Locality and nonlocality in the interaction-free measurement

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Abstract. We present a paradox involving a particle and a mirror. They exchange a nonlocal quantity, modular angular momentum L_z mod $2\hbar$, but there seems to be no local interaction between them that allows such an exchange. We demonstrate that the particle and mirror *do* interact locally via a weak local current $\langle L_z \bmod 2\hbar\rangle_w$. In this sense, we transform the "interaction-free measurement" of Elitzur and Vaidman, in which two local quantities (the positions of a photon and a bomb in the two arms of a Mach-Zehnder interferometer) interact nonlocally, into a thought experiment in which two nonlocal quantities (the weak modular angular momentum of the particle and of the mirror) interact locally.

1 The quantum Cheshire Cat

So-called "weak values" [1] have taken their place alongside eigenvalues and expectation values as possible measured values in quantum mechanics. But while an ordinary ensemble suffices for measuring eigenvalues and expectation values, weak values require a "pre- and post-selected" (PPS) ensemble. Though unconventional, a PPS ensemble with initial state $|\psi_{in}\rangle$ and final state $|\psi_{fin}\rangle$ is (in principle) easy to prepare: we measure an operator that has $|\psi_{in}\rangle$ as an eigenstate, and then an operator with $|\psi_{fin}\rangle$ as an eigenstate, on as many systems as we like; and then we keep only those systems with those respective eigenstates. In between, we measure whatever we like, but with a measurement interaction weak enough to be consistent with the PPS ensemble. If the interaction is weak enough, the result of measuring an operator A is the weak value $\langle A\rangle_w$ of A:

$$\langle A\rangle_w = \frac{\langle\psi_{fin}|A|\psi_{in}\rangle}{\langle\psi_{fin}|\psi_{in}\rangle} \quad . \tag{1}$$

In this way, weak values enable us to *answer* questions about quantum systems that we otherwise cannot even *ask*.

An example of a weak value is the "quantum Cheshire cat" [2, 3], named after the Cheshire Cat in *Alice in Wonderland* [4] who could disappear while leaving its grin behind. In the weak-value version, a photon takes one path through a Mach-Zehnder interferometer while its net polarization vanishes on that path but not on the other. In this experiment, the photon and its polarization separate at a well-defined moment as the Cat passes through the first beam-splitter of the interferometer. There is also [5] an experiment in which the separation is continuous: the Cat is confined to one side of a potential

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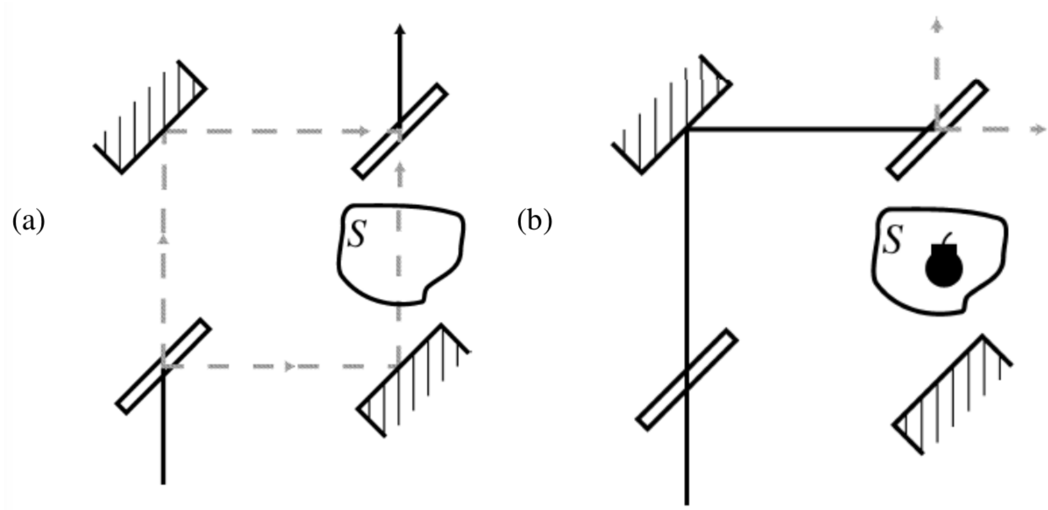

Figure 1. The Elitzur-Vaidman interaction-free measurement. (a) With a well defined relative phase between the arms, photons exit the interferometer in one direction only. (b) A bomb, by not exploding, provides which-path information.

barrier, while its grin tunnels through to the other side. Here, we apply this continuous Cheshire Cat separation to the “interaction-free measurement" (IFM) of Elitzur and Vaidman [6], where it challenges us to reconsider whether the effect—a paradigm of quantum nonlocality—is nonlocal after all.

