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The analogy of equation of rotation in complex plane with the Dirac equation, and its foundation

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Abstract The Three Wave Hypothesis (TWH) has been proposed by Horodecki in 1981. Sanduk attributed TWH to a classical kinematical model of two rolling circles in 2007. In a previous project in 2012, it is shown that the position vector of a point in a system of two rolling circles can be transformed to a complex vector under the effect of partial observation. The present work tries to develop this concept of transformation. Under this transformation, it is found that the kinematical equations of the motion of point can be transformed to equations analogise the relativistic quantum mechanics equations. Many analogies have been found and are listed in a comparison table. These analogies may sagest that both of the quantum mechanics and the special relativity are emergent, and are of the same origin.

1. Introduction

During the eighties of last century, R. Horodecki proposed Three Wave Hypothesis (TWH) where the particle is related to waves. This hypothesis implies that a massive particle is an intrinsically spatially as well as temporally extended non-linear wave phenomenon [1,2].
The TWH is based on an assumption that, in a Lorentz frame where the particle is at rest it can be associated with an intrinsic nondispersive Compton wave. When the particle moves with velocity v (relative to the lab frame), it will be associated with the three waves: the superluminal de Broglie wave (of wavelength λ_B), a subluminal dual wave (of wavelength λ_D), and a transformed Compton wave (of wavelength λ_C) [1, 2].

Considering this system of waves in angular form and in single representation (instead of two dispersion relations of the de Broglie wave and dual wave) exhibits similarities with a system of two perpendicular circles [3,4]. Thus, the combination of the three waves may form a classical kinematical bevel gear model. However, the main problem of TWH is lack of experimental evidence for the superluminal dual wave.

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1.1 The position vector and the complex vector

A mathematical relationship between the position vector of a point in rolling circles system and a complex vector has been explained via a transformation under partial observation effect [5]. The position vector of a point in this system is a real quantity. This project [5] propose a concept of partial observation to transfer the position vector to a complex function. The partial observation is based on inability to distinguish the complete system due to resolution limit. The work was not in quantum mechanics.

1.2 The aim of the work

In present work, we try to use the classical concepts mentioned above to formulate a model analogous to the relativistic quantum mechanics. Thus this approach does not base on the quantum postulates (wave function, and Dirac equation), but its goal is to go toward them. At the end, we will demonstrate a comparison table to show the conventional relativistic quantum mechanics equations and the obtained equations by the present work.

2. Kinematical model

Based on 2012 work [5], the system of two rolling circles is shown in Figure 1. The circle of interest is guided circle of radius a_1 and the second circle is the guiding circle of radius a_2. Position vector ($\boldsymbol{r}$) of point P is

$$\boldsymbol{r} = b\left\{\cos(\vartheta - \phi + \beta) \pm \sqrt{-sin^2(\vartheta - \phi + \beta) + (\frac{a_1}{b})^2}\right\}. \tag{1}$$

where a_1,b are the radius of the circle, and the norm of the position vector of the circle centre (C_1) respectively. The angles ϑ, ϕ, and β are as shown in Figure 1.

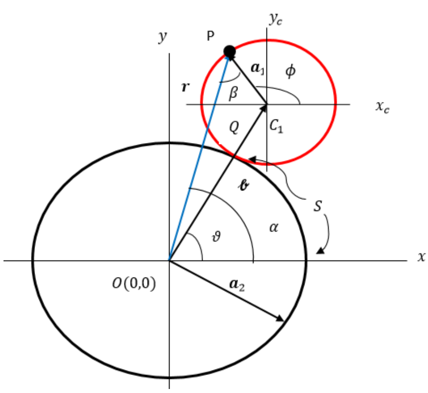

Figure 1. The real model. Rolling circles model.

The guiding circle is of radius

$$a_2 = b - a_1\,, \tag{2}$$

and its centre coordinates are (0,0). Let the ratio $\boldsymbol{a_1}/b$ in Eq.(1) is

$$X \equiv \left(\frac{|\boldsymbol{a}_1|}{|\boldsymbol{\ell}|}\right)^2 = \left(\frac{a_1}{\ell}\right)^2. \quad (3)$$

Point Q is the point of contact between the two circles. For generality, let $a_1 < a_2$. The ratio of the system is:

$$\frac{a_2}{a_1} = \frac{\phi}{\vartheta} = \frac{\omega_1}{\omega_2} = \mu > 1\,. \quad (4)$$

