$f_1(1285) \rightarrow e^+e^-$ decay and direct $f_1$ production in $e^+e^-$ collisions

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Abstract. The width of the $f_1(1285) \rightarrow e^+e^-$ decay is calculated in the vector meson dominance model. The result depends on the relative phase between two coupling constants describing $f_1 \rightarrow \rho^0\gamma$ decay. The width $\Gamma(f_1 \rightarrow e^+e^-)$ is estimated to be $\approx 0.07 - 0.19$ eV. Direct $f_1$ production in $e^+e^-$ collisions is discussed, and the $e^+e^- \rightarrow f_1 \rightarrow a_0\pi \rightarrow \eta\pi\pi$ cross section is calculated. Charge asymmetry in the $e^+e^- \rightarrow \eta\pi^+\pi^-$ reaction due to interference between $e^+e^- \rightarrow f_1$ and $e^+e^- \rightarrow \eta\rho$ amplitudes is studied.

1 Introduction

High-luminosity electron-positron colliders are powerful tools for measuring electronic widths of hadronic resonances with positive charge parity, $C = +1$. The idea of such measurements was put forward many years ago [1, 2]. Several experiments in search of direct production of $C$-even resonances in $e^+e^-$ collisions were performed, and very low upper limits on the leptonic widths of $\eta'$, $f_2(1270)$, $a_2(1320)$, and $X(3872)$ mesons were set [3–6]: $\Gamma(\eta'(958) \rightarrow e^+e^-) < 0.002$ eV (90% CL), $\Gamma(f_2(1270) \rightarrow e^+e^-) < 0.11$ eV (90% CL), $\Gamma(a_2(1320) \rightarrow e^+e^-) < 0.56$ eV (90% CL). The explanation of the smallness of the leptonic widths of $C$-even resonances is that corresponding decays proceed via two virtual photons and therefore are suppressed by a factor of $\alpha^4$, where $\alpha$ is the fine structure constant.

In this paper we consider $1^{++}$ meson $f_1(1285)$, its decay into the $e^+e^-$ pair, and its direct production in $e^+e^-$ collisions. The process $e^+e^- \rightarrow f_1 \rightarrow mesons$ is still not measured and may be studied at the VEPP-2000 $e^+e^-$ collider in experiments with the SND and CMD-3 detectors.

There is a quite extensive list of literature on the production of $1^{++}$ resonances in $e^+e^-$ annihilation. The direct production of $1^{++}$ states through the neutral current was evaluated many years ago in the nonrelativistic quarkonium model [7]. The calculation of the width $\Gamma(\chi_{c1} \rightarrow e^+e^-)$ was performed in the quarkonium and vector meson dominance models (VMD) [8]. There are also some recent papers devoted to $X(3872)$ and $\chi_{c1}$ decays into the $e^+e^-$ pair and their production in $e^+e^-$ collisions (see [9–12] and references therein). The production of $1^{++}$ resonances $R$ in two-photon collisions ($e^+e^- \rightarrow e^+e^-R$) was also extensively studied both theoretically [13–16] and experimentally [17–19].

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2 Simple estimate of $f_1 \rightarrow e^+e^-$ decay width

The electron and positron produced in the $f_1 \rightarrow e^+e^-$ decay are ultrarelativistic in the $f_1$ rest frame. So, in this frame $e^+$ and $e^-$ can be considered as massless and having the definite helicities. In Ref. [22] the width of the $\chi_c \rightarrow \gamma\gamma^*$ decay was calculated with the finite lepton mass, and it was found that the mass effects are negligible.

There is only one $P$- and $C$-even invariant amplitude for the $f_1 \rightarrow e^+e^-$ decay, which is written as

$$M(f_1 \rightarrow e^+e^-) = F_A \alpha^2 \bar{e}_\mu \gamma^\mu \gamma^5 v,$$

where $\bar{e}_\mu$ is the $C$-even axial vector describing the $f_1$ meson, $\bar{u}\gamma^\mu \gamma^5 v = f_A$ is the axial current, and $F_A$ is the dimensionless coupling constant. Since $f_1$ meson is $C$-even, it decays into $e^+e^-$ via two virtual photons as depicted in Fig. 1. This explains the origin of the factor $\alpha^2$ in (1).

![Figure 1. One-loop diagram of the $f_1 \rightarrow e^+e^-$ decay with two intermediate photons.](image)

Using the amplitude (1) it is easy to calculate the decay width

$$\Gamma(f_1 \rightarrow e^+e^-) = \frac{\alpha^4 |F_A|^2}{12\pi} m_f,$$

where $m_f$ is the $f_1$ mass, $m_f = 1282.0$ MeV [21].

For a naive estimate, it is natural to assume that the coupling constant $F_A$ is of the order of unity, $|F_A| \sim 1$. In (1) we have already written explicitly the small factor $\alpha^2$, and there are not any additional small factors. So, we obtain that $\Gamma(f_1 \rightarrow e^+e^-) \sim 0.1$ eV. In what follows we calculate this width in a certain model and find that this simple estimate is correct by the order of magnitude.

3 Model-independent description of $f_1 \rightarrow e^+e^-$ amplitude

To calculate the width $\Gamma(f_1 \rightarrow e^+e^-)$ more accurately, we should know the amplitude of the $f_1 \rightarrow \gamma^*\gamma^*$ transition (see Fig. 1). This amplitude must be symmetric with respect to the permutation of virtual photons and must vanish when both photons are on shell ($f_1 \rightarrow \gamma\gamma$ decay is forbidden by the Landau-Yang theorem [22]). We choose the $f_1 \rightarrow \gamma^*\gamma^*$ amplitude in the following form based on amplitudes used, e.g., in Refs. [8, 13, 14]:

$$M(f_1 \rightarrow \gamma^*\gamma^*) = \frac{\alpha}{m_f^2} F_1(q_1^2, q_2^2) i \epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^\nu q_2^\rho e_2^\sigma \epsilon(q_1 - q_2)_\nu +$$

$$+ \frac{\alpha}{m_f^2} \left[ F_2(q_1^2, q_2^2) i \epsilon_{\mu\nu\rho\sigma} q_1^\mu e_1^\nu \tilde{q}_{1\lambda}^\rho e_{2\sigma}^\sigma \tilde{q}_{2\lambda}^\nu \right] + F_2(q_1^2, q_2^2) i \epsilon_{\mu\nu\rho\sigma} q_2^\mu e_2^\nu \tilde{q}_{1\lambda}^\rho e_{2\sigma}^\sigma \tilde{q}_{1\lambda}^\nu \left[ q_1^\nu e_1^\lambda q_1^\lambda - e_1^\nu q_1^\lambda \right],$$

(3)
where $e_1$, $e_2$, and $\vec{e}$ are the polarization vectors of the first photon, second photon, and $f_1$ meson, respectively.

Because of the Bose symmetry form factor $F_1(q_1^2, q_2^2)$ must be antisymmetric, $F_1(q_1^2, q_2^2) = -F_1(q_2^2, q_1^2)$. As it should be, the amplitude (3) vanishes when both photons are on shell. Indeed, the first term vanishes because of $F_1(0, 0) = 0$, while all terms in the last line of (3) vanish because $q_2^2 = 0$ and $e^4 q_2 = 0$ for real photons.

We substitute this $f_1 \to \gamma^*\gamma^*$ amplitude into the expression for the one-loop diagram (see Fig. 1) and perform straightforward calculation in the Feynman gauge. This leads to the following expression for the $f_1 \to e^+e^-$ amplitude:

$$M(f_1 \to e^+e^-) = -\frac{16\pi\alpha^2}{m_f^2} \tilde{e}^\mu \gamma^\nu \gamma^5 \gamma^5 \int \frac{d^4k}{(2\pi)^4} \frac{k_{\mu}k_{\nu}k_{\lambda}}{k^2 q_1^2 q_2^2} F_1(q_1^2, q_2^2) -$$

$$- \frac{8\pi\alpha^2}{m_f^2} \tilde{e}^\mu \gamma^\nu \gamma^5 \int \frac{d^4k}{(2\pi)^4} \frac{k_{\mu}k_{\nu}}{k^2 q_1^2 q_2^2} \left[ F_2(q_1^2, q_2^2)q_1^2 + F_2(q_2^2, q_1^2)q_1^2 \right] +$$

$$+ \frac{4\pi\alpha^2}{m_f^2} \tilde{e}^\mu \gamma^\nu \gamma^5 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 q_1^2 q_2^2} \left[ F_2(q_1^2, q_2^2) \left[ k^2(p_1 p_2 + p_1 k - p_2 k) - 2q_2^2(p_1 k) + 2q_2^2 k^2 \right] + \right.$$

$$\left. + F_2(q_2^2, q_1^2) \left[ k^2(p_1 p_2 + p_1 k - p_2 k) + 2q_1^2(p_2 k) + 2q_1^2 k^2 \right] \right], \quad (4)$$

where $q_1 = p_1 - k$ and $q_2 = p_2 + k$.

### 4 Constants of $f_1 \to \rho^0\gamma$ decay from experimental data

One cannot calculate the width $\Gamma(f_1 \to e^+e^-)$ in a model-independent way, because the explicit form of functions $F_1$ and $F_2$ in (4) is unknown. So, we have to choose some reasonable model. We assume that the main contribution to the amplitude $M(f_1 \to e^+e^-)$ comes from the diagram depicted in Fig. 2, where both virtual photons are coupled with the $f_1$ meson via intermediate $\rho^0$ mesons.

Experimental data show that one of the main $f_1$ decay channels, $f_1 \to 4\pi [B(f_1 \to 4\pi) \approx 33\%]$, proceeds mainly via the intermediate $\rho\rho$ state [23]. Other evidence of this mechanism is a large (5.5%) branching ratio of radiative $f_1 \to \rho^0\gamma$ decay [21]. So, the assumption that $f_1\rho^0\rho^0$ coupling gives the main contribution to the amplitude $M(f_1 \to e^+e^-)$ looks quite reasonable.

![Figure 2](image-url)  
**Figure 2.** The VMD mechanism of the $f_1 \to e^+e^-$ decay with two intermediate $\rho^0$ mesons.

Some parameters of the model can be constrained from experimental data on $f_1 \to \rho^0\gamma$ decay. The corresponding amplitude can be obtained from (3), where all particles should be considered on
shell. For \( q_1^2 = m_p^2 \) (here \( m_p = 775.26 \) MeV [21] is the \( \rho^0 \) mass), \( q_2^2 = 0 \), and \( e_1 q_1 = e_2 q_2 = 0 \) we obtain

\[
M(f_1 \rightarrow \rho^0 \gamma) = \frac{\alpha^2}{m_f^2} g_1 i \epsilon_{\mu \nu \rho \sigma} p^\mu e^{\nu} q^\rho e^{\sigma}(p - q)_\tau - \frac{\alpha m_\rho^2}{m_f^2} g_2 i \epsilon_{\mu \nu \rho \sigma} e^{\nu} q^\rho e^{\sigma},
\]

(5)

where \( e, \epsilon, \) and \( \bar{\epsilon} \) are polarization vectors of photon, \( \rho^0 \), and \( f_1 \), respectively; \( p \) and \( q \) are momenta of \( \rho^0 \) and photon. This amplitude contains two complex coupling constants, \( g_1 \) and \( g_2 \), because there are two different polarization states. The first state is when \( \rho^0 \) meson polarization is longitudinal \((L)\) in the \( f_1 \) rest frame, and the second one is when \( \rho^0 \) meson polarization is transversal \((T)\). In the expression (5) the coupling constant \( g_1 \) corresponds to the \( T \) polarization state of \( \rho^0 \), and \( g_2 \) corresponds to a combination of \( L \) and \( T \) polarization states.

Now it is straightforward to calculate the width of \( f_1 \rightarrow \rho^0 \gamma \) decay,

\[
\Gamma(f_1 \rightarrow \rho^0 \gamma) = \frac{\alpha^2}{96\pi} m_f (1 - \xi)^3 \left[ (1 - \xi^2) |g_1|^2 + \xi (1 + \xi) |g_2|^2 + 2 \xi (1 - \xi) |g_1||g_2| \cos \delta \right],
\]

(6)

where \( \xi = m_p^2 / m_f^2 \approx 0.37 \). Since the parameters \( g_1 \) and \( g_2 \) do not correspond to different polarization states, the interference term does not vanish after summation over polarizations, and expression (6) contains \( \delta = \phi_1 - \phi_2 \), which is the relative phase of the complex constants \( g_1 \) and \( g_2 \).

The expression (6) represents one relation between three unknown parameters \( g_1 \), \( g_2 \), and \( \delta \). One more relation can be derived from the polarization experiments. The ratio of the contributions of two \( \rho^0 \) helicity states, \( r = \rho_{LL}/\rho_{TT} = 3.9 \pm 0.9 \pm 1.0 \), was determined in the VES experiment [24] from the analysis of angular distributions in the reaction \( f_1 \rightarrow \rho^0 \gamma \rightarrow \pi^0 \pi^- \gamma \),

\[
|M(f_1 \rightarrow \rho^0 \gamma \rightarrow \pi^0 \pi^- \gamma)|^2 \sim \rho_{LL} \cos^2 \theta + \rho_{TT} \sin^2 \theta,
\]

(7)

where \( \rho_{LL} \) and \( \rho_{TT} \) are density matrix elements corresponding to longitudinal and transverse \( \rho^0 \) mesons, respectively; \( \theta \) is the angle between \( \pi^+ \) and \( \gamma \) momenta in the \( \rho^0 \) rest frame.

Calculation of \( |M(f_1 \rightarrow \rho^0 \gamma \rightarrow \pi^0 \pi^- \gamma)|^2 \) with the amplitude (5) leads to the following ratio of the coefficients at \( \cos^2 \theta \) and \( \sin^2 \theta \):

\[
r = \frac{2 \xi |g_2|^2}{(1 - \xi^2) |g_1|^2 + \xi^2 |g_2|^2 + 2 \xi (1 - \xi) |g_1||g_2| \cos \delta},
\]

(8)

which equals to \( \rho_{LL}/\rho_{TT} \) from Ref. [24].

Recently CLAS Collaboration at Jefferson Laboratory (JLab) published the results of the first measurements of the \( f_1 \) meson in the photoproduction reaction \( \gamma p \rightarrow f_1 p \) off a proton target [25]. The first estimate of the cross section of this reaction in the JLab kinematics and suggestion that its measurement with the CLAS detector is possible were reported in Ref. [26].

According to the data of CLAS Collaboration, the branching ratio \( \mathcal{B}(f_1 \rightarrow \rho^0 \gamma) \) equals \( (2.5 \pm 0.9)\% \), which is substantially smaller than the PDG value, \( (5.5 \pm 1.3)\% \). A few theoretical models consistent with this result are proposed already [27–29]. The total \( f_1 \) width measured by CLAS Collaboration \( \Gamma_{CLAS} = (18.4 \pm 1.4) \) MeV is also considerably smaller than the PDG value, \( \Gamma_f = (24.1 \pm 1.0) \) MeV. Therefore, below we present the results of our calculations for PDG averages and CLAS Collaboration values as well.

Using experimental result \( r = 3.9 \pm 0.9 \pm 1.0 \) [24] we find the magnitude of coupling constant \( g_2 \),

\[
|g_2| = 1.5 \pm 0.2, \quad |g_2|^{CLAS} = 0.87 \pm 0.18.
\]

(9)

It is seen that values of this constant obtained for PDG and CLAS data are essentially different.
It is impossible to extract the magnitude of the constant \( g_1 \) and/or the phase \( \delta \) from the experimental data. Taking into account that \(-1 \leq \cos \delta \leq 1\), we obtain

\[
0.16 \leq \alpha |g_1| \leq 1.9, \quad 0.09 \leq \alpha |g_1|^{\text{CLAS}} \leq 1.1. \tag{10}
\]

The upper and lower limits on \( |g_1| \) correspond to \( \delta = \pi \) and \( \delta = 0 \), respectively.

It is seen that there is a large uncertainty in the value of \( |g_1| \). Indeed, \( |g_1| \) could be of the same order of magnitude as \( |g_2| \), if \( \delta \) is close to \( \pi \), and of the order of magnitude smaller, if \( \delta \) is close to 0. Moreover, quite large experimental uncertainties in the polarization experiment [24] allow one to speculate that \( |g_1| \) could be very small or even negligible. Indeed, in the case \( g_1 = 0 \) one obtains from (8) that \( r = 2/\xi \approx 5.5 \), which is not very far from the central value \( r = 3.9 \).

There are some papers concerning \( f_1 \rightarrow \rho^0 \gamma \) decay, where the amplitude \( M(f_1 \rightarrow \rho^0 \gamma) \) is parametrized only by one constant [26, 28, 29]. These models correspond to our model at \( g_1 = 0 \).

There are also papers where the amplitude \( M(f_1 \rightarrow \rho^0 \gamma) \) is parametrized by two constants; see, e.g., Ref. [30]. Therein two relations between parameters of \( f_1 \rightarrow \rho^0 \gamma \) decay are obtained using \( \Gamma(f_1 \rightarrow \rho^0 \gamma) \) and \( r = \rho_{1L}/\rho_{1T} \). In this paper the \( f_1 \) meson is considered as the molecular state and \( f_1 \rightarrow \rho^0 \gamma \) decay is studied in the chiral effective field theory.

## 5 Calculation of \( f_1 \rightarrow e^+e^- \) decay width

Now let us consider \( f_1 \rightarrow \pi^+\pi^-\pi^+\pi^- \) decay. Experimental data indicate that the main contribution to it is given by the intermediate state with two virtual \( \rho \) mesons [23] (see Fig. 3). The vertex \( f_{1\rho\rho} \) contains the form factors of our model. Certainly, these form factors should meet the requirements that the result of the calculation of the \( f_1 \rightarrow \pi^+\pi^-\pi^+\pi^- \) decay width should be in a good agreement with the experimental value.

![Feynman diagrams](image)

**Figure 3.** Feynman diagrams of \( f_1 \rightarrow \pi^+\pi^-\pi^+\pi^- \) decay.

We parametrize the \( \rho^0 \rightarrow \pi^+\pi^- \) amplitude as

\[
M(\rho^0 \rightarrow \pi^+\pi^-) = i f_{\rho\pi\pi} e_\mu p^\mu, \tag{11}
\]

where \( e_\mu \) is the polarization vector of the \( \rho^0 \) meson, and \( p^\mu \) is the momentum of the \( \pi^+ \) meson. We neglect the \( q^2 \) dependence of \( f_{\rho\pi\pi} \) and obtain the following value:

\[
f_{\rho\pi\pi} = \left( \frac{192\pi \Gamma(\rho^0 \rightarrow \pi^+\pi^-)}{m_\rho(1 - 4m_\pi^2/m_\rho^2)^{3/2}} \right)^{1/2} \approx 11.9, \tag{12}
\]
where \( m_\rho = 775.26 \text{ MeV} \), \( \Gamma(\rho^0 \rightarrow \pi^+\pi^-) \approx \Gamma_\rho = 147.8 \text{ MeV} \), and \( m_\pi = 139.57 \text{ MeV} \) is the mass of \( \pi^\pm \) mesons.

Taking into account that form factor \( F_1(q_1^2, q_2^2) \) is antisymmetric, and all other requirements, we write the form factors \( F_1 \) and \( F_2 \) as

\[
F_1(q_1^2, q_2^2) = \frac{g_\rho \sqrt{m_\rho}}{(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)} (q_2^2 - q_1^2)
\]

(13)

\[
F_2(q_1^2, q_2^2) = \frac{g_\rho \sqrt{m_\rho}}{(q_1^2 - m_\rho^2 + i m_\rho \Gamma_\rho)} (-q_2^2 + q_1^2)
\]

(14)

where \( \Gamma_\rho = 147.8 \text{ MeV} \) are \( \rho^0 \)-meson width, \( g_\rho \) is the coupling constant of the transition \( \rho^0 \rightarrow \gamma^* \), \( M(\rho^0 \rightarrow \gamma^*) = g_\rho \sqrt{m_\rho} e^{\mu \nu} \), so \( g_\rho = \sqrt{3 \Gamma(\rho^0 \rightarrow e^+e^-)/am_\rho} \approx 0.06 \).

The result of numerical calculation is shown in Fig. 4.

\[
\begin{align*}
\mathcal{B}(f_1 \rightarrow \pi^+\pi^-\pi^+\pi^-) &\approx (11.0^{+0.7}_{-0.8})
\end{align*}
\]

**Figure 4.** The branching ratio of the \( f_1 \rightarrow \pi^+\pi^-\pi^+\pi^- \) decay for the certain choice of the form factors \( F_1(q_1^2, q_2^2) \) and \( F_2(q_1^2, q_2^2) \); see Eqs. (13) and (14). The solid line corresponds to the \( f_1 \rightarrow \pi^+\pi^-\pi^+\pi^- \) branching ratio calculated using the central values: \( r = 3.9 \) and \( \mathcal{B}(f_1 \rightarrow \rho^0\gamma) = 5.5\% \) or \( \mathcal{B}^{\text{CLAS}}(f_1 \rightarrow \rho^0\gamma) = 2.5\% \). Dashed and dotted lines indicate 1\( \sigma \) deviations for \( \mathcal{B}(f_1 \rightarrow \pi^+\pi^-\pi^+\pi^-) \). The shaded horizontal band denotes the value allowed experimentally, \( \mathcal{B}(f_1 \rightarrow \pi^+\pi^-\pi^+\pi^-) = (11.0^{+0.7}_{-0.8})\% \).

It is seen from Fig. 4 that we still cannot derive the exact value of the phase \( \delta \) in our model because of large uncertainties of the model parameters. Therefore, in what follows we treat \( \delta \) as a free parameter.

To calculate the \( f_1 \rightarrow e^+e^- \) branching ratio, we substitute the expressions for \( F_1(q_1^2, q_2^2) \) and \( F_2(q_1^2, q_2^2) \) into (4) and perform the numerical calculations; then, comparing the answer with (1), we obtain the following result for the constant \( F_A \):

\[
F_A \approx -\alpha g_1 (0.22 + 0.25 i) - \alpha g_2 (0.75 + 0.57 i)
\]

(15)

It is convenient to express complex numbers \( g_1 \) and \( g_2 \) in polar form as \( g_1 = |g_1|e^{i\phi_1} \) and \( g_2 = |g_2|e^{i\phi_2} \), respectively. Then using \( \delta = \phi_1 - \phi_2 \) one can write the absolute square of the constant \( F_A \) as

\[
|F_A|^2 \approx |e^{i\delta} \cdot \alpha|g_1| \cdot (0.22 + 0.25 i) + \alpha|g_2| \cdot (0.75 + 0.57 i)|^2.
\]

(16)
Since \( \alpha|g_2| \leq 1 \) and \( \alpha|g_1| \leq 1 \) [see (9) and (10)], then \( |F_A| \sim 1 \), as expected. In particular, for central values of \( |g_1| \) and \( |g_2| \) we get \( |F_A| \approx 1.12 \) for \( \delta = 0.7\pi \), \( |F_A| \approx 1.28 \) for \( \delta = 1.3\pi \), and \( |F_A|^{\text{CLAS}} \approx 0.85 \) for \( \delta = 0 \).

Now it is straightforward to calculate the branching ratio \( \mathcal{B}(f_1 \to e^+e^-) \) as a function of \( \delta \). Corresponding plots are shown in Fig. 5.

![Figure 5](https://doi.org/10.1051/epjconf/201818202106)

**Figure 5.** The branching ratio \( \mathcal{B}(f_1 \to e^+e^-) \) as a function of the relative phase \( \delta \) in our model. The solid line corresponds to \( \mathcal{B}(f_1 \to e^+e^-) \) calculated for the central values: \( r = 3.9 \) and \( \mathcal{B}(f_1 \to \rho^0\gamma) = 5.5\% \) or \( \mathcal{B}^{\text{CLAS}}(f_1 \to \rho^0\gamma) = 2.5\% \). The dashed and dotted lines indicate 1\( \sigma \) deviations from the \( \mathcal{B}(f_1 \to e^+e^-) \) central value.

It is seen from Figs. 4 and 5 that in our model the branching ratio \( \mathcal{B}(f_1 \to e^+e^-) \) should be taken in the range from \( 3 \cdot 10^{-9} \) for \( \delta \approx \pi \) to \( 8 \cdot 10^{-9} \) for \( \delta = 0 \), and from \( 4 \cdot 10^{-9} \) for \( \delta^{\text{CLAS}} = 0 \) to \( 5 \cdot 10^{-9} \) for \( \delta^{\text{CLAS}} = \pm 0.3\pi \),

\[
\mathcal{B}(f_1 \to e^+e^-) \approx (3 - 8) \cdot 10^{-9}, \quad \mathcal{B}^{\text{CLAS}}(f_1 \to e^+e^-) \approx (4 - 5) \cdot 10^{-9},
\]

and the corresponding decay width is

\[
\Gamma(f_1 \to e^+e^-) \approx 0.07 - 0.19 \text{ eV}, \quad \Gamma^{\text{CLAS}}(f_1 \to e^+e^-) \approx 0.07 - 0.10 \text{ eV}.
\]

The values of the branching ratio and the decay width obtained for CLAS data lie in a more narrow interval than the corresponding values obtained for PDG data. However, both ranges of \( \Gamma(f_1 \to e^+e^-) \) values are in good agreement with the naive estimate \( \Gamma \approx 0.1 \text{ eV} \) (see the end of Sec. 2).

### 6 Estimate of \( e^+e^- \to f_1 \to \eta\pi\pi \) cross section

Let us estimate the cross section of the process \( e^+e^- \to f_1 \to \eta\pi\pi \), which can be used for the study of direct \( f_1 \) production in \( e^+e^- \) collisions. Here, the \( f_1 \to \eta\pi\pi \) decay proceeds mainly (approximately with 70\% probability [21]) through the intermediate \( a_0(980) \) meson; see Figs. 6 and 7.

The cross section of the \( e^+e^- \to f_1 \to a_0\pi \) process can easily be calculated,

\[
\sigma(e^+e^- \to f_1 \to a_0\pi) = \frac{12\pi}{m_f^2} \mathcal{B}(f_1 \to a_0\pi) \mathcal{B}(f_1 \to e^+e^-),
\]

where the center-of-mass energy equals the mass of the \( f_1 \) meson, \( \sqrt{s} = m_f \). Using the experimental value for the branching ratio \( \mathcal{B}(f_1 \to a_0\pi) = 0.36 \pm 0.07 \) and the result of our calculations (17) for
The values of cross sections obtained for PDG and CLAS data are in reasonable agreement with the following estimates:

\[ \sigma + \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi \approx 7.8 - 30 \text{ pb}, \quad \sigma_{\text{CLAS}}^{+} \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi \approx 10 - 20 \text{ pb}. \] (20)

Assuming that the \( a_{0} \) meson decays only into the \( \eta\pi \) final state and using the isospin symmetry, we obtain the following estimates:

\[ \sigma^{+} \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi^{\pm} \rightarrow \eta\pi^{\pm} \approx 5.2 - 20 \text{ pb}, \quad \sigma_{\text{CLAS}}^{+} \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi^{\pm} \rightarrow \eta\pi^{\pm} \approx 7 - 13.3 \text{ pb}, \]
\[ \sigma^{+} \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi^{0} \rightarrow \eta\pi^{0} \approx 2.6 - 10 \text{ pb}, \quad \sigma_{\text{CLAS}}^{+} \pi^{0} \rightarrow f_{1} \rightarrow a_{0}\pi^{0} \rightarrow \eta\pi^{0} \approx 3.5 - 6.7 \text{ pb}. \] (21) (22)

It is seen that the values of cross sections obtained for PDG and CLAS data are in reasonable agreement with uncertainties. However, the values of \( \sigma_{\text{CLAS}}^{+} \) lie in a narrower range. Therefore, one can hope that future precise experiments could make it possible to distinguish between \( \sigma_{\text{PDG}}^{+} \) and \( \sigma_{\text{CLAS}}^{+} \).

### 7 Charge asymmetry in \( e^{+}e^{-} \rightarrow \eta\pi^{+}\pi^{-} \) process

Though the cross section of the \( e^{+}e^{-} \rightarrow f_{1} \rightarrow \eta\pi^{0}\pi^{0} \) process is twice less than that of \( e^{+}e^{-} \rightarrow f_{1} \rightarrow \eta\pi^{+}\pi^{-} \), the former is more convenient for the study of direct \( f_{1} \) production in \( e^{+}e^{-} \) collisions.

Indeed, the \( e^{+}e^{-} \rightarrow \eta\pi^{0}\pi^{0} \) reaction proceeds only through two-photon annihilation, since \( C \) parity of the \( \eta\pi^{0}\pi^{0} \) final state is positive. Therefore, there is no background from one-photon annihilation, and the \( e^{+}e^{-} \rightarrow f_{1} \rightarrow \eta\pi^{0}\pi^{0} \) cross section can be measured directly. According to the estimate (22), the lower bound on this cross section is quite small, but it can be measured in a special experiment at the VEPP-2000 collider in Novosibirsk.

In contrast, the \( e^{+}e^{-} \rightarrow \eta\pi^{+}\pi^{-} \) reaction proceeds mainly through one-photon annihilation, which is described quite well by the VMD model with intermediate \( \rho^{0}(1450) \) and \( \rho^{0}(770) \) mesons [31], as depicted in Fig. 8. The annihilation \( e^{+}e^{-} \rightarrow \rho \rightarrow \eta\pi^{+}\pi^{-} \) was studied theoretically in Ref. [32].
The measured $e^+e^\to\eta\pi^+\pi^-$ Born cross section is about 500 pb at $\sqrt{s} = m_f$ [31]. According to the estimate (21), the $e^+e^\to f_1 \to a_0^+\pi^\pm \to \eta\pi^+\pi^-$ cross section constitutes only several percent of the total $e^+e^\to \eta\pi^+\pi^-$ cross section, and its measurement is a rather complicated task. One possibility to overcome this difficulty is to investigate the two-photon annihilation channel $e^+e^\to f_1 \to \eta\pi^+\pi^-$ through C-odd effects, which arise from the interference of C-odd one-photon and C-even two-photon amplitudes.

This interference is $P$- and C-odd, therefore it does not contribute to the total cross section, but it can lead to the change asymmetry in the differential cross section. Indeed, calculation shows that after integration over azimuthal angle $\phi_\pi$ the interference term is an odd function of $\cos \theta_\eta$ and $\cos \theta_\pi$. Here $\theta_\eta$ is the angle between $\eta$ meson 3-momentum and $e^+$ beam axis in the $e^+e^-$ center-of-mass frame, and $\theta_\pi$ is the angle between $\pi^+$ meson and $\eta$ meson 3-momenta in the $\pi^+\pi^-$ center-of-mass system. Therefore, if we consider events with $\theta_\eta$ in a definite interval $d\cos \theta_\eta$, then the interference term has opposite signs for $\cos \theta_\pi$ and $\cos (\pi - \theta_\pi) = -\cos \theta_\pi$. Physically it means that the number of $\pi^+$ mesons propagating in some direction $\theta_\pi$ differs from the number of $\pi^-$ mesons propagating in the same direction.

Let us define the change asymmetry in the $e^+e^\to \eta\pi^+\pi^-$ process as

$$A = \frac{\sigma_{\text{tot}}(\cos \theta_\pi > 0) - \sigma_{\text{tot}}(\cos \theta_\pi < 0)}{\sigma_{\text{tot}}(\cos \theta_\pi > 0) + \sigma_{\text{tot}}(\cos \theta_\pi < 0)}_{\cos \theta_\eta > 0},$$

where $\sigma_{\text{tot}} = \sigma_1 + \sigma_2 + \sigma_{\text{int}}$ is the total cross section. Condition $\cos \theta_\eta > 0$ is chosen here quite arbitrarily, so for real experiment one can redefine asymmetry in another $\theta_\eta$ range based on experimental conditions.

The interference term contains one additional free parameter $\phi$, which is the relative phase arising from the complex coupling constants,

$$F_A g_{f_1 a_0 g_{\omega\eta} f_{\pi\pi}} = |F_A g_{f_1 a_0 g_{\omega\eta} f_{\pi\pi}}| e^{i\phi}. \quad (24)$$

Using the values $\phi_{a_0^+}(770) = 0$, $\phi_{\rho^0(1450)} = \pi$ [31] we perform numerical calculations of the change asymmetry (23) for $\delta = 0.7\pi$, $\delta = 1.3\pi$, and $\delta_{\text{CLAS}} = 0$. The dependence of the change asymmetry $A$ on the relative phase $\phi$ is shown in Fig. 9.

It is seen that the change asymmetry in the $e^+e^\to \eta\pi^+\pi^-$ process may be quite large, up to $\pm10\%$ for $\phi \approx \mp\pi/2$.

### 8 Conclusion

We calculate the width of the $f_1(1285) \to e^+e^-$ decay in the vector meson dominance model, where both virtual photons are coupled with the $f_1$ meson via the intermediate $\rho^0$ mesons; see Fig. 2.
We assume that this is the main mechanism of the decay, and such an assumption is based on the experimental data on $f_1 \rightarrow 4\pi$ and $f_1 \rightarrow \rho^0\gamma$ decays [21, 23]. In our model the decay width, $\Gamma(f_1 \rightarrow e^+e^-)$, depends on the relative phase $\delta$ between two coupling constants describing the $f_1 \rightarrow \rho^0\gamma$ decay. This phase is not fixed unambiguously from the experimental data. Therefore, the width can only be estimated as $\Gamma(f_1 \rightarrow e^+e^-) \approx 0.07 - 0.19$ eV using the PDG data [21], and as $\Gamma^{\text{CLAS}}(f_1 \rightarrow e^+e^-) \approx 0.07 - 0.10$ eV using the CLAS Collaboration data [25]. The corresponding branching ratio is $\mathcal{B}(f_1 \rightarrow e^+e^-) = (3 - 8) \cdot 10^{-9}$ and $\mathcal{B}^{\text{CLAS}}(f_1 \rightarrow e^+e^-) \approx (4 - 5) \cdot 10^{-9}$.

The process of direct $f_1$ production in $e^+e^-$ collisions, $e^+e^- \rightarrow f_1 \rightarrow mesons$, is still not measured due to smallness of the corresponding cross section. Now it can be studied at modern high-luminosity colliders, e.g., at VEPP-2000 in Novosibirsk. We estimate the $e^+e^- \rightarrow f_1 \rightarrow \eta\pi\pi$ cross section and find it to be $\sigma(e^+e^- \rightarrow f_1 \rightarrow a_0^0\pi^0 \rightarrow \eta\pi^+\pi^-) \approx 5.2 - 20$ pb ($\sigma^{\text{CLAS}} \approx 7 - 13.3$ pb) for the $\eta\pi^+\pi^-$ final state, and $\sigma(e^+e^- \rightarrow f_1 \rightarrow a_0^0\pi^0 \rightarrow \eta\pi^0\pi^0) \approx 2.6 - 10$ pb ($\sigma^{\text{CLAS}} \approx 3.5 - 6.7$ pb) for the $\eta\pi^0\pi^0$ final state. The latter process, $e^+e^- \rightarrow f_1 \rightarrow \eta\pi^0\pi^0$, is more convenient to study, because the $e^+e^- \rightarrow \eta\rho^0\pi^0$ reaction proceeds only through two-photon annihilation. Therefore, there is no background from one-photon annihilation, and the $e^+e^- \rightarrow f_1 \rightarrow \eta\rho^0\pi^0$ cross section can be measured directly. In our model the lower bound on this cross section is quite small, $\approx 3$ pb. However, even such a small cross section can be measured in a special experiment at the VEPP-2000 $e^+e^-$ collider in Novosibirsk.

In contrast, the reaction $e^+e^- \rightarrow \eta\pi^+\pi^-$ proceeds mainly through one-photon annihilation. Therefore, measurement of the cross section of the two-photon channel, $e^+e^- \rightarrow f_1 \rightarrow \eta\pi^+\pi^-$, is a rather complicated task, because of the background from one-photon annihilation. One possibility to overcome this difficulty is to investigate the charge asymmetry which arises from the interference of $C$-odd one-photon and $C$-even two-photon amplitudes. We calculate this asymmetry in the $e^+e^- \rightarrow \eta\pi^+\pi^-$ reaction for some values of parameters in our model. It turns out that the magnitude of the charge asymmetry is quite uncertain. It depends on the relative phase $\phi$ and may be quite large, up to $\pm 10\%$.

We hope that in the nearest future our predictions will be tested in precise experiments at $e^+e^-$ colliders. Such experiments could allow us to obtain values of free parameters of our model, $\delta$ and $\phi$, as well as to define more accurately $\mathcal{B}(f_1 \rightarrow \rho^0\gamma)$, $\Gamma_f$, and $r$, measured by now with quite large uncertainties.

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