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$f_1(1285) \to e^+e^-$ decay and direct f_1 production in e^+e^- collisions

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Abstract. The width of the $f_1(1285) \to e^+e^-$ decay is calculated in the vector meson dominance model. The result depends on the relative phase between two coupling constants describing $f_1 \to \rho^0\gamma$ decay. The width $\Gamma(f_1 \to e^+e^-)$ is estimated to be $\simeq 0.07 - 0.19$ eV. Direct f_1 production in e^+e^- collisions is discussed, and the $e^+e^- \to f_1 \to a_0\pi \to \eta\pi\pi$ cross section is calculated. Charge asymmetry in the $e^+e^- \to \eta\pi^+\pi^-$ reaction due to interference between $e^+e^- \to f_1$ and $e^+e^- \to \eta\rho^0$ amplitudes is studied.

1 Introduction

High-luminosity electron-positron colliders are powerful tools for measuring electronic widths of hadronic resonances with positive charge parity, $C = +1$. The idea of such measurements was put forward many years ago [1, 2]. Several experiments in search of direct production of C-even resonances in e^+e^- collisions were performed, and very low upper limits on the leptonic widths of η', $f_2(1270)$, $a_2(1320)$, and $X(3872)$ mesons were set [3–6]: $\Gamma(\eta'(958) \to e^+e^-) < 0.002$ eV (90% CL), $\Gamma(f_2(1270) \to e^+e^-) < 0.11$ eV (90% CL), $\Gamma(a_2(1320) \to e^+e^-) < 0.56$ eV (90% CL). The explanation of the smallness of the leptonic widths of C-even resonances is that corresponding decays proceed via two virtual photons and therefore are suppressed by a factor of α^4, where α is the fine structure constant.

In this paper we consider 1^{++} meson $f_1(1285)$, its decay into the e^+e^- pair, and its direct production in e^+e^- collisions. The process $e^+e^- \to f_1 \to mesons$ is still not measured and may be studied at the VEPP-2000 e^+e^- collider in experiments with the SND and CMD-3 detectors.

There is a quite extensive list of literature on the production of 1^{++} resonances in e^+e^- annihilation. The direct production of 1^{++} states through the neutral current was evaluated many years ago in the nonrelativistic quarkonium model [7]. The calculation of the width $\Gamma(\chi_1 \to e^+e^-)$ was performed in the quarkonium and vector meson dominance models (VMD) [8]. There are also some recent papers devoted to $X(3872)$ and χ_{c1} decays into the e^+e^- pair and their production in e^+e^- collisions (see [9–12] and references therein). The production of 1^{++} resonances R in two-photon collisions ($e^+e^- \to e^+e^-R$) was also extensively studied both theoretically [13–16] and experimentally [17–19].

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2 Simple estimate of $f_1 \to e^+e^-$ decay width

The electron and positron produced in the $f_1 \to e^+e^-$ decay are ultrarelativistic in the f_1 rest frame. So, in this frame e^+ and e^- can be considered as massless and having the definite helicities. In Ref. [20] the width of the $\chi_{c1} \to l^+l^-$ decay was calculated with the finite lepton mass, and it was found that the mass effects are negligible.

There is only one P- and C-even invariant amplitude for the $f_1 \to e^+e^-$ decay, which is written as

$$M(f_1 \to e^+e^-) = F_A \alpha^2 \widetilde{e}_\mu \bar{u}\gamma^\mu\gamma^5 v, \tag{1}$$

where $\widetilde{e}_\mu$ is the C-even axial vector describing the f_1 meson, $\bar{u}\gamma^\mu\gamma^5 v = j_A^\mu$ is the axial current, and F_A is the dimensionless coupling constant. Since f_1 meson is C-even, it decays into e^+e^- via two virtual photons as depicted in Fig. 1. This explains the origin of the factor α^2 in (1).

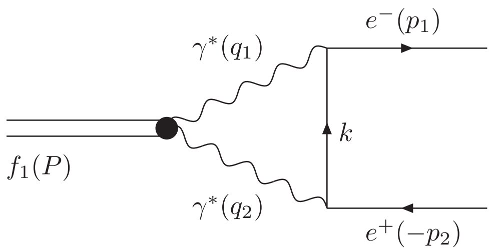


Figure 1. One-loop diagram of the $f_1 \to e^+e^-$ decay with two intermediate photons.

Using the amplitude (1) it is easy to calculate the decay width

$$\Gamma(f_1 \to e^+e^-) = \frac{\alpha^4 |F_A|^2}{12\pi} m_f, \tag{2}$$

where m_f is the f_1 mass, $m_f = 1282.0$ MeV [21].

For a naive estimate, it is natural to assume that the coupling constant F_A is of the order of unity, $|F_A| \sim 1$. In (1) we have already written explicitly the small factor α^2, and there are not any additional small factors. So, we obtain that $\Gamma(f_1 \to e^+e^-) \sim 0.1$ eV. In what follows we calculate this width in a certain model and find that this simple estimate is correct by the order of magnitude.