2 The Cheshire Cat and interaction-free measurements

Recall that Elitzur and Vaidman imagined a bomb so sensitive that any interaction makes it explode. To detect such a bomb in a region $\mathcal{S}$, we build a Mach-Zehnder interferometer with one of its arms crossing the region $\mathcal{S}$. Fig. 1(a) shows the interferometer with no bomb in the region $\mathcal{S}$. A half-silvered mirror splits the incident photon beam into two equal parts. The parts recombine at another half-silvered mirror. By adjusting the length of each arm, we can make the parts of the beam interfere constructively in one direction and destructively in the other. Then all the photons leave the interferometer in the same direction. If, however, a detector records which path the photon actually takes through the interferometer, interference disappears (according to the complementarity principle) and the photon may leave the interferometer in either direction. If there is a bomb in the region $\mathcal{S}$ and it does not explode, it records the fact that the photon did not pass through $\mathcal{S}$. The photon went through the other arm of the interferometer. Then interference disappears and the photon may come out either way. (See Fig. 1(b).) So if we see a single photon leave the interferometer in the direction of destructive interference, we have detected the bomb without exploding it. What is so striking about this effect is that the bomb is revealed by a photon that, we can be quite sure, never came near it.

But, analogously, we can consider a one-dimensional cavity of length $2L$ with a finite potential barrier positioned symmetrically at its center. To the left of the barrier is, initially, a single neutron. (See Fig. 2(a).) If the right end of the cavity were not blocked, the neutron would ultimately tunnel through the barrier and leave the left side completely. However, the neutron reflects completely from the right end of the cavity. Let us assume that the neutron initially approaches the potential barrier in

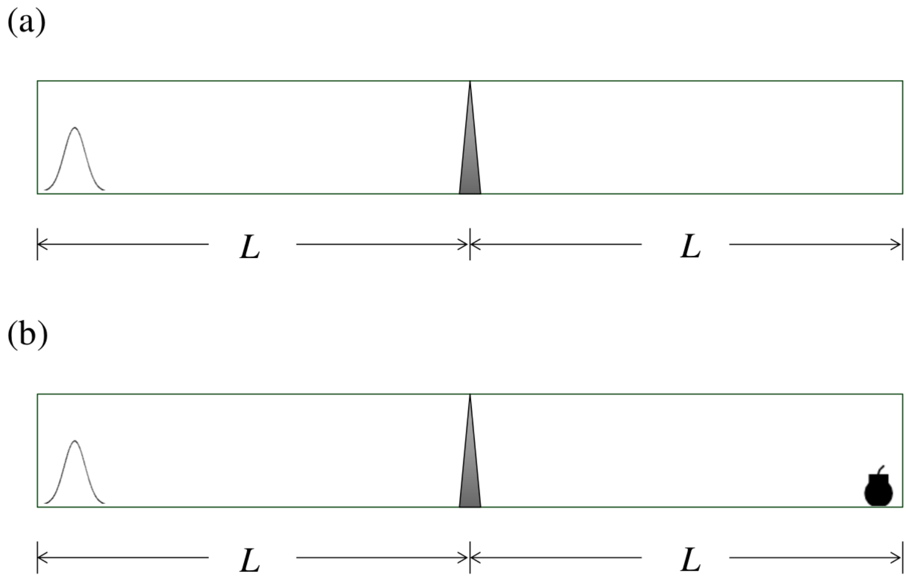

Figure 2. One-dimensional cavity of length $2L$ with a partition at its center, (a) without and (b) with a bomb at the right end.

