarrow D \ell \nu$ has also been measured in fully reconstructed events by the Belle collaboration [27], They have performed a fit to the CLN parametrization, which has two free parameters, the form factor at zero recoil $\mathcal{G}(1)$ and the linear slope ρ^2 . The fit has been used to determine $\eta_{EW} \mathcal{G}(1) |V_{cb}|$, that, divided by the form-factor normalization $\mathcal{G}(1)$ found by the FNAL/MILC Collaboration [24], gives $\eta_{EW} |V_{cb}| = (40.12 \pm 1.34) \times 10^{-3}$ [27]. Assuming $\eta_{EW} \simeq 1.0066$, it translates into [27]

$$|V_{cb}| = (39.86 \pm 1.33) \times 10^{-3} \quad (12)$$

The Belle Collaboration also obtain a slightly more precise result (2.8% vs. 3.3%) by exploiting lattice data at non-zero recoil and performing a combined fit to the BGL form factor. It yields $\eta_{EW} |V_{cb}| = (41.10 \pm 1.14) \times 10^{-3}$ which translates into [27]

$$|V_{cb}| = (40.83 \pm 1.13) \times 10^{-3} \quad (13)$$

assuming once again $\eta_{EW} \simeq 1.0066$.

The latest lattice results, as well as [24, 25], Belle [27] and Babar [26] data, have been used in a global fit in the BGL parametrization which gives, in agreement with previous results [9]

$$|V_{cb}| = (40.49 \pm 0.97) \times 10^{-3} \quad (14)$$

In [9] differences on BGL, CLN, and BCL parameterizations are discussed.

3 Inclusive $|V_{cb}|$ determination

In inclusive $B \rightarrow X_c \ell \nu_l$ decays, the final state X_c is an hadronic state originated by the charm quark. There is no dependence on the details of the final state, and quark-hadron duality is generally assumed. Sufficiently inclusive quantities (typically the width and the first few moments of kinematic distributions) can be expressed as a double series in α_s and Λ_{QCD}/m , in the framework of the Heavy Quark Expansion (HQE), schematically indicated as

$$\Gamma(B \rightarrow X_c \ell \nu) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left[c_3 \langle O_3 \rangle + c_5 \frac{\langle O_5 \rangle}{m_b^2} + c_6 \frac{\langle O_6 \rangle}{m_b^3} + O \left(\frac{\Lambda_{QCD}^4}{m_b^4}, \frac{\Lambda_{QCD}^5}{m_b^3 m_c^2}, \dots \right) \right] \quad (15)$$

Here c_d ($d = 3, 5, 6 \dots$) are short distance coefficients, calculable in perturbation theory as a series in the strong coupling α_s , and O_d denote local operators of (scale) dimension d . The hadronic expectation values of the operators $\langle O_d \rangle$ encode the nonperturbative corrections and can be parameterized in terms of HQE parameters, whose number grows with powers of Λ_{QCD}/m_b . Similar expansions give the moments of distributions of charged-lepton energy, hadronic invariant mass and hadronic energy.

Let us observe that the first order in the series corresponds to the parton order, while terms of order Λ_{QCD}/m_b are absent. At order $1/m_b^0$ in the HQE, that is at the parton level, the perturbative corrections

up to order α_s^2 to the width and to the moments of the lepton energy and hadronic mass distributions are known completely (see Refs. [28–32] and references therein). The terms of order $\alpha_s^{n+1}\beta_0^n$, where β_0 is the first coefficient of the QCD β function, $\beta_0 = (33 - 2n_f)/3$, have also been computed following the Brodsky-Lepage-Mackenzie (BLM) procedure [29, 33].

The next order is Λ_{QCD}^2/m_b^2 , and at this order the HQE includes two operators, called the kinetic energy and the chromomagnetic operator, μ_π^2 and μ_G^2 . Perturbative corrections to the coefficients of the kinetic operator [34, 35] and the chromomagnetic operator [36–38] have been evaluated at order α_s^2 .

Neglecting perturbative corrections, i.e. working at tree level, contributions to various observables have been computed at order $1/m_b^3$ [39] and estimated at order $1/m_b^{4,5}$ [40–42].