3 Model-independent description of $f_1 \to e^+e^-$ amplitude

To calculate the width $\Gamma(f_1 \to e^+e^-)$ more accurately, we should know the amplitude of the $f_1 \to \gamma^*\gamma^*$ transition (see Fig. 1). This amplitude must be symmetric with respect to the permutation of virtual photons and must vanish when both photons are on shell ($f_1 \to \gamma\gamma$ decay is forbidden by the Landau-Yang theorem [22]). We choose the $f_1 \to \gamma^*\gamma^*$ amplitude in the following form based on amplitudes used, e.g., in Refs. [8, 13, 14]:

$$M(f_1 \to \gamma^*\gamma^*) = \frac{\alpha}{m_f^2} F_1(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} q_2^\rho e_2^{*\sigma} \widetilde{e}^\tau (q_1 - q_2)_\tau +$$
$$+ \frac{\alpha}{m_f^2} \left\{ F_2(q_1^2, q_2^2) i\epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^{*\nu} \widetilde{e}^\rho \left[q_2^\sigma e_2^{*\lambda} q_{2\lambda} - e_2^{*\sigma} q_2^2 \right] + F_2(q_2^2, q_1^2) i\epsilon_{\mu\nu\rho\sigma} q_2^\mu e_2^{*\nu} \widetilde{e}^\rho \left[q_1^\sigma e_1^{*\lambda} q_{1\lambda} - e_1^{*\sigma} q_1^2 \right] \right\}, \tag{3}$$

where e_1, e_2, and $\widetilde{e}$ are the polarization vectors of the first photon, second photon, and f_1 meson, respectively.

Because of the Bose symmetry form factor $F_1(q_1^2, q_2^2)$ must be antisymmetric, $F_1(q_1^2, q_2^2) = -F_1(q_2^2, q_1^2)$. As it should be, the amplitude (3) vanishes when both photons are on shell. Indeed, the first term vanishes because of $F_1(0,0) = 0$, while all terms in the last line of (3) vanish because $q^2 = 0$ and $e^\lambda q_\lambda = 0$ for real photons.

We substitute this $f_1 \to \gamma^*\gamma^*$ amplitude into the expression for the one-loop diagram (see Fig. 1) and perform straightforward calculation in the Feynman gauge. This leads to the following expression for the $f_1 \to e^+e^-$ amplitude:

$$
\begin{aligned}
M(f_1 \to e^+e^-) = &-\frac{16\pi i\alpha^2}{m_f^2}\widetilde{e}^\mu P^\nu \bar{u}\gamma^\lambda\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu k_\lambda}{k^2 q_1^2 q_2^2} F_1(q_1^2, q_2^2) - \\
&- \frac{8\pi i\alpha^2}{m_f^2}\widetilde{e}^\mu \bar{u}\gamma^\nu\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{k^2 q_1^2 q_2^2}\left\{F_2(q_1^2, q_2^2) q_2^2 + F_2(q_2^2, q_1^2) q_1^2\right\} + \\
&+ \frac{4\pi i\alpha^2}{m_f^2}\widetilde{e}_\mu \bar{u}\gamma^\mu\gamma^5 v \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 q_1^2 q_2^2}\left\{F_2(q_1^2, q_2^2)\left[k^2(p_1p_2 + p_1k - p_2k) - 2q_2^2(p_1k) + 2q_2^2k^2\right] + \right.\\
&\left. + F_2(q_2^2, q_1^2)\left[k^2(p_1p_2 + p_1k - p_2k) + 2q_1^2(p_2k) + 2q_1^2k^2\right]\right\}, \quad (4)
\end{aligned}
$$

where $q_1 = p_1 - k$ and $q_2 = p_2 + k$.

4 Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

One cannot calculate the width $\Gamma(f_1 \to e^+e^-)$ in a model-independent way, because the explicit form of functions F_1 and F_2 in (4) is unknown. So, we have to choose some reasonable model. We assume that the main contribution to the amplitude $M(f_1 \to e^+e^-)$ comes from the diagram depicted in Fig. 2, where both virtual photons are coupled with the f_1 meson via intermediate ρ^0 mesons.

Experimental data show that one of the main f_1 decay channels, $f_1 \to 4\pi$ [$\mathcal{B}(f_1 \to 4\pi) \approx 33\%$], proceeds mainly via the intermediate $\rho\rho$ state [23]. Other evidence of this mechanism is a large (5.5%) branching ratio of radiative $f_1 \to \rho^0\gamma$ decay [21]. So, the assumption that $f_1\rho^0\rho^0$ coupling gives the main contribution to the amplitude $M(f_1 \to e^+e^-)$ looks quite reasonable.

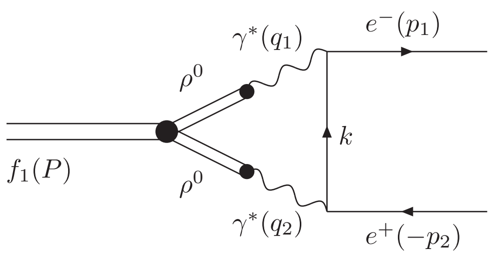


Figure 2. The VMD mechanism of the $f_1 \to e^+e^-$ decay with two intermediate ρ^0 mesons.