a gaussian wave packet with momentum p_0 which is large compared to its uncertainty: $p_0 \gg \Delta p$. The neutron hits the barrier, and we can approximate its immediate evolution there via the matrix

$$U(\epsilon) = \begin{pmatrix} \cos\epsilon & i\sin\epsilon \\ i\sin\epsilon & \cos\epsilon \end{pmatrix} \quad , \tag{2}$$

where ϵ is, for now, arbitrary.[1] Note that $U(\epsilon)$ is unitary, as it must be, and that $[U(\epsilon)]^2 = U(2\epsilon)$. It is straightforward to prove, by induction, that $[U(\epsilon)]^n = U(n\epsilon)$. This equation has the following application. Let us assume that at time $t = 0$, the neutron is at the left end of the cavity. It takes a time Lm/p_0 (where m is the neutron mass) to reach the finite potential barrier. When the neutron reaches the barrier, it passes through with amplitude $i\sin\epsilon$ and reflects with amplitude $\cos\epsilon$. Both the transmitted and reflected wave packets reflect completely off the right and left ends, respectively, of the cavity, and meet again at the potential barrier after an additional time $2Lm/p_0$. They continue to do so every $2Lm/p_0$, and at time $t_n = 2nLm/p_0$, the amplitude for the neutron to be at the left end of the cavity is given by the first diagonal element of $U(n\epsilon)$, which is $\cos(n\epsilon)$. Hence by choosing $n\epsilon = \pi/2$, we can guarantee that the neutron will be on the *right* side of the potential barrier, at the right end of the cavity; it happens at time $T \equiv t_N$ such that $N = \pi/2\epsilon$, namely $T = \pi Lm/\epsilon p_0$. At time $t = 2T$ the neutron will again be fully at the left end of the cavity, at time $t = 3T$ at the right end again, and so on.

And now—continuing with the analogy—let there be a bomb at the right end of the cavity, as in Fig. 2(b). Again, the bomb is so sensitive that any interaction makes it explode. The unitary matrix

[1] If the diagonal elements of $U(\epsilon)$ are real, then the off-diagonal elements must be imaginary, as the following argument shows: Let the barrier be a δ-function potential located at $z = 0$. Without loss of generality we can let the incident neutron wave be e^{ikx}, the reflected wave be re^{-ikx} and the transmitted wave be te^{ikx}. Continuity of the wave function at $z = 0$ requires $1 + r = t$. Conservation of current requires $1 = |r|^2 + |t|^2$. Then $|1 + r|^2 = |t|^2$, hence $r + r^* = 0$ and r is pure imaginary.

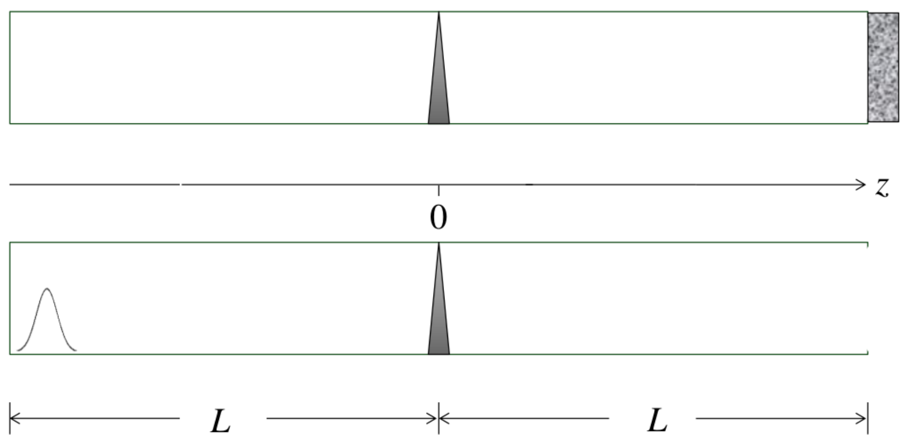

Figure 3. Two parallel cavities, with a single neutron wave packet (here in the lower cavity) and a single mirror (closing the upper cavity).

$U(\epsilon)$ still describes the immediate evolution when the neutron first hits the potential barrier, and we specialize to the case where ϵ is very small, such that $\cos\epsilon \approx 1 - \epsilon^2/2$ and $\sin\epsilon \approx \epsilon$. Now, for each interval of time $2Lm/p_0$ (during which the neutron, on the left side, hits the barrier *once*), the amplitude that the neutron tunnels through the barrier and hits the bomb is ϵ, up to a phase, and the probability that the bomb does not explode is therefore proportional to $1 - \epsilon^2$. For a given finite time T, we can take N as large as we like; and since $N = \pi/2\epsilon$, the total probability for the bomb not to explode is $(1-\epsilon^2)^N = (1-\pi^2/4N^2)^N \approx e^{-\pi^2/4N}$, which approaches 1 in the limit $N \to \infty$. So the bomb never explodes! But—from the fact that at time T we can find the neutron on the *left* of the potential barrier—we can be sure that the bomb is indeed on the *right*. Here is the IFM in a new guise close to the quantum Zeno effect [7].