Eq.1 can be formulated as a kinematical model:

$$\boldsymbol{r} = (\boldsymbol{a}_2 + \boldsymbol{\ell}\sqrt{X})\left\{\cos\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\}, \quad (5)$$

where,

$$\vartheta = \frac{\boldsymbol{s}}{\boldsymbol{a}_2} = \boldsymbol{k}_2\cdot\boldsymbol{s}\,, \quad (6a)$$

and $\boldsymbol{s}$, ω_1, and ω_β represent the arc length made by point Q , the angular velocity of point P, and the time variation of angle β . In terms of X and from Eq. (2), ℓ can be written as:

$$\boldsymbol{\ell} = \boldsymbol{a}_2 + \boldsymbol{\ell}\sqrt{X}\,. \quad (6b)$$

Eq. (5) gives full description of the location of the point. Then, from Eq. (5), the velocity equation of point P is:

$$\begin{aligned}\frac{\partial \boldsymbol{r}(r,t,X)}{\partial t} &= \frac{\partial(\boldsymbol{a}_2 + \boldsymbol{\ell}\sqrt{X})}{\partial t}\left\{cos\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) \pm \sqrt{-sin^2\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}\right\} \\ &+ (\boldsymbol{a}_2 + \boldsymbol{\ell}\sqrt{X})\left\{(\omega_1 - \omega_\beta)\, sin\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right)\right. \\ &\left.\pm \frac{(\omega_1 - \omega_\beta)\, sin(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t) cos\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + \frac{\partial X}{\partial t}}{\sqrt{-sin^2\left(\boldsymbol{k}_2\cdot\boldsymbol{s} - \omega_1 t + \omega_\beta t\right) + X}}\right\}. \end{aligned} \quad (7)$$

3. Empiricist approach

To deal with the system as a physical object, testing (measuring) in a lab is an essential approach to prove its physical existence (positivism). In classical physics, the observable distinguishability is related to the optical resolution (Rayleigh criterion). The spatial resolution (d_λ) is the minimum linear distance between two distinguishable points [6].

The system is fully observed (seen) when its dimensions are larger than the spatial resolution (d_λ) and the angular frequencies (ω_λ) or:

$$d_\lambda \ll a_1 \ll a_2 \text{ , and } \omega_1 \gg \omega_2 \gg \omega_\lambda. \tag{8}$$

The system is said to be a classical physical system, and all its quantities are said to be physical and can be measured. The lab observer obeys these conditions.

3.1 Partial observation

The concept of partial observation and elimination has been proposed in Ref. [5]. Physically, $a = 0$ (*elimination*) suggests that no distinguishable spatial extent can be observed (empirical observation), in other words, the value of the parameter is less than the Rayleigh criterion (spatial resolution) [7].
Based on the resolution limit, the system is partially resolved under the condition

$$a_1 \ll d_\lambda \ll a_2 \text{ ,} \tag{9}$$

and according to the ratio (4), it temporally resolved under the condition

$$\omega_1 \gg \omega_\lambda \gg \omega_2 = \frac{\partial \vartheta}{\partial t}. \tag{10}$$

That implies $\mu \gg 1$. Practically, e*limination* can be considered missing observation, indistinguishable data, etc. Thus, for partial observation $a_1 = \omega_2 = 0$ and the zero quantity is a practical approximation.

Inequalities (9, 10) describes the inability to resolve a_1 and ω_2 (missing the data), whereas a_2 and $\omega_1$can be resolved. Partial resolution refers to the inability to completely resolve of the kinematical system by using *monochromatic* light (λ, f). d_λ is related to the wavelength so as ω_λ is related to f. Then, the measured quantities are:

$$\boldsymbol{a}_1 \neq \boldsymbol{a}_{1m} = 0 \text{ , and } X_m = 0 \text{ ,} \tag{11a}$$

and

$$\boldsymbol{\mathcal{b}} \rightarrow \boldsymbol{a}_2 = \boldsymbol{a}_{2m}, \text{ and } \omega_1 = \omega_\vartheta = \omega_{1m} \text{ .} \tag{11b}$$

The subscript m indicates resolved (measured) values.

When a_1 can not be detected, then the angle β can not be detected as well. Since $\alpha = \phi - \beta$, then:

$$\omega_{\phi m} = \omega_{\alpha m} = \omega_{1m} \text{ .} \tag{12}$$

Where the existence of angle β is related to the recognition of the guided circle of a_1.