Starting at order Λ_{QCD}/m_b^3 , terms with an infrared sensitivity to the charm mass, appear, at this order as a $\log m_c$ contribution [43–45]. At higher orders these contributions, sometimes dubbed intrinsic charm contribution, in form of powers of Λ_{QCD}/m_c have to be considered as well. Indeed, roughly speaking, since $m_c^2 \sim O(m_b\Lambda_{QCD})$ and $\alpha_s(m_c) \sim O(\Lambda_{QCD})$, contributions of order $\Lambda_{QCD}^5/m_b^3 m_c^2$ and $\alpha_s(m_c)\Lambda_{QCD}^4/m_b^2 m_c^2$ are expected comparable in size to contributions of order Λ_{QCD}^4/m_b^4 . The HQE parameters are affected by the particular theoretical framework (scheme) that is used to define the quark masses.

In HQE the number of nonperturbative parameters grows with the order in $1/m_b$. At leading order, the matrix elements can be reduced to one, while at dimension-four heavy-quark symmetries and the equations of motion ensure that the forward matrix elements of the operators can be expressed in terms of the matrix elements of higher dimensional operators. The first nontrivial contributions appear at dimension five, where two independent parameters, $\mu_{\pi,G}^2$, are needed, and two independent parameters, $\rho_{D,LS}^3$, are also needed at dimension six. At dimension seven and eight, nine and eighteen independent matrix elements appear, respectively, and for higher orders one has an almost factorial increase of the number of independent parameters. These parameters depend on the heavy quark mass, although sometimes the infinite mass limits of these parameters is taken.

The rates and the spectra are very sensitive to m_b . The physical pole mass definition for heavy quark masses is not a reasonable choice, because of problems in the convergence of perturbative series for the decay rates [46, 47]. Other possibilities are the use of “short-distance” mass definitions, such as the kinetic scheme [48], the 1S scheme [49], or the \overline{MS} mass, $m_b^{\overline{MS}}(m_b)$. The 1S scheme eliminates the b quark pole mass by relating it to the perturbative expression for the mass of the 1S state of the Υ system. In the kinetic scheme, the so-called “kinetic mass” $m_b^{kin}(\mu)$ is the mass entering the non-relativistic expression for the kinetic energy of a heavy quark, and is defined using heavy-quark sum rules. The alternative are short-distance mass definitions, as the \overline{MS} masses. However, the scale m_b for $m_b^{\overline{MS}}(m_b)$ is generally considered unnaturally high for B decays, while $m_b^{\overline{MS}}(\mu)$ at smaller scales ($\mu \sim 1$ GeV) is under poor control.

A global fit is a simultaneous fit to HQE parameters, quark masses and absolute values of CKM matrix elements obtained by measuring spectra plus all available moments. The semileptonic moments alone determine only a linear combination of m_b and m_c , and additional input is required to allow a precise determination of m_b . This additional information can come from the radiative $B \rightarrow X_s \gamma$ moments or from precise determinations of the charm quark mass. The HFLAV global fit [14] employs as experimental inputs the (truncated) moments of the lepton energy E_l^n (in the B rest frame) and the m_X^n momenta in the hadron spectra in $B \rightarrow X_c \ell \nu$. It is performed in the kinetic scheme, includes 6 non-perturbative parameters ($m_{b,c}$, $\mu_{\pi,G}^2$, $\rho_{D,LS}^3$) and the charm mass as the additional constraint, yielding

$$|V_{cb}| = (42.19 \pm 0.78) \times 10^{-3} \tag{16}$$

In the same kinetic scheme, another global fit, including the complete power corrections up to $O(\alpha_s \Lambda_{QCD}^2/m_b^2)$, has been performed, giving the estimate $|V_{cb}| = (42.21 \pm 0.78) \times 10^{-3}$ [50]. More recently, the effect of including $1/m_b^{4,5}$ corrections in the global fit has been also analyzed, in the so-called Lowest-Lying State Approximation (LLSA), which assumes that the lowest lying heavy meson states saturate a sum-rule for the insertion of a heavy meson state sum [41, 42, 51]. The LLSA was used because of the large number of new parameters, in order to provide loose constraints on the higher power matrix elements. A resulting global fit to the semileptonic moments in the LLSA gives the estimate [51]

$$|V_{cb}| = (42.11 \pm 0.74) \times 10^{-3} \quad (17)$$

Indirect $|V_{cb}|$ estimates from CKMfitter [52], using a frequentist statistical approach, and UTfit [53] Collaborations, adopting instead a Bayesian approach, are reported in Table 1.

Let us mention that this year a method to non-perturbatively calculate the forward-scattering matrix elements relevant to inclusive semi-leptonic B meson decays on lattice has been proposed [54].

Table 1. Status of exclusive and inclusive $|V_{cb}|$ determinations

| Exclusive decays | $ V_{cb} \times 10^3$ |
|--|--|
| $\bar{B} \rightarrow D^* l \bar{\nu}$ | |
| Grinstein et al. 2017 (Belle data, BGL) [11] | $41.9^{+2.0}_{-1.9}$ |
| Bigi et al. 2017 (Belle data, BGL) [10] | $41.7^{+2.0}_{-2.1}$ |
| Belle 2017 (CLN) [18] | 37.04 ± 1.3 |
| FLAG 2016 [15] | $39.27 \pm 0.49_{\text{exp}} \pm 0.56_{\text{latt}}$ |
| HFLAV 2016 (FNAL/MILC 2014 $\omega = 1$) [14] | $39.05 \pm 0.47_{\text{exp}} \pm 0.58_{\text{th}}$ |
| HFAG 2012 (Sum Rules) [21–23] | $41.6 \pm 0.6_{\text{exp}} \pm 1.9_{\text{th}}$ |
| $\bar{B} \rightarrow D l \bar{\nu}$ | |
| Global fit 2016 [9] | 40.49 ± 0.97 |
| Belle 2015 (CLN) [24, 27] | 39.86 ± 1.33 |
| Belle 2015 (BGL) [24, 25, 27] | 40.83 ± 1.13 |
| FNAL/MILC 2015 (Lattice $\omega \neq 1$) [24] | $39.6 \pm 1.7_{\text{exp+QCD}} \pm 0.2_{\text{QED}}$ |
| HPQCD 2015 (Lattice $\omega \neq 1$) [25] | $40.2 \pm 1.7_{\text{latt+stat}} \pm 1.3_{\text{sys}}$ |
| Inclusive decays | |
| HFLAV 2016 [14] | 42.19 ± 0.78 |
| Gambino et al. 2016 [51] | 42.11 ± 0.74 |
| Indirect fits | |
| UTfit 2017 [53] | 42.7 ± 0.7 |
| CKMfitter 2016 (3σ) [52] | $41.81^{+0.91}_{-1.81}$ |

4 Exclusive $|V_{ub}|$ determination

The parameter $|V_{ub}|$ is the less precisely known among the modules of the CKM matrix elements. The CKM-suppressed decay $B \rightarrow \pi \ell \nu$ with light final leptons is the typical exclusive channel used to extract $|V_{ub}|$. It is well-controlled experimentally and several measurements have been performed by both BaBar and Belle collaborations [55–61].

Commonly used non-perturbative approaches to form factor calculations are lattice QCD (LQCD) and light-cone sum rules (LCSR). At low q^2 , i.e. when the mass of the B-meson must be balanced by

a large pion momentum in order to transfer a small momentum to the lepton pair, lattice computations present large discretization errors and very large statistical errors. The high q^2 region is much more accessible to the lattice. On the other side, the low q^2 region is the range of applicability of LCSR.

The lattice determinations of $f_+(q^2)$ in the $B \rightarrow \pi l \nu$ channel, based on unquenched simulations, have been obtained by the HPQCD [62], the Fermilab/MILC [63, 64] and the RBC/UKQCD [65] collaborations. The Fermilab/MILC collaboration has evaluated the form factor $f_+(q^2 = 20 \text{ GeV}^2)$ with an uncertainty going down to 3.4%. Leading contribution to the uncertainty come from the chiral-continuum extrapolation fit, including statistical and heavy-quark discretization errors.

In 2016 the HPQCD collaboration has presented 2+1+1-flavor results for $B \rightarrow \pi l \nu$ decay at zero recoil, with the u/d quark masses going down to their physical values, for the first time; they also calculated the scalar factor f_0 form at zero recoil to 3% precision [66].

At large recoil (small q^2), direct LCSR calculations of the semi-leptonic form factors are available, which have benefited by progress in pion distribution amplitudes, next-to-leading and leading higher order twists and QCD corrections (see e.g. Refs. [67–71] and references within).

Branching fraction measurements of semileptonic B decays are possible using several different experimental techniques that differ in the way the companion B meson is reconstructed. In untagged analyses, the signal B meson is reconstructed, with the exception of the escaped neutrino. The 4-momentum of the companion B meson is inclusively determined by adding up the 4-momenta of all the remaining charged tracks and neutral clusters in the event. Since the initial state $\Upsilon(4S)$ is well-known, the missing 4-momentum can be identified with the neutrino 4-momentum, if neutrino is the only missing particle in the event. In tagged analyses, the companion B meson is fully reconstructed in either a semileptonic or an hadronic way. The available state-of-the-art experimental input consists of three untagged measurements by BaBar [58, 60] and Belle [59], and the two tagged Belle measurements [61]. The most recent analysis is the Belle hadronic tagged analysis [61], performed in 2013, which gives a branching ratio of $\mathcal{B}(B^0 \rightarrow \pi^- l^+ \nu) = (1.49 \pm 0.09_{\text{stat}} \pm 0.07_{\text{sys}}) \times 10^{-4}$, whose uncertainty is not very far from the more precise results from untagged measurements. By employing their measured partial branching fractions, and combining LCSR, lattice points and the BCL [8] parametrization, the Belle collaboration extracts the value $|V_{ub}| = (3.52 \pm 0.29) \times 10^{-3}$ [61].

The HFLAV $|V_{ub}|$ determination comes from a combined fit of a $B \rightarrow \pi$ form factor parameterization to theory predictions and the average q^2 spectrum in data. The theory input included in the fit are the results from the FLAG lattice average [15] and the light-cone sum rule result at $q^2 = 0 \text{ GeV}^2$ [68]. For the form factor parametrization, the BCL parametrization is used [8] with 3+1 parameters, i.e. 3 parameters for the coefficients in the BCL expansion and one normalization parameter for $|V_{ub}|$. The results of the combined fit are [14]

$$|V_{ub}| = (3.67 \pm 0.09 \pm 0.12) \times 10^{-3} \quad (18)$$

where the first error comes from the experiment and the second one from theory.

The FLAG Collaboration performs a constrained BCL fit of the vector and scalar form factors, together with the combined experimental datasets, finding [15]

$$|V_{ub}| = (3.73 \pm 0.14) \times 10^{-3} \quad (19)$$

The previous $|V_{ub}|$ estimates, together with recent estimates given by Fermilab/MILC [64] and RBC/UKQCD [65] Collaborations, have been reported in Table 2.

Other exclusive meson decays induced by $b \rightarrow u \ell \bar{\nu}_l$ transitions at the quark level are $B \rightarrow \rho/\omega \ell \bar{\nu}_l$ decays. The LCSR computation of the needed form factors has allowed different estimates of $|V_{ub}|$; recent values have also been reported in Table 2. Let us observe that the values extracted by $B \rightarrow$

Table 2. Status of exclusive $|V_{ub}|$ determinations and indirect fits.

| Exclusive decays | $ V_{ub} \times 10^3$ |
|--|---|
| $\bar{B} \rightarrow \pi \bar{\nu}_l$ | |
| HFLAV (FLAG+LCSR, BCL) 2016 [14] | $3.67 \pm 0.09 \pm 0.12$ |
| FLAG 2016 [15] | 3.73 ± 0.14 |
| Fermilab/MILC 2015 [64] | 3.72 ± 0.16 |
| RBC/UKQCD 2015 [65] | 3.61 ± 0.32 |
| $\bar{B} \rightarrow \omega \bar{\nu}_l$ | |
| Bharucha et al. 2016 (LCSR) [72] | $3.31 \pm 0.19_{\text{exp}} \pm 0.30_{\text{th}}$ |
| $\bar{B} \rightarrow \rho \bar{\nu}_l$ | |
| Bharucha et al. 2016 (LCSR) [72] | $3.29 \pm 0.09_{\text{exp}} \pm 0.20_{\text{th}}$ |
| $\Lambda_b \rightarrow p \mu \nu_\mu$ | |
| HFLAV (combined fit excl B) [14, 73] | 3.50 ± 0.13 |
| Indirect fits | |
| UTfit (2017) [53] | 3.61 ± 0.12 |
| CKMfitter (2016, 3σ) [52] | $3.71^{+0.24}_{-0.19}$ |

$\rho/\omega \ell \bar{\nu}_l$ decays appear to be systematically lower than the ones extracted by $B \rightarrow \pi \ell \nu$ decays. Values of $|V_{ub}|$ can also be extracted by $B \rightarrow \pi \pi \ell \nu$ decays [74].

The $B_s \rightarrow K^{(*)} \ell \nu$ decays have not been measured yet; however, they can become an additional channel to extract $|V_{ub}|$, since they are expected to be within the reach of future B -physics facilities [65, 75–78].

Another channel depending on $|V_{ub}|$ is the baryonic semileptonic $\Lambda_b^0 \rightarrow p \mu^- \bar{\nu}_\mu$ decay. At the end of Run I, LHCb has measured the probability of this decay relative to the channel $\Lambda_b^0 \rightarrow \Lambda_c^+ \mu^- \bar{\nu}_\mu$ [79]. This result has been combined with the ratio of form factors computed using lattice QCD with 2+1 flavors of dynamical domain-wall fermions [80], enabling the first determination of the ratio of CKM elements $|V_{ub}|/|V_{cb}|$ from baryonic decays [79]. The value of $|V_{ub}|$ depends on the choice of the value of $|V_{cb}|$. A combined fit from HFLAV for $|V_{ub}|$ and $|V_{cb}|$ that includes the constraint from LHCb, and the determination of $|V_{ub}|$ and $|V_{cb}|$ from exclusive B meson decays, gives [14, 73]

$$|V_{ub}| = (3.50 \pm 0.13) \times 10^{-3} \quad (20)$$

Indirect determination of $|V_{ub}|$ by the UTfit [53] and the CKMfitter [52] collaborations have also been reported in Table 2.

Finally, let us mention that in 2016 Belle has presented the first experimental result on $B \rightarrow \pi \tau \nu$, with an upper limit compatible with the SM [81].

5 Inclusive $|V_{ub}|$ determination

The extraction of $|V_{ub}|$ from inclusive decays requires to address theoretical issues absent in the inclusive $|V_{cb}|$ determination, since the experimental cuts, needed to reduce the background, enhance the relevance of the so-called threshold region in the phase space. Several theoretical schemes are available, which are tailored to analyze data in the threshold region, but differ in their treatment of perturbative corrections and the parametrization of non-perturbative effects. We limit to compare four theoretical different approaches, which have been recently analyzed by BaBar [82], Belle [83] and HFAG [16] collaborations, that is: ADFR by Aglietti, Di Lodovico, Ferrera and Ricciardi [84–86]; BLNP by Bosch, Lange, Neubert and Paz [87–89]; DGE, the dressed gluon exponentiation, by

Andersen and Gardi [90]; GGOU by Gambino, Giordano, Ossola and Uraltsev [91]¹. Although conceptually quite different, all these approaches lead to roughly consistent results when the same inputs are used and the theoretical errors are taken into account. The HFLAV estimates [14], together with the latest estimates by BaBar [82, 93] and Belle [83], are reported in Table 3.

Table 3. Status of inclusive $|V_{ub}|$ determinations.

| Inclusive decays | $ V_{ub} \times 10^3$ | | | |
|------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| | ADFR [84–86] | BNLP [87–89] | DGE [90] | GGOU [91] |
| HFLAV 2016 [14] | $4.08 \pm 0.13^{+0.18}_{-0.12}$ | $4.44 \pm 0.15^{+0.21}_{-0.22}$ | $4.52 \pm 0.16^{+0.15}_{-0.16}$ | $4.52 \pm 0.15^{+0.11}_{-0.14}$ |
| BaBar 2011 [82] | $4.29 \pm 0.24^{+0.18}_{-0.19}$ | $4.28 \pm 0.24^{+0.18}_{-0.20}$ | $4.40 \pm 0.24^{+0.12}_{-0.13}$ | $4.35 \pm 0.24^{+0.09}_{-0.10}$ |
| Belle 2009 [83] | $4.48 \pm 0.30^{+0.19}_{-0.19}$ | $4.47 \pm 0.27^{+0.19}_{-0.21}$ | $4.60 \pm 0.27^{+0.11}_{-0.13}$ | $4.54 \pm 0.27^{+0.10}_{-0.11}$ |

The BaBar and Belle estimates in Table 3 refer to the value extracted by the most inclusive measurement, namely the one based on the two-dimensional fit of the $M_X - q^2$ distribution with no phase space restrictions, except for $p_l^* > 1.0$ GeV. This selection allow to access approximately 90% of the total phase space [93]. The BaBar collaboration also reports measurements of $|V_{ub}|$ in other regions of the phase space [82], but the values reported in Table 3 are the most precise. When averaged, the ADFR value is lower than the one obtained with the other three approaches, and closer to the exclusive values; this difference disappears if we restrict to the BaBar and Belle results quoted in Table 3. By taking the arithmetic average of the results obtained from these four different QCD predictions of the partial rate the Babar collaboration gives [82] $|V_{ub}| = (4.33 \pm 0.24_{\text{exp}} \pm 0.15_{\text{th}}) \times 10^{-3}$.

By comparing the results in Table 2 and 3, we observe a tension between exclusive and inclusive determinations, of the order of $2 - 3\sigma$, according to the chosen values. Belle II is expected, at about 50 ab^{-1} , to decrease experimental errors on both inclusive and exclusive $|V_{ub}|$ determinations up to about 2% [94].

A new measurement [95] from BABAR based on the inclusive electron spectrum determines the partial branching fraction and $|V_{ub}|$ for $E_e > 0.8$ GeV. This analysis shows clearly that the partial branching fraction has substantial model dependence when the kinematic acceptance includes regions dominated by $B \rightarrow X_c \ell \nu$ background.

6 Exclusive decays into heavy leptons

In the SM the couplings to the W^\pm bosons are assumed to be universal for all leptons. This universality can be tested in semileptonic B meson decays involving a τ lepton, which might be sensitive to a possible charged Higgs boson or other BSM processes. The ratio of branching fractions (the denominator is the average for $\ell \in \{e, \mu\}$)

$$R_{D^{(*)}} \equiv \frac{\mathcal{B}(B \rightarrow D^{(*)} \tau \nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)} \ell \nu_\ell)} \quad (21)$$

is typically used instead of the absolute branching fraction of $B \rightarrow D^{(*)} \tau \nu_\tau$ decays to cancel uncertainties common to the numerator and the denominator. These include the CKM matrix element and several theoretical uncertainties on hadronic form factors and experimental reconstruction effects.

¹Recently, artificial neural networks have been used to parameterize the shape functions and extract $|V_{ub}|$ in the GGOU framework [92]. The results are in good agreement with the original paper.

In the standard model values for $R_{D^*}^{SM}$ can be calculated by means of HQE [96], while the most recent computation of R_D^{SM} uses a fit to lattice and experimental data [9]

$$R_{D^*}^{SM} = 0.252 \pm 0.003 \quad (22)$$

$$R_D^{SM} = 0.299 \pm 0.003 \quad (23)$$

In the standard model, estimates by lattice collaborations have become available in 2015 [24, 25]

$$R_{D^*}^{HPQCD} = 0.300 \pm 0.008 \quad (24)$$

$$R_D^{FL/MLC} = 0.299 \pm 0.011 \quad (25)$$

The previous values are all in agreement among them and with older R_D^{SM} determinations [97, 98].

Exclusive semi-tauonic B decays were first observed by the Belle Collaboration in 2007 [99]. Subsequent analysis by Babar and Belle [100–102] measured branching fractions above, although consistent with, the SM predictions. In 2012–2013 Babar has measured $R_{D^{(*)}}$ by using its full data sample [103, 104], and reported a significant excess over the SM expectation, confirmed in 2016 by the first measurement of R_{D^*} using the semileptonic tagging method (Belle [105]).

In 2015 a confirmation came also by the LHCb collaboration, who has studied the decay $\bar{B} \rightarrow D^{*+} \tau \bar{\nu}_\tau$ with $D^{*+} \rightarrow D^0 \pi^+$ and $\tau \rightarrow \mu \nu_\tau \bar{\nu}_\mu$ in pp collisions [106].

Most recently, the Belle collaboration has reported a new measurement in the hadronic τ decay modes which is statistically independent of the previous Belle measurements, with a different background composition, giving [107]

$$R_{D^*} = 0.270 \pm 0.035^{+0.028}_{-0.025} \quad (26)$$

where the first errors are statistical and the second ones systematic. This result is consistent with the theoretical predictions of the SM in Ref. [96] within 0.6σ standard deviations. They also report the first measurement of the τ lepton polarization in the decay $\bar{B} \rightarrow D^* \tau^- \bar{\nu}$ [107], which is again compatible with SM expectations [108].

By averaging the most recent measurements [102–107], including results from LHCb presented at FPCP 2017 [109], the HFLAV Collaboration has found [110]

$$R_{D^*} = 0.304 \pm 0.013 \pm 0.007 \quad (27)$$

$$R_D = 0.407 \pm 0.039 \pm 0.024 \quad (28)$$

where the first uncertainty is statistical and the second one is systematic. R_D and R_{D^*} exceed the SM values by about 2σ and 3σ , respectively. If one consider both deviations, the tension rises to about 4σ . At Belle II a better understanding of backgrounds tails under the signal and a reduction of the uncertainty to 3% for R_{D^*} and 5% for R_D is expected at 5 ab^{-1} .

While R_B is defined as the ratio of branching fractions of decays that occur at tree level in the SM at the lowest perturbative order, the observable R_K is defined as the ratio of branching fractions of rare decays, starting at one loop order in the SM, that is

$$R_{K^{(*)}[q_{\min}^2, q_{\max}^2]} = \frac{\mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-)_{q^2 \in [q_{\min}^2, q_{\max}^2]}}{\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)_{q^2 \in [q_{\min}^2, q_{\max}^2]}} \quad (29)$$

where $R_{K^{(*)}}$ is measured over specific ranges for the squared di-lepton invariant mass q^2 (in GeV^2).

Let us compare experimental data and theoretical determinations, and express their tension in terms of σ

$$\begin{aligned}
 R_{K[1,6]}^{\text{exp}} &= 0.745_{-0.074}^{+0.090} \pm 0.036 [111] & R_K^{\text{th}} &= 1.00 \pm 0.01 [113, 114] & 2.8 \sigma \\
 R_{K^*[0.045,1.1]}^{\text{exp}} &= 0.66_{-0.07}^{+0.11} \pm 0.03 [112] & R_{K^*[0.045,1.1]}^{\text{th}} &= 0.922 \pm 0.022 [114] & 2.7 \sigma \\
 R_{K^*[1.1,6.0]}^{\text{exp}} &= 0.69_{-0.07}^{+0.11} \pm 0.05 [112] & R_{K^*[1.1,6.0]}^{\text{th}} &= 1.000 \pm 0.006 [114] & 3.0 \sigma
 \end{aligned} \tag{30}$$

In the experimental data the first errors are statistical and the second ones systematic. The impact of radiative corrections has been estimated not to exceed a few % [113].

The alleged breaking of lepton-flavour universality suggested by most of the data is quite large, and several theoretical models have been tested against the experimental results. A welcome feature of measurements in the τ sector is the capacity of putting stringent limits on new physics models (see e.g. [115–122]). In particular, the simultaneous interpretation of the deviation of R_D and R_D^* in terms of the two Higgs doublet model II (2HDMII) seems to be ruled out [103]. This is also particularly interesting since this corresponds to the Higgs sector of commonly used supersymmetric models.

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