3 A local IFM?

To test further the notion that the IFM can be local, let us now consider a variation of this experiment. Let there be *two* one-dimensional cavities, identical to the one above, parallel to each other and to the z axis, symmetrically above and below the z axis, both having length $2L$ and a barrier in the middle. (See Fig. 3.) However, these two cavities *differ* from the one cavity in the following respect: although the left ends (at $z = -L$) of the cavities are closed, and the neutron reflects perfectly from them, the right ends (at $z = L$) are *open*.

We now “populate" the two cavities with a single neutron (with horizontal coordinate z) and a single mirror (with horizontal coordinate Z) that is much heavier that the neutron and perfectly reflects it. We define neutron states $\psi_U^{(\nu)}(z)$ and $\psi_D^{(\nu)}(z)$ to have support only for $z \approx 0$ and only in the upper and lower cavities, respectively. Likewise, we define mirror states $\psi_U^{(\mu)}(Z)$ and $\psi_D^{(\mu)}(Z)$ to have support only for $Z \approx 0$ and only in the upper and lower cavities, respectively.

In particular, if the the neutron is in a state $\psi_U^{(\nu)}(z + L)$, i.e. in the left (closed) end of the upper cavity, and the mirror is in the state $\psi_U^{(\mu)}(Z - L)$, i.e. in the right (open) end of the upper cavity, then we can ignore the lower cavity, and the physics reduces to the physics of the single cavity above. But suppose we prepare the neutron and mirror in an initial state $\Psi_{in}(z, Z, 0)$, where

$$\Psi_{in}(z, Z, 0) = \frac{1}{2}\left[\psi_U^{(\nu)}(z+L) + \psi_D^{(\nu)}(z+L)\right]\left[\psi_U^{(\mu)}(Z-L) + \psi_D^{(\mu)}(Z-L)\right] \quad . \tag{3}$$

Now $\Psi_{in}(z, Z, 0)$ is a product of the neutron and mirror wave functions, and is invariant under interchange of U and D, both for the wave functions $\left[\psi_U^{(\nu)}(z+L) + \psi_D^{(\nu)}(z+L)\right]/\sqrt{2}$ and $\left[\psi_U^{(\mu)}(Z-L) + \psi_D^{(\mu)}(Z-L)\right]/\sqrt{2}$ of the neutron and mirror separately, and for their product. Formally, we can write $e^{iL_z^{(\nu)}\pi/\hbar}\Psi_{in}(z, Z, 0) = \Psi_{in}(z, Z, 0) = e^{iL_Z^{(\mu)}\pi/\hbar}\Psi_{in}(z, Z, 0)$ where $L_z^{(\nu)}$ and $L_Z^{(\mu)}$ are the angular momentum operators for the neutron and mirror, respectively. We denote the total angular momentum as $L_z = L_z^{(\nu)} + L_Z^{(\mu)}$ and define a *modular* [8] angular momentum $L_z \bmod \hbar$. Since $e^{iL_z^{(\nu)}\pi/\hbar}e^{iL_Z^{(\mu)}\pi/\hbar}$ applied to $\Psi_{in}(z, Z, 0)$ equals $\Psi_{in}(z, Z, 0)$, we obtain that $L_z \bmod 2\hbar = 0$.

How does the state $\Psi_{in}(z, Z, 0)$ evolve in time? In particular, what is the state of the neutron and mirror at time $T \equiv t_N = \pi L m/\epsilon p_0$? The answer to this question is contained in our previous calculations. Expanding the product in $\Psi_{in}(z, Z, 0)$, we get a sum of four terms:

$$\begin{aligned}\Psi_{in}(z, Z, 0) = \quad & \tfrac{1}{2} \quad \psi_U^{(\nu)}(z+L)\psi_U^{(\mu)}(Z-L) + \frac{1}{2}\psi_U^{(\nu)}(z+L)\psi_D^{(\mu)}(Z-L) \\ & + \frac{1}{2}\psi_D^{(\nu)}(z+L)\psi_U^{(\mu)}(Z-L) + \frac{1}{2}\psi_D^{(\nu)}(z+L)\psi_D^{(\mu)}(Z-L) \quad .\end{aligned} \tag{4}$$

Each term tells a story, and the first and last term tell similar stories that differ qualitatively from the stories of the intermediate two terms. For in the first and last terms, there is a mirror at the right end of the cavity to reflect the neutron; and in such cases, as we know, the chance of finding the neutron in the left half of the cavity, whether the upper or the lower, vanishes at time T. For the inner two terms, the mirror is located where it has no influence on the neutron—which therefore can eventually escape from whichever cavity it initially inhabited. But the neutron escapes on a time scale much longer than T, namely $N^2\epsilon^2$, which is of order NT. At time $t = T$, the neutron is virtually assured to be at the left end of the cavity. Thus a natural post-selection is a projection of $\Psi_{in}(z, Z, T)$ onto the left side of either cavity, i.e. the projection

$$\Psi_{fin}(z, Z, T) = \psi_U^{(\nu)}(z+L)\psi_D^{(\mu)}(Z-L)/\sqrt{2} + \psi_D^{(\nu)}(z+L)\psi_U^{(\mu)}(Z-L)/\sqrt{2} \quad , \tag{5}$$

which leaves the neutron *entangled* with the mirror. But note, the post-selection did not create the entanglement: the neutron and the mirror states were entangled already *before* the post-selection, as a result of their evolution in time.

Note that our post-selection, which projected the entangled state onto the left side of either cavity, could not have affected either $L_z^{(\nu)} \bmod 2\hbar$ and $L_Z^{(\mu)} \bmod 2\hbar$, since it commutes with them. Explicitly, let us define two projection operators, Π_{UL} and Π_{DL}, that project the neutron wave function onto the left side (from the left wall to the barrier) of the upper and lower cavities, respectively. Then we have $e^{iL_z^{(\nu)}\pi/\hbar}\Pi_{UL} = \Pi_{DL}$ and $e^{iL_z^{(\nu)}\pi/\hbar}\Pi_{DL} = \Pi_{UL}$, hence $e^{iL_z^{(\nu)}\pi/\hbar}$ and $L_z^{\nu} \bmod 2\hbar$ commute with $\Pi_{UL} + \Pi_{DL}$. At the same time, $e^{iL_Z^{(\mu)}\pi/\hbar}$ and $L_Z^{\mu} \bmod 2\hbar$ commute with $\Pi_{UL} + \Pi_{DL}$ because they act on orthogonal subspaces. (L_Z^{μ} does not act on the neutron at all.) Therefore projection of the overall state onto the left side of the cavities cannot affect its symmetry under the operators $e^{iL_z^{(\nu)}\pi/\hbar}$ and $e^{iL_Z^{(\mu)}\pi/\hbar}$ in any way, and non-conservation of $L_z^{\nu} \bmod 2\hbar$ and $L_Z^{\mu} \bmod 2\hbar$ could not be traced to post-selection of the left side of the cavities.

Thus, what is striking about this result is that the entangled state is *not* an eigenstate of the modular angular momentum of either the neutron or the mirror, i.e. it is not an eigenstate of either $e^{iL_z^{(\nu)}\pi/\hbar}$ or $e^{iL_Z^{(\mu)}\pi/\hbar}$; but it is still an eigenstate of the total modular angular momentum $L_z \bmod 2\hbar$. We thus conclude that $L_z^{(\nu)}$ mod $2\hbar$ and $L_Z^{(\mu)}$ mod $2\hbar$ were not separately conserved during the interaction of the neutron and the mirror, yet their sum was conserved. This conclusion would not be a paradox if it were not for the fact that, according to the post-selection, the neutron and mirror could never have met; for while the mirror was stationed at the right end of the cavity, the neutron was confined to the left end! If the neutron and the mirror did interact—as apparently they did, since they exchanged modular angular momentum—they apparently did so *nonlocally*—which challenges our proposed *local* interpretation of interaction-free measurements.

Here is a contrasting experiment. Equations (2-3) define the state $\Psi_{in}(z, Z, 0)$, i.e. the combined state of the neutron and mirror at time $t = 0$. This state is an eigenvector of $e^{iL_z^{(\nu)}\pi/\hbar}$ and of $e^{iL_z^{(\mu)}\pi/\hbar}$, both with eigenvalue 1. What is $\Psi_{in}(z, Z, 2T)$, i.e. their combined state at time $t = 2T$? At that time, we know, the neutron has either never leaked to the right (for the two middle terms of Eq. (4)) or has leaked to the right and leaked back to the left. So does $\Psi_{in}(z, Z, 2T)$ equal $\Psi_{in}(z, Z, 0)$, up to an overall phase? No, it does not, because the first and fourth terms have acquired a phase factor of -1. That is, $t = 2T$ corresponds to $n = 2N = \pi/\epsilon$, hence $\cos(n\epsilon) = \cos\pi = -1$. Combining the four terms of $\Psi_{in}(z, Z, 2T)$ into a product, we obtain

$$\Psi_{in}(z, Z, 2T) = \frac{1}{2}\left[\psi_U^{(\nu)}(z+L) - \psi_D^{(\nu)}(z+L)\right]\left[\psi_U^{(\mu)}(Z-L) - \psi_D^{(\mu)}(Z-L)\right] \quad , \tag{6}$$

i.e. it is an eigenvector of $e^{iL_z^{(\nu)}\pi/\hbar}$ and of $e^{iL_z^{(\mu)}\pi/\hbar}$, both with eigenvalue -1. Thus indeed the neutron and the mirror have exchanged modular L_z (subject to the constraint of conservation of total modular L_z). There is no paradox here, because the neutron has reached the mirror and returned. But how do we explain the exchange of modular L_z when we post-select the state $\Psi_{fin}(z, Z, T)$ at time T?

4 Nonlocal modular currents

Local or nonlocal? In this section we will show explicitly how the neutron and mirror exchange modular momentum—and revisit the question of whether the exchange is local or nonlocal.

As noted in the previous section, the sum of the modular angular momenta of the neutron and the mirror, expressed as $e^{i\pi L_z^{(\nu)}/\hbar}e^{i\pi L_z^{(\mu)}/\hbar}$, is always conserved; both the pre- and post-selected states are eigenstates of this operator, with eigenvalue 1. In addition, the pre-selected state is trivially an eigenstate of both $e^{i\pi L_z^{(\nu)}/\hbar}$ and $e^{i\pi L_z^{(\mu)}/\hbar}$ separately, while the post-selected state is an eigenstate of neither, since either operator turns the post-selected state

$$\Psi_{fin}(z, Z, T) = \psi_U^{(\nu)}(z+L)\psi_D^{(\mu)}(Z-L)/\sqrt{2} + \psi_D^{(\nu)}(z+L)\psi_U^{(\mu)}(Z-L)/\sqrt{2} \quad , \tag{7}$$

into

$$\psi_U^{(\nu)}(z+L)\psi_U^{(\mu)}(Z-L)/\sqrt{2} + \psi_D^{(\nu)}(z+L)\psi_D^{(\mu)}(Z-L)/\sqrt{2} \quad , \tag{8}$$

which is *orthogonal* to $\Psi_{fin}(z, Z, T)$. Thus the weak expectation values of both $e^{i\pi L_z^{(\nu)}/\hbar}$ and $e^{i\pi L_z^{(\mu)}/\hbar}$ vanish, indicating total uncertainty about both $L_z^{(\nu)}$ mod $2\hbar$ and $L_z^{(\mu)}$ mod $2\hbar$, although their sum is still 0 mod $2\hbar$. Since, at the pre-selection, both the neutron and the mirror had well defined modular angular momenta and, at the post-selection, neither did, they must have exchanged uncertain quantities of modular angular momentum. But how?

A first glimpse of how to resolve this paradox comes if we consider the post-selected state and its time evolution. The post-selected state represents a neutron at the left end of one cavity times a mirror at the right end of the other cavity (and superposes the two ways this product state can arise). Since, in each term, the neutron is to the left of the barrier and there is no mirror to the right of the

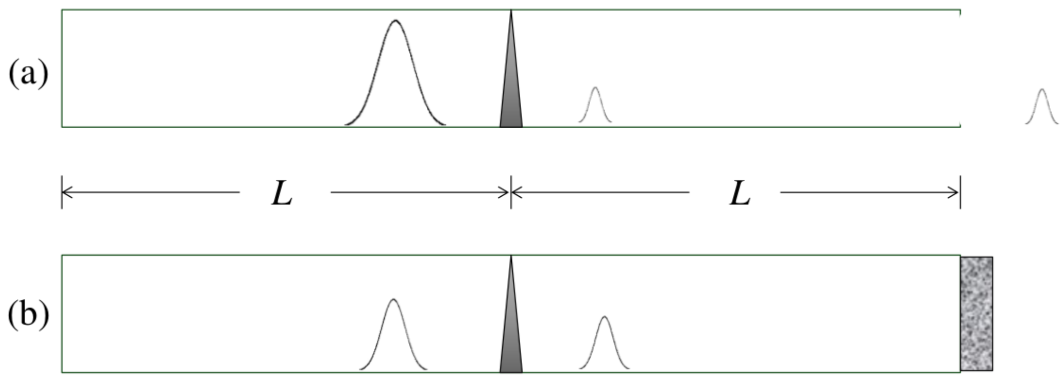

Figure 4. (a) A term in the post-selected state $\Psi_{fin}(z, Z, T)$ of Eq. (5). Since no mirror is present to reflect the neutron from the right end of the cavity, the wave packets to the right of the partition have amplitudes of order ϵ and move away from the partition. (In the time-reversed account, they approach the partition.) (b) A term in the evolution of the pre-selected state $\Psi_{in}(z, Z, 0)$ of Eq. (4). The mirror reflects the neutron which, over a time T, passes completely through the partition.

same barrier, we know from the previous sections that a series of wave packets will "leak" through the barrier and escape from the right of the same cavity; but since the amplitude of each "leakage" is of order ϵ, it proves to be negligible over the time scale T we consider. (See Fig. 4.) However, it is worth noting that the time evolution relevant to our calculation is not only the evolution *forwards* in time after $t = T$, but also the evolution *backwards* in time for $t \leq T$. This evolution must be the time reverse of the evolution we considered in previous sections; in particular, it must include wave packets entering the cavity from the *right* (wherever there is no mirror in the way) and *approaching* the barrier from the right. For concreteness, it is useful to break up the evolution into pairs of time intervals of duration Lm/p_0. In the forward time evolution, a neutron on the right of the barrier approaches the barrier in the *first* of these two intervals, and leaves the barrier, moving to the right, in the second interval. By contrast, a neutron on the left end of the cavity produces order-ϵ wave packets only in the *second* of the two Lm/p_0 intervals; in the first interval, it is still on the way to the barrier on its right.

Now the *forwards* evolution of the post-selected state in the *second* interval translates into a *backwards* evolution of the post-selected state in the *first* interval, with order-ϵ wave packets moving to the left. Thus in the first interval of each pair of intervals, we have a coincidence between an order-ϵ wave packet moving to the left in one cavity and a generally larger wave packet moving to the left in the other cavity. These wave packets have the same z coordinate. Since they are in different cavities, the weak value of the projection on each wave packet's position vanishes, but the weak value of $e^{i\pi L_z^{(v)}/\hbar}$ does not vanish! A "weak" current of modular angular momentum flows from the right end of the cavities towards the barrier! Here is a qualitative explanation of how the mirror and the neutron can exchange modular angular momentum, even when the neutron never reaches the mirror. Can we make this qualitative explanation quantitative? To show that we can, let us calculate the change in modular angular momentum *to the left of the barrier* during each paired interval. The pre-selected state evolves forward in time such that the weak value of $e^{i\pi L_z^{(v)}/\hbar}$, calculated over the wave packet to the left of the barrier (in the closed cavity), is $\cos(n\epsilon)$ at time $t_n = 2nLm/p_0$. Thus during every interval of time $t_{n+1} - t_n = 2Lm/p_0$, the region to the left of the barrier gains $\cos([n+1]\epsilon) - \cos(n\epsilon) = \cos(n\epsilon)\cos(\epsilon) - \sin(n\epsilon)\sin(\epsilon) - \cos(n\epsilon)$, which equals $-\epsilon\sin(n\epsilon)$ up to terms of order ϵ^2. During the same interval of time, the weak value of $e^{i\pi L_z^{(\nu)}}$ equals the weak value of $e^{i\pi L_z^{(\mu)}/\hbar}$ since, as noted above, both operators turn the post-selected state $\Psi_{fin}(z, Z, T)$ of Eq. (5) into the state of Eq. (6). However, if we want to *localize* the modular angular momentum, i.e. to determine its location, then we can multiply $e^{i\pi L_z^{(\nu)}/\hbar}$ (or $e^{i\pi L_z^{(\mu)}/\hbar}$) by an operator that projects onto a region of interest. In fact, we have already done so above, in calculating the modular angular momentum to the left of the barrier, onto which we implicitly projected $e^{i\pi L_z^{(\nu)}/\hbar}$. Two additional regions are relevant. First, in the region to the right of the barrier there is a time-dependent region of overlap between the order-ϵ wave packet in the channel without a mirror and the $\sin(n\epsilon)$ wave packet in the channel with a mirror. Multiplying them together in the formula for the *local* weak value of $e^{i\pi L_z^{(\nu)}/\hbar}$ (i.e. the weak value at the location of the wave packet), we obtain $\epsilon \sin(n\epsilon)$, which precisely accounts for the change in modular momentum to the left of the barrier. Second, there is the region at the right end of the channel, the (possible) location of the mirror. The wave function of the mirror has support only there, near $Z = L$. Since this is also the third region of interest, the entire wave function of the mirror is included, and projection onto that region changes nothing. Also, the inner product of $\psi_U^{(\mu)}(Z-L)$ or $\psi_D^{(\mu)}(Z-L)$ with itself yields only a factor 1. However, the weak value of $e^{i\pi L_z^{(\mu)}/\hbar}$ depends also on the pre- and post-selected states of the neutron; these are the same pre- and post-selected states as in the calculation of $e^{i\pi L_z^{(\nu)}/\hbar}$. Consequently, the weak value of $e^{i\pi L_z^{(\mu)}/\hbar}$ must always equal the weak value of $e^{i\pi L_z^{(\nu)}/\hbar}$. At first, this result may seem to be a contradict conservation of modular L_z: how can the sum of $L_z^{(\nu)}$ and $L_z^{(\mu)}$ be invariant if they are the same? But it is conservation of modular L_z that requires this result. Namely, conservation of $L_z^{(\nu)} + L_z^{(\mu)}$ mod $2\hbar$ requires that $\cos\left(\pi[L_z^{(\nu)} + L_z^{(\mu)}]/\hbar\right)$ always vanish (mod $2\hbar$), hence $L_z^{(\nu)} = -L_z^{(\mu)}$ (mod $2\hbar$), i.e. they have opposite signs; but since the cosine function ignores the sign of the arguments, $e^{i\pi L_z^{(\nu)}/\hbar}$ and $e^{i\pi L_z^{(\mu)}/\hbar}$ are the same [9]. For the same reason, the modular angular momentum of the mirror can decrease while the modular angular momentum of the neutron increases, even though the weak values of $e^{i\pi L_z^{(\nu)}/\hbar}$ and $e^{i\pi L_z^{(\mu)}/\hbar}$ both *increase*.

We conclude that, in the sense of this current of modular L_z mediating locally between the neutron and the mirror, the "interaction-free measurement" is local; but what is local here is a nonlocal dynamical quantity—modular angular momentum—that has itself become disconnected from the neutron and the mirror! No measurements in either cavity alone could ever reveal the value of the conserved current of L_z mod $2\hbar$. In this sense, we have inverted the interaction-free measurement of Elitzur and Vaidman [6]: in their original thought experiment, two *local* quantities (the positions of a photon and a bomb in the two arms of a Mach-Zehnder interferometer) interact *nonlocally*; in the present thought experiment, two *nonlocal* quantities (the modular angular momentum of a neutron and of a mirror) interact *locally* via their weak currents.

Acknowledgements

This publication was made possible through the support of grants from the John Templeton Foundation (Project ID 43297)and from the Israel Science Foundation (grant no. 1190/13). The opinions expressed in this publication are those of the authors and do not necessarily reflect the views of either of these supporting foundations.

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[7] See Y. Aharonov and D. Rohrlich, *op. cit.*, Sect. 12.1.

[8] Y. Aharonov, H. Pendleton and A. Petersen, *Int. J. Theor. Phys.* **2**, 213 (1969). See also Y. Aharonov and D. Rohrlich, *op. cit.*, Chap. 5.

[9] See Y. Aharonov and D. Rohrlich, *op. cit.*, Eq. (6.11).