Substitution of Eqs. (11) and (12) into Eq. (5) yields:

$$\boldsymbol{\mathcal{Z}}\,(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m}\left\{\cos\,(\boldsymbol{k}_{2m}\cdot\boldsymbol{s} - \omega_{1m}t) \pm \sqrt{-sin^2(\boldsymbol{k}_{2m}\cdot\boldsymbol{s} - \omega_{1m}t)}\right\}. \tag{13}$$

The real $\boldsymbol{r}$ transforms to complex $\boldsymbol{\mathcal{Z}}$. This form of Eq. (13) can be rewritten as (in Eulerian form):

$$\boldsymbol{\mathcal{Z}}(\boldsymbol{s}, t, 0) = \boldsymbol{a}_{2m} \exp \pm i\,(\boldsymbol{k}_{2m}\cdot\boldsymbol{s} - \omega_{1m}t). \tag{14}$$

The equation (14) represents a complex vector. The combination of the real vector ($\boldsymbol{a}_{2m}$) and the complex phase factor in Eq. (14) ($\exp \pm i\,(\boldsymbol{k}_{2m}\cdot \boldsymbol{s}-\omega_{1m}t)$) forms the complex vector.

3.2 The system as seen by the Lab observer

The partial observation serves as a filter, see Figure 2. This filter makes a separation between two different worlds, the real full deterministic world (mathematical) and the physical world (observable) of the complex vector. What the lab observer (Figure 3) can see is not a point and not a sinusoidal wave in real space. For him/her there is a collapse of a real system to a complex mathematical form, which has no physical meaning. In such a case, the lab observer may use some of the quantum mechanics techniques (postulates):

1- *Born's postulate(probabilistic interpretation of the wave function)*
2- *Correspondence principle (operators technique)*
3- *Von Neumann's postulate (wave function collapse)*

The only possible utilisation of this complex function is by using quantum mechanics technique, like a statistical technique (Born rule), and the point location is probable. Note Figure3.

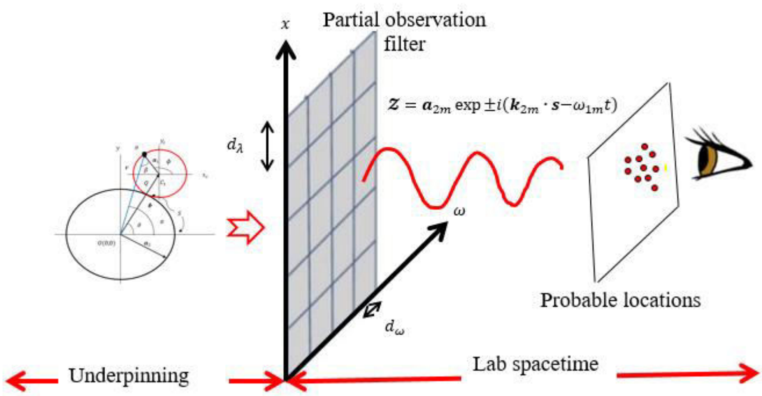

Figure 2. The system and the lab observation.

The lab observer can recognise the observable parameters of the system (a_{2m}, ω_{1m}). According to Eq. (14) the lab observation can be considered as a combination of sinusoidal wave (probability wave) and point. With aid of the above postulates, the lab observer can recognise probable locations as in Figure 3.

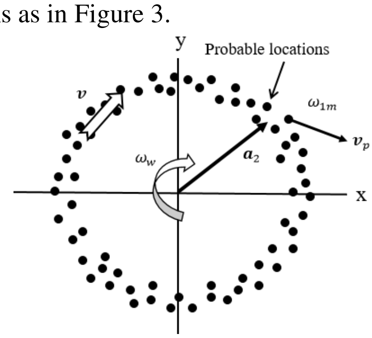

Figure 3. The observable probable locations.

3.3 Kinematic equations of the Lab observer

The complex vector can not be regarded as a physical expression. Thus, one may find in the statistical approach of the quantum mechanics a useful approach here as well. The concept of probable location of the point well be considered by the lab observer.