Some parameters of the model can be constrained from experimental data on $f_1 \to \rho^0\gamma$ decay. The corresponding amplitude can be obtained from (3), where all particles should be considered on shell. For $q_1^2 = m_\rho^2$ (here $m_\rho = 775.26$ MeV [21] is the ρ^0 mass), $q_2^2 = 0$, and $e_1 q_1 = e_2 q_2 = 0$ we obtain

$$M(f_1 \to \rho^0\gamma) = \frac{\alpha}{m_f^2} g_1 i\epsilon_{\mu\nu\rho\sigma} p^\mu \epsilon^{*\nu} q^\rho e^{*\sigma} \widetilde{e}^\tau (p-q)_\tau - \frac{\alpha m_\rho^2}{m_f^2} g_2 i\epsilon_{\mu\nu\rho\sigma} \widetilde{e}^\mu \epsilon^{*\nu} q^\rho e^{*\sigma}, \tag{5}$$

where e, ϵ, and $\widetilde{e}$ are polarization vectors of photon, ρ^0, and f_1, respectively; p and q are momenta of ρ^0 and photon. This amplitude contains two complex coupling constants, g_1 and g_2, because there are two different polarization states. The first state is when ρ^0 meson polarization is longitudinal (L) in the f_1 rest frame, and the second one is when ρ^0 meson polarization is transversal (T). In the expression (5) the coupling constant g_1 corresponds to the T polarization state of ρ^0, and g_2 corresponds to a combination of L and T polarization states.

Now it is straightforward to calculate the width of $f_1 \to \rho^0\gamma$ decay,

$$\Gamma(f_1 \to \rho^0\gamma) = \frac{\alpha^2}{96\pi} m_f (1-\xi)^3 \left[(1-\xi)^2 |g_1|^2 + \xi(1+\xi)|g_2|^2 + 2\xi(1-\xi)|g_1||g_2|\cos\delta\right], \tag{6}$$

where $\xi = m_\rho^2/m_f^2 \approx 0.37$. Since the parameters g_1 and g_2 do not correspond to different polarization states, the interference term does not vanish after summation over polarizations, and expression (6) contains $\delta = \phi_1 - \phi_2$, which is the relative phase of the complex constants g_1 and g_2.

The expression (6) represents one relation between three unknown parameters g_1, g_2, and δ. One more relation can be derived from the polarization experiments. The ratio of the contributions of two ρ^0 helicity states, $r = \rho_{LL}/\rho_{TT} = 3.9 \pm 0.9 \pm 1.0$, was determined in the VES experiment [24] from the analysis of angular distributions in the reaction $f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma$,

$$|M(f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma)|^2 \sim \rho_{LL}\cos^2\theta + \rho_{TT}\sin^2\theta, \tag{7}$$

where ρ_{LL} and ρ_{TT} are density matrix elements corresponding to longitudinal and transverse ρ^0 mesons, respectively; θ is the angle between π^+ and γ momenta in the ρ^0 rest frame.

Calculation of $|M(f_1 \to \rho^0\gamma \to \pi^+\pi^-\gamma)|^2$ with the amplitude (5) leads to the following ratio of the coefficients at $\cos^2\theta$ and $\sin^2\theta$:

$$r = \frac{2\xi|g_2|^2}{(1-\xi)^2|g_1|^2 + \xi^2|g_2|^2 + 2\xi(1-\xi)|g_1||g_2|\cos\delta}, \tag{8}$$

which equals to ρ_{LL}/ρ_{TT} from Ref. [24].

Recently CLAS Collaboration at Jefferson Laboratory (JLab) published the results of the first measurements of the f_1 meson in the photoproduction reaction $\gamma p \to f_1 p$ off a proton target [25]. The first estimate of the cross section of this reaction in the JLab kinematics and suggestion that its measurement with the CLAS detector is possible were reported in Ref. [26].

According to the data of CLAS Collaboration, the branching ratio $\mathcal{B}(f_1 \to \rho^0\gamma)$ equals $(2.5 \pm 0.9)\%$, which is substantially smaller than the PDG value, $(5.5 \pm 1.3)\%$. A few theoretical models consistent with this result are proposed already [27–29]. The total f_1 width measured by CLAS Collaboration $\Gamma_f^{CLAS} = (18.4 \pm 1.4)$ MeV is also considerably smaller than the PDG value, $\Gamma_f = (24.1 \pm 1.0)$ MeV. Therefore, below we present the results of our calculations for PDG averages and CLAS Collaboration values as well.

Using experimental result $r = 3.9 \pm 0.9 \pm 1.0$ [24] we find the magnitude of coupling constant g_2,

$$\alpha|g_2| = 1.5 \pm 0.2, \qquad \alpha|g_2|^{CLAS} = 0.87 \pm 0.18. \tag{9}$$

It is seen that values of this constant obtained for PDG and CLAS data are essentially different.

It is impossible to extract the magnitude of the constant g_1 and/or the phase δ from the experimental data. Taking into account that $-1 \leq \cos\delta \leq 1$, we obtain

$$0.16 \lesssim \alpha|g_1| \lesssim 1.9, \qquad 0.09 \lesssim \alpha|g_1|^{CLAS} \lesssim 1.1. \tag{10}$$

The upper and lower limits on $|g_1|$ correspond to $\delta = \pi$ and $\delta = 0$, respectively.

It is seen that there is a large uncertainty in the value of $|g_1|$. Indeed, $|g_1|$ could be of the same order of magnitude as $|g_2|$, if δ is close to π, and of the order of magnitude smaller, if δ is close to 0. Moreover, quite large experimental uncertainties in the polarization experiment [24] allow one to speculate that $|g_1|$ could be very small or even negligible. Indeed, in the case $g_1 = 0$ one obtains from (8) that $r = 2/\xi \approx 5.5$, which is not very far from the central value $r = 3.9$.

There are some papers concerning $f_1 \to \rho^0\gamma$ decay, where the amplitude $M(f_1 \to \rho^0\gamma)$ is parametrized only by one constant [26, 28, 29]. These models correspond to our model at $g_1 = 0$.

There are also papers where the amplitude $M(f_1 \to \rho^0\gamma)$ is parametrized by two constants; see, e.g., Ref. [30]. Therein two relations between parameters of $f_1 \to \rho^0\gamma$ decay are obtained using $\Gamma(f_1 \to \rho^0\gamma)$ and $r = \rho_{LL}/\rho_{TT}$. In this paper the f_1 meson is considered as the molecular state and $f_1 \to \rho^0\gamma$ decay is studied in the chiral effective field theory.

5 Calculation of $f_1 \to e^+e^-$ decay width

Now let us consider $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay. Experimental data indicate that the main contribution to it is given by the intermediate state with two virtual ρ mesons [23] (see Fig. 3). The vertex $f_1\rho\rho$ contains the form factors of our model. Certainly, these form factors should meet the requirements that the result of the calculation of the $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay width should be in a good agreement with the experimental value.

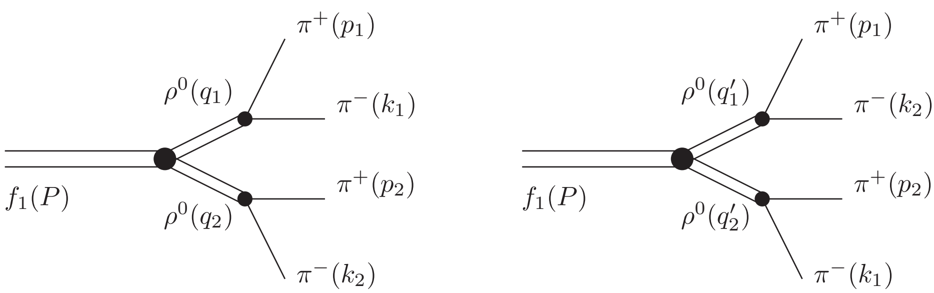


Figure 3. Feynman diagrams of $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay.

We parametrize the $\rho^0 \to \pi^+\pi^-$ amplitude as

$$M(\rho^0 \to \pi^+\pi^-) = i f_{\rho\pi\pi} e_\mu p^\mu, \tag{11}$$

where e_μ is the polarization vector of the ρ^0 meson, and p^μ is the momentum of the π^+ meson. We neglect the q^2 dependence of $f_{\rho\pi\pi}$ and obtain the following value:

$$f_{\rho\pi\pi} = \left(\frac{192\pi\Gamma(\rho^0 \to \pi^+\pi^-)}{m_\rho(1 - 4m_\pi^2/m_\rho^2)^{3/2}}\right)^{1/2} \approx 11.9, \tag{12}$$

where $m_\rho = 775.26$ MeV, $\Gamma(\rho^0 \to \pi^+\pi^-) \approx \Gamma_\rho = 147.8$ MeV, and $m_\pi = 139.57$ MeV is the mass of $\pi^\pm$ mesons.

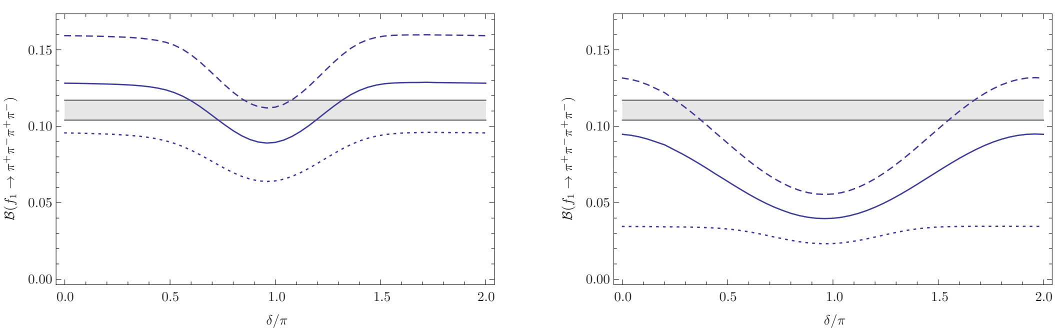
Taking into account that form factor $F_1(q_1^2, q_2^2)$ is antisymmetric, and all other requirements, we write the form factors F_1 and F_2 as

$$F_1(q_1^2, q_2^2) = \frac{g_{\rho\gamma} g_1 (m_\rho^2 - i m_\rho \Gamma_\rho)(q_2^2 - q_1^2)}{(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(q_2^2 - m_\rho^2 + i m_\rho \Gamma_\rho)}, \tag{13}$$

$$F_2(q_1^2, q_2^2) = \frac{g_{\rho\gamma} g_2 (m_\rho^2 - i m_\rho \Gamma_\rho)(-m_\rho^2)}{(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(q_2^2 - m_\rho^2 + i m_\rho \Gamma_\rho)}, \tag{14}$$

where $\Gamma_\rho = 147.8$ MeV are ρ^0-meson width, $g_{\rho\gamma}$ is the coupling constant of the transition $\rho^0 \to \gamma^*$, $M(\rho^0 \to \gamma^*) = g_{\rho\gamma}(q^2 g_{\mu\nu} - q_\mu q_\nu)\epsilon^\mu e^{*\nu}$, so $g_{\rho\gamma} = \sqrt{3\Gamma(\rho^0 \to e^+e^-)/\alpha m_\rho} \approx 0.06$.

The result of numerical calculation is shown in Fig. 4.



PDG data CLAS Collaboration data

Figure 4. The branching ratio of the $f_1 \to \pi^+\pi^-\pi^+\pi^-$ decay for the certain choice of the form factors $F_1(q_1^2, q_2^2)$ and $F_2(q_1^2, q_2^2)$; see Eqs. (13) and (14). The solid line corresponds to the $f_1 \to \pi^+\pi^-\pi^+\pi^-$ branching ratio calculated using the central values: $r = 3.9$ and $\mathcal{B}(f_1 \to \rho^0\gamma) = 5.5\%$ or $\mathcal{B}^{CLAS}(f_1 \to \rho^0\gamma) = 2.5\%$. Dashed and dotted lines indicate 1σ deviations for $\mathcal{B}(f_1 \to \pi^+\pi^-\pi^+\pi^-)$. The shaded horizontal band denotes the value allowed experimentally, $\mathcal{B}(f_1 \to \pi^+\pi^-\pi^+\pi^-) = \left(11.0^{+0.7}_{-0.6}\right)\%$.

It is seen from Fig. 4 that we still cannot derive the exact value of the phase δ in our model because of large uncertainties of the model parameters. Therefore, in what follows we treat δ as a free parameter.

To calculate the $f_1 \to e^+e^-$ branching ratio, we substitute the expressions for $F_1(q_1^2, q_2^2)$ and $F_2(q_1^2, q_2^2)$ into (4) and perform the numerical calculations; then, comparing the answer with (1), we obtain the following result for the constant F_A:

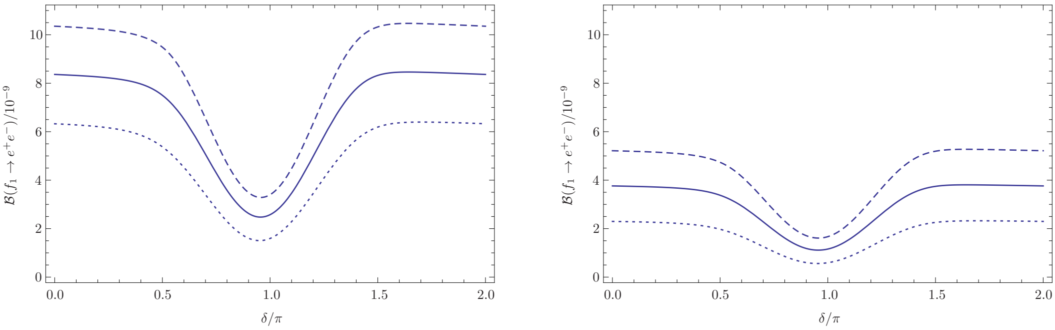
$$F_A \simeq -\alpha g_1 (0.22 + 0.25i) - \alpha g_2 (0.75 + 0.57i)\,. \tag{15}$$

It is convenient to express complex numbers g_1 and g_2 in polar form as $g_1 = |g_1| \cdot e^{i\phi_1}$ and $g_2 = |g_2| \cdot e^{i\phi_2}$, respectively. Then using $\delta = \phi_1 - \phi_2$ one can write the absolute square of the constant F_A as

$$|F_A|^2 \simeq \left| e^{i\delta} \cdot \alpha |g_1| \cdot (0.22 + 0.25i) + \alpha |g_2| \cdot (0.75 + 0.57i) \right|^2. \tag{16}$$

Since $\alpha|g_2| \sim 1$ and $\alpha|g_1| \lesssim 1$ [see (9) and (10)], then $|F_A| \sim 1$, as expected. In particular, for central values of $|g_1|$ and $|g_2|$ we get $|F_A| \simeq 1.12$ for $\delta = 0.7\pi$, $|F_A| \simeq 1.28$ for $\delta = 1.3\pi$, and $|F_A|^{CLAS} \simeq 0.85$ for $\delta = 0$.

Now it is straightforward to calculate the branching ratio $\mathcal{B}(f_1 \to e^+e^-)$ as a function of δ. Corresponding plots are shown in Fig. 5.



PDG data CLAS Collaboration data

Figure 5. The branching ratio $\mathcal{B}(f_1 \to e^+e^-)$ as a function of the relative phase δ in our model. The solid line corresponds to $\mathcal{B}(f_1 \to e^+e^-)$ calculated for the central values: $r = 3.9$ and $\mathcal{B}(f_1 \to \rho^0\gamma) = 5.5\%$ or $\mathcal{B}^{CLAS}(f_1 \to \rho^0\gamma) = 2.5\%$. The dashed and dotted lines indicate 1σ deviations from the $\mathcal{B}(f_1 \to e^+e^-)$ central value.

It is seen from Figs. 4 and 5 that in our model the branching ratio $\mathcal{B}(f_1 \to e^+e^-)$ should be taken in the range from $3 \cdot 10^{-9}$ for $\delta \simeq \pi$ to $8 \cdot 10^{-9}$ for $\delta = 0$, and from $4 \cdot 10^{-9}$ for $\delta^{CLAS} = 0$ to $5 \cdot 10^{-9}$ for $\delta^{CLAS} \simeq \pm 0.3\pi$,

$$\mathcal{B}(f_1 \to e^+e^-) \simeq (3-8) \cdot 10^{-9}, \qquad \mathcal{B}^{CLAS}(f_1 \to e^+e^-) \simeq (4-5) \cdot 10^{-9}, \tag{17}$$

and the corresponding decay width is

$$\Gamma(f_1 \to e^+e^-) \simeq 0.07 - 0.19 \text{ eV}, \qquad \Gamma^{CLAS}(f_1 \to e^+e^-) \simeq 0.07 - 0.10 \text{ eV}. \tag{18}$$

The values of the branching ratio and the decay width obtained for CLAS data lie in a more narrow interval than the corresponding values obtained for PDG data. However, both ranges of $\Gamma(f_1 \to e^+e^-)$ values are in good agreement with the naive estimate $\Gamma \sim 0.1$ eV (see the end of Sec. 2).

6 Estimate of $e^+e^- \to f_1 \to \eta\pi\pi$ cross section

Let us estimate the cross section of the process $e^+e^- \to f_1 \to \eta\pi\pi$, which can be used for the study of direct f_1 production in e^+e^- collisions. Here, the $f_1 \to \eta\pi\pi$ decay proceeds mainly (approximately with 70% probability [21]) through the intermediate $a_0(980)$ meson; see Figs. 6 and 7.

The cross section of the $e^+e^- \to f_1 \to a_0\pi$ process can easily be calculated,

$$\sigma(e^+e^- \to f_1 \to a_0\pi) = \frac{12\pi}{m_f^2}\mathcal{B}(f_1 \to a_0\pi)\mathcal{B}(f_1 \to e^+e^-), \tag{19}$$

where the center-of-mass energy equals the mass of the f_1 meson, $\sqrt{s} = m_f$. Using the experimental value for the branching ratio $\mathcal{B}(f_1 \to a_0\pi) = 0.36 \pm 0.07$ and the result of our calculations (17) for

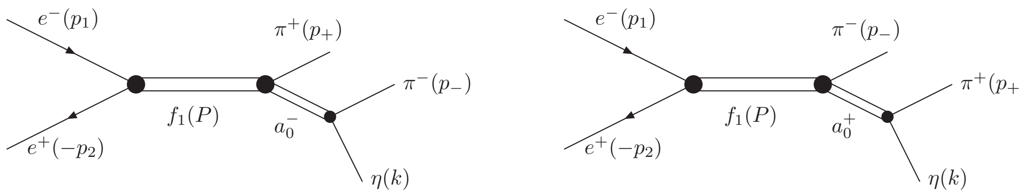


Figure 6. The diagrams for e^+e^- annihilation into the $\eta\pi^+\pi^-$ final state via the intermediate f_1 and a_0 mesons.

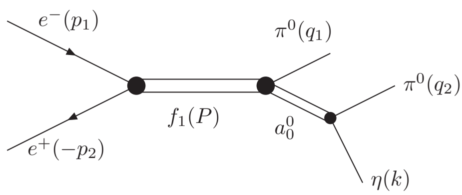


Figure 7. The diagram for e^+e^- annihilation into the $\eta\pi^0\pi^0$ final state via the intermediate f_1 and a_0 mesons.

$\mathcal{B}(f_1 \to e^+e^-)$, we obtain

$$\sigma(e^+e^- \to f_1 \to a_0\pi) \simeq 7.8 - 30 \text{ pb}, \qquad \sigma^{CLAS}(e^+e^- \to f_1 \to a_0\pi) \simeq 10 - 20 \text{ pb}. \tag{20}$$

Assuming that the a_0 meson decays only into the $\eta\pi$ final state and using the isospin symmetry $\mathcal{B}(f_1 \to a_0^\pm\pi^\mp \to \eta\pi^+\pi^-) = 2\,\mathcal{B}(f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) = 2/3\,\mathcal{B}(f_1 \to a_0\pi \to \eta\pi\pi)$, we obtain the following estimates:

$$\sigma(e^+e^- \to f_1 \to a_0^\pm\pi^\mp \to \eta\pi^+\pi^-) \simeq 5.2\text{–}20 \text{ pb}, \quad \sigma^{CLAS}(e^+e^- \to f_1 \to a_0^\pm\pi^\mp \to \eta\pi^+\pi^-) \simeq 7\text{–}13.3 \text{ pb}, \tag{21}$$

$$\sigma(e^+e^- \to f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) \simeq 2.6 - 10 \text{ pb}, \quad \sigma^{CLAS}(e^+e^- \to f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) \simeq 3.5 - 6.7 \text{ pb}. \tag{22}$$

It is seen that the values of cross sections obtained for PDG and CLAS data are in reasonable agreement within uncertainties. However, the values of σ^{CLAS} lie in a narrower range. Therefore, one can hope that future precise experiments could make it possible to distinguish between σ^{PDG} and σ^{CLAS}.

7 Charge asymmetry in $e^+e^- \to \eta\pi^+\pi^-$ process

Though the cross section of the $e^+e^- \to f_1 \to \eta\pi^0\pi^0$ process is twice less than that of $e^+e^- \to f_1 \to \eta\pi^+\pi^-$, the former is more convenient for the study of direct f_1 production in e^+e^- collisions. Indeed, the $e^+e^- \to \eta\pi^0\pi^0$ reaction proceeds only through two-photon annihilation, since C parity of the $\eta\pi^0\pi^0$ final state is positive. Therefore, there is no background from one-photon annihilation, and the $e^+e^- \to f_1 \to \eta\pi^0\pi^0$ cross section can be measured directly. According to the estimate (22), the lower bound on this cross section is quite small, but it can be measured in a special experiment at the VEPP-2000 collider in Novosibirsk.

In contrast, the $e^+e^- \to \eta\pi^+\pi^-$ reaction proceeds mainly through one-photon annihilation, which is described quite well by the VMD model with intermediate $\rho'(1450)$ and $\rho^0(770)$ mesons [31], as depicted in Fig. 8. The annihilation $e^+e^- \to \rho \to \eta\pi^+\pi^-$ was studied theoretically in Ref. [32].

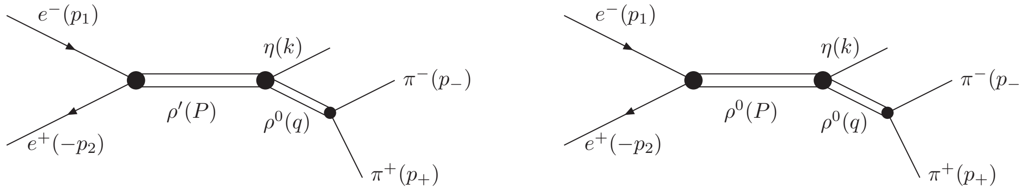


Figure 8. The diagrams for e^+e^- annihilation into the $\eta\pi^+\pi^-$ final state via intermediate vector $\rho'(1450)$ and $\rho^0(770)$ mesons.

The measured $e^+e^- \to \eta\pi^+\pi^-$ Born cross section is about 500 pb at $\sqrt{s} = m_f$ [31]. According to the estimate (21), the $e^+e^- \to f_1 \to a_0^\pm\pi^\mp \to \eta\pi^+\pi^-$ cross section constitutes only several percent of the total $e^+e^- \to \eta\pi^+\pi^-$ cross section, and its measurement is a rather complicated task. One possibility to overcome this difficulty is to investigate the two-photon annihilation channel $e^+e^- \to f_1 \to \eta\pi^+\pi^-$ through C-odd effects, which arise from the interference of C-odd one-photon and C-even two-photon amplitudes.

This interference is P- and C-odd, therefore it does not contribute to the total cross section, but it can lead to the *charge asymmetry* in the differential cross section. Indeed, calculation shows that after integration over azimuthal angle ϕ_π the interference term is an odd function of $\cos\theta_\eta$ and $\cos\theta_\pi$. Here θ_η is the angle between η meson 3-momentum and e^+ beam axis in the e^+e^- center-of-mass frame, and θ_π is the angle between π^+ meson and η meson 3-momenta in the $\pi^+\pi^-$ center-of-mass system. Therefore, if we consider events with θ_η in a definite interval $d\cos\theta_\eta$, then the interference term has opposite signs for $\cos\theta_\pi$ and $\cos(\pi - \theta_\pi) = -\cos\theta_\pi$. Physically it means that the number of π^+ mesons propagating in some direction θ_π differs from the number of π^- mesons propagating in the same direction.

Let us define the charge asymmetry in the $e^+e^- \to \eta\pi^+\pi^-$ process as

$$A = \left.\frac{\sigma_{tot}(\cos\theta_\pi > 0) - \sigma_{tot}(\cos\theta_\pi < 0)}{\sigma_{tot}(\cos\theta_\pi > 0) + \sigma_{tot}(\cos\theta_\pi < 0)}\right|_{\cos\theta_\eta > 0}, \tag{23}$$

where $\sigma_{tot} = \sigma_1 + \sigma_2 + \sigma_{int}$ is the total cross section. Condition $\cos\theta_\eta > 0$ is chosen here quite arbitrarily, so for real experiment one can redefine asymmetry in another θ_η range based on experimental conditions.

The interference term contains one additional free parameter ϕ, which is the relative phase arising from the complex coupling constants,

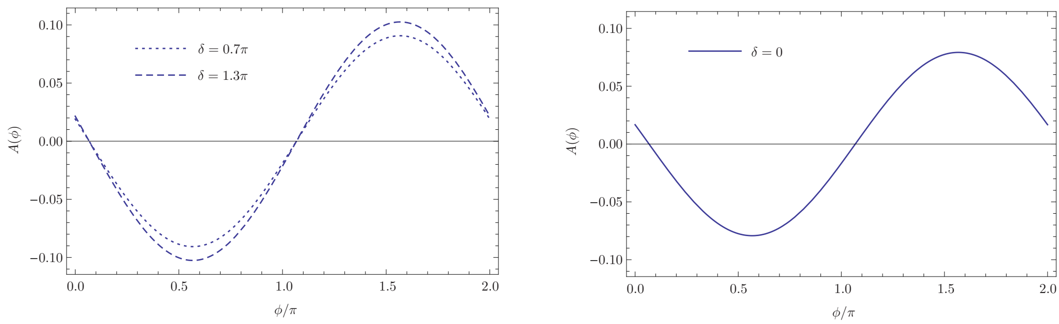
$$F_A g_{f\pi a} g_{a\pi\eta} f^*_{\rho\pi\pi} = |F_A g_{f\pi a} g_{a\pi\eta} f_{\rho\pi\pi}| e^{i\phi}. \tag{24}$$

Using the values $\phi_{\rho^0(770)} = 0$, $\phi_{\rho'(1450)} = \pi$ [31] we perform numerical calculations of the charge asymmetry (23) for $\delta = 0.7\pi$, $\delta = 1.3\pi$, and $\delta^{CLAS} = 0$. The dependence of the charge asymmetry A on the relative phase ϕ is shown in Fig. 9.

It is seen that the charge asymmetry in the $e^+e^- \to \eta\pi^+\pi^-$ process may be quite large, up to $\pm 10\%$ for $\phi \simeq \mp\pi/2$.

8 Conclusion

We calculate the width of the $f_1(1285) \to e^+e^-$ decay in the vector meson dominance model, where both virtual photons are coupled with the f_1 meson via the intermediate ρ^0 mesons; see Fig. 2.



PDG data CLAS Collaboration data

Figure 9. The charge asymmetry A as a function of the relative phase ϕ for different values of the phase δ.

We assume that this is the main mechanism of the decay, and such an assumption is based on the experimental data on $f_1 \to 4\pi$ and $f_1 \to \rho^0\gamma$ decays [21, 23]. In our model the decay width, $\Gamma(f_1 \to e^+e^-)$, depends on the relative phase δ between two coupling constants describing the $f_1 \to \rho^0\gamma$ decay. This phase is not fixed unambiguously from the experimental data. Therefore, the width can only be estimated as $\Gamma(f_1 \to e^+e^-) \simeq 0.07 - 0.19$ eV using the PDG data [21], and as $\Gamma^{CLAS}(f_1 \to e^+e^-) \simeq 0.07 - 0.10$ eV using the CLAS Collaboration data [25]. The corresponding branching ratio is $\mathcal{B}(f_1 \to e^+e^-) \simeq (3-8)\cdot 10^{-9}$ and $\mathcal{B}^{CLAS}(f_1 \to e^+e^-) \simeq (4-5)\cdot 10^{-9}$.

The process of direct f_1 production in e^+e^- collisions, $e^+e^- \to f_1 \to mesons$, is still not measured due to smallness of the corresponding cross section. Now it can be studied at modern high-luminosity colliders, e.g., at VEPP-2000 in Novosibirsk. We estimate the $e^+e^- \to f_1 \to \eta\pi\pi$ cross section and find it to be $\sigma(e^+e^- \to f_1 \to a_0^{\pm}\pi^{\mp} \to \eta\pi^+\pi^-) \simeq 5.2 - 20$ pb ($\sigma^{CLAS} \simeq 7 - 13.3$ pb) for the $\eta\pi^+\pi^-$ final state, and $\sigma(e^+e^- \to f_1 \to a_0^0\pi^0 \to \eta\pi^0\pi^0) \simeq 2.6 - 10$ pb ($\sigma^{CLAS} \simeq 3.5 - 6.7$ pb) for the $\eta\pi^0\pi^0$ final state. The latter process, $e^+e^- \to f_1 \to \eta\pi^0\pi^0$, is more convenient to study, because the $e^+e^- \to \eta\pi^0\pi^0$ reaction proceeds only through two-photon annihilation. Therefore, there is no background from one-photon annihilation, and the $e^+e^- \to f_1 \to \eta\pi^0\pi^0$ cross section can be measured directly. In our model the lower bound on this cross section is quite small, ~ 3 pb. However, even such a small cross section can be measured in a special experiment at the VEPP-2000 e^+e^- collider in Novosibirsk.

In contrast, the reaction $e^+e^- \to \eta\pi^+\pi^-$ proceeds mainly through one-photon annihilation. Therefore, measurement of the cross section of the two-photon channel, $e^+e^- \to f_1 \to \eta\pi^+\pi^-$, is a rather complicated task, because of the background from one-photon annihilation. One possibility to overcome this difficulty is to investigate the charge asymmetry which arises from the interference of C-odd one-photon and C-even two-photon amplitudes. We calculate this asymmetry in the $e^+e^- \to \eta\pi^+\pi^-$ reaction for some values of parameters in our model. It turns out that the magnitude of the charge asymmetry is quite uncertain. It depends on the relative phase ϕ and may be quite large, up to $\pm 10\%$.

We hope that in the nearest future our predictions will be tested in precise experiments at e^+e^- colliders. Such experiments could allow us to obtain values of free parameters of our model, δ and ϕ, as well as to define more accurately $\mathcal{B}(f_1 \to \rho^0\gamma)$, Γ_f, and r, measured by now with quite large uncertainties.

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