For lab observer who is assumed to be under the conditions of partial observation (Eqs. (13), and (14)), the equation of velocity Eq. (7) becomes (Appendix I):

$$i\frac{\partial \boldsymbol{Z}}{\partial t} = (-i\, v\boldsymbol{A}\cdot\boldsymbol{\nabla} + B\omega_{1m})\boldsymbol{Z}\,, \tag{15}$$

where $\boldsymbol{A}$ and B are coefficients related to the rotation of the system (Appendix I). Equation (15) is a complex velocity equation. The properties of the coefficients are shown in Appendix II. As shown in Figure 1 and Eq.(1), B (± 1) is related to the rotation $((\hat{e}_\alpha)$ of the position vector $\boldsymbol{r}$ of point P, and $\boldsymbol{A}$ (Eq. (5-A)) is related to the rotation direction $(\hat{e}_\vartheta)$ of $\boldsymbol{a}_2$. **O**ne can say that the cross product of the unit vectors is non-commutative:

$$\hat{e}_\vartheta \times \hat{e}_\alpha + \hat{e}_\alpha \times \hat{e}_\vartheta = 0, \tag{16}$$

and that may be the physics behind the non-commutative Dirac coefficients. Thus for the lab observer, the definitions of $\boldsymbol{A}$ and B are changed to $\boldsymbol{A}'$ and B'.
With aid of the coefficients properties $\boldsymbol{A}'$ and B'. (same as those of relativistic quantum mechanics), the time differentiation of Eq. (15) is

$$\frac{\partial^2 \boldsymbol{Z}}{\partial t^2} = (-v^2\,\nabla^2 + \omega_{1m}^2)\boldsymbol{Z}\,. \tag{17}$$

Eq. (17) looks like a complex acceleration equation for the complex velocity equation.

3.4 The seen system by the lab observer

Equations (13) and (14) are for point trajectory and sinusoidal wave in the complex plane. Thus, the lab observer with aid of quantum techniques may recognise the point and wave features together. The velocity v that appears in Eqs. (15, and 17), is (Eq. (2-A):

$$v = a_{2m'}\,\omega_{w'}\,. \tag{18}$$

The quantities with the prime symbol are the observed quantities by the lab observer. The point parameters are v_p (the observable *point* velocity) and $\omega_{1m'}$ whereas wave properties are $a_{2m'}$ (Eq. (18)) and $\omega_{w'}$.

Duo to the equivalence of point and sinusoidal wave, then for the lab observer:

$$v_p\,\omega_{1m'} \equiv \frac{v^2}{a_{2m'}} = v\,\omega_{w'}\;, \tag{19}$$

The left-hand side is related to the point and the right-hand side is for the wave. Eq. (19) can be formulated as a ratio for that point wave combination:

$$\frac{v}{v_p} = \frac{a_{2m'}\,\omega_{w'}}{v_p} = \frac{\omega_{1m'}}{{\omega_w}'} = \mu_L\,. \tag{20}$$

From Eq. (20) and Eq. (18) one can say that the velocity of the point is

$$v_p = \frac{v^2}{\omega_{1m\prime}\ a_{2m\prime}}\ , \tag{21}$$

and

$$v^2 = v_p \omega_{1m\prime}\, a_{2m\prime} = v_p\ v_{ph} \tag{22}$$

where v_{ph} phase velocity and is related to the phase of Eq. (14).

3.5 Lab transformation

With aid of operator postulate, Eq.(17) shows:

$$\omega^2 = -v^2\, \nabla^2 + \omega_{1m}^2 = \omega_{1m'}^2\, . \tag{23}$$

From Eq. (23), with aid of Eqs. (5-A), and (21), one can obtain:

$$\frac{\omega_{1m}}{\sqrt{1 - \frac{{v_p}^2}{v^2}}} = \omega_{1m'}\ . \tag{24}$$

The same for $\omega_{w\prime}$,

$$\frac{\omega_w}{\sqrt{1 - \frac{{v_p}^2}{v^2}}} = \omega_{w\prime}\, . \tag{25}$$

From Eqs. (24) and (21) we find

$$\frac{v^2}{\omega_{1m}\ v_p} \sqrt{1 - \frac{{v_p}^2}{v^2}} = a_{2m} \sqrt{1 - \frac{{v_p}^2}{v^2}} \equiv a_{2m\prime}\, . \tag{26}$$

According to Eqs. (19) , (25) and (26)

$$v = a_{2m'}\, \omega_{w\prime} = a_{2m} \sqrt{1 - \frac{{v_p}^2}{v^2}} \frac{\omega_w}{\sqrt{1 - \frac{{v_p}^2}{v^2}}} = a_{2m}\omega_w\, . \tag{27}$$

Thus, the velocity v is invariant. The $\omega_{1m\prime}$ is a real quantity, then from Eq. (24) we can say that $v_p < v < v_{ph}$.

4. Remarks

4.1 Comparisons

The similarity of the above-obtained equations' forms with the conventional equations of relativistic quantum mechanics and special relativity are shown in Table 1.

Table 1. Comparisons of the equations of conventional quantum mechanics and special relativity with the analogical model forms.

Conventional definition	Conventional equations of quantum mechanics and special relativity	Analogical model forms
Dirac wave function	$\psi_D = u_D \exp i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t)$	$\boldsymbol{Z} = \boldsymbol{a}_{2m} \exp \pm i(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t)$
Dirac equation	$i\dfrac{\partial \psi}{\partial t} = (-ic\boldsymbol{\alpha} \cdot \nabla + \beta\omega)\psi$	$i\dfrac{\partial \boldsymbol{Z}}{\partial t} = (-iv\boldsymbol{A} \cdot \nabla + B\omega_{1m})\boldsymbol{Z}$
The coefficients	$\boldsymbol{\alpha}$ and β	$\boldsymbol{A}$ and B
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_j + \boldsymbol{\alpha}_j\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}_\theta \cdot \boldsymbol{A}_\varphi + \boldsymbol{A}_\varphi \cdot \boldsymbol{A}_\theta = 0$
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i + \boldsymbol{\alpha}_i\boldsymbol{\alpha}_i = 2$	$\boldsymbol{A}_\theta \cdot \boldsymbol{A}_\theta + \boldsymbol{A}_\theta \cdot \boldsymbol{A}_\theta = 2$
Property	$\boldsymbol{\alpha}_i^2 = \beta^2 = 1$	$\boldsymbol{A}^2 = B^2 = 1$
Klein-Gordon equation	$\dfrac{\partial^2 \psi}{\partial t^2} = [-c^2\nabla^2 + \omega^2]\psi$	$\dfrac{\partial^2 \boldsymbol{Z}}{\partial t^2} = [-v^2\nabla^2 + {\omega_{1m}}^2]\boldsymbol{Z}$
Light speed	$c < v$	$v = a_{2m}\omega_w < v_p$
Relativistic mass (angular frequency) $\times \dfrac{\hbar}{c^2}$	$\omega = \dfrac{\omega_\circ}{\sqrt{1 - \dfrac{v^2}{c^2}}}$	$\omega_{1m\prime} = \dfrac{\omega_{1m}}{\sqrt{1 - \dfrac{{v_p}^2}{v^2}}}$
Length contraction	$\Delta L = \Delta L_\circ \sqrt{1 - \dfrac{v^2}{c^2}}$	$a_{2m\prime} = a_{2m}\sqrt{1 - \dfrac{{v_p}^2}{v^2}}$
de Broglie equation	$\lambda = \dfrac{\hbar}{m\,v} = \dfrac{c^2}{\omega v}$	$a_{2m'} = \dfrac{v^2}{\omega_{1m'} v_p}$,

4.2 Dose the system is a fine structure?

Our terminology for the lab observer corresponds to the relativistic observer, then any unobservable is out of the scope of special relativity. The classical electron radius (r_e) is out of the scope of relativity, and our a_1 is out of the scope of lab observation. Then we can assume that

$$r_e = 2.81794092 \times 10^{-15}\, m \equiv a_1 \tag{28}$$

For the ground state of election in hydrogen atom, Bohr radius (r_B) is within the scope of relativity, and our a_2 is observable for the lab observer. So, we can assume as well that:

$$r_B = 0.529177249\ \times 10^{-10} m \equiv a_2 \tag{29}$$

Then, the system ratio (Eq, (4)) may be equal to:

$$\frac{a_1}{a_2} = \frac{r_e}{r_B} = 5.325136191 \times 10^{-5} = (0.00729735075)^2 = \alpha^2 \tag{30}$$

where α is the fine structure constant. and accordingly:

$$\frac{1}{\alpha^2} = \mu = 18778.87441 \tag{31}$$

This may agree with the approximation of partial observation.

Electron velocity in ground state of the hydrogen atom (v) may correspond to our v_p

$$v_p \equiv 2.1876961417 \ \times 10^6 \ m/s \tag{32}$$

The light speed (c) may correspond to our v as shown above. Then from lab the ratio (Eq.(20))

$$\frac{v_p}{v} = \frac{2.1876961417 \ \times 10^6}{2.99792458 \times 10^8} = 0.007297368838 \tag{33}$$

and

$$\frac{v_p}{v} = \frac{1}{\mu_L} = \alpha \tag{34}$$

Then $\mu_L \equiv 137.0356936$, which is equivalent to coupling constants for the electromagnetic force.

4.3 is the relativistic quantum mechanics an emergent?

The obtained complex vector, the complex velocity, and complex acceleration equations may throw light on the origin of the first and second postulates of quantum mechanics.

4.4 Is the spacetime emergent?

The similarity between the special relativity equations and the equations of the analogize model as shown in Table 1 may lead to say that the spacetime is related to the same underpinning that of the quantum mechanics. Then, the spacetime is not just emergent but is related to the same origin of the quantum mechanics. The rolling circles model under the partial observation may show unification of the special relativity with quantum mechanics.

Appendix I

Under partial observation conditions, Eq. (7) becomes:

$$\begin{aligned}\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = {} & \frac{\partial \boldsymbol{a}_{2m}}{\partial t}\left\{cos(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) \pm \sqrt{-sin^2(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)}\right\} \\ & + \boldsymbol{a}_{2m}\left\{\omega_{1m}\, sin(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) \right. \\ & \left. \pm \frac{\omega_{1m}\sin(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)cos(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)}{i\, sin(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)}\right\}.\end{aligned} \tag{1-A}$$

The vector differentiation is

$$\frac{\partial \boldsymbol{a}_{2m}}{\partial t} = a_{2m}\omega\hat{\boldsymbol{e}}_\theta = v\ \hat{\boldsymbol{e}}_\theta\ . \tag{2-A}$$

Then, Eq. (1-A) becomes

$$i\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = (i\, v\hat{\boldsymbol{e}}_\theta \cdot \boldsymbol{k}_{2m})\, \boldsymbol{a}_{2om}\{\, cos(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) \pm\ i\, sin(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)\} + \boldsymbol{a}_{2m}\omega_{1m}\{i\ sin(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) \pm cos(\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t)\}, \qquad (3\text{-A})$$

In exponential form, Eq. (3-A) becomes:

$$i\frac{\partial \boldsymbol{r}(r,t,0)}{\partial t} = (i\, v\hat{\boldsymbol{e}}_\theta \cdot \boldsymbol{k_{2m}})\, \boldsymbol{a}_{2m} \exp i \pm (\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) + B\omega_{1m}\boldsymbol{a}_{2m} \exp i \pm (\boldsymbol{k}_{2m}\cdot \boldsymbol{s} - \omega_{1m}t) \qquad (4\text{-A})$$

Regarding that

$$\pm i\, \boldsymbol{\nabla} = \frac{1}{\boldsymbol{a_{2m}}} = \boldsymbol{k_{2m}}\text{, (operator)} \qquad (5\text{-A})$$
$$B = \pm 1,$$
and
$$\boldsymbol{A} = \mp\, i\, \hat{\boldsymbol{e}}_\theta\ ,$$

and with the aide of Eq. (5-A) then Eq. (4-A) becomes

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-i\, v\boldsymbol{A}\cdot \boldsymbol{\nabla} + B\omega_{1m})\boldsymbol{\mathcal{Z}}\,. \qquad (6\text{-A})$$

Appendix II

The dot product of two perpendicular $\boldsymbol{A}$ is

$$(\pm i\hat{\boldsymbol{e}}_\theta)\cdot (\pm i\hat{\boldsymbol{e}}_\varphi) = \|\pm i\hat{\boldsymbol{e}}_\varphi\|\ \|\pm i\hat{\boldsymbol{e}}_\theta\| = 0\,, \qquad (7\text{-A})$$

The property of $\boldsymbol{A}$ for two perpendicular planes of rotation becomes

$$(\pm i\hat{\boldsymbol{e}}_\theta)\cdot (\pm i\hat{\boldsymbol{e}}_\varphi) + (\pm i\hat{\boldsymbol{e}}_\varphi)\cdot (\pm i\hat{\boldsymbol{e}}_\theta) = 0\,, \qquad (8\text{-A})$$

The same for

$$(\pm i\hat{\boldsymbol{e}}_\theta)\cdot (\pm i\hat{\boldsymbol{e}}_\theta) + (\pm i\hat{\boldsymbol{e}}_\varphi)\cdot (\pm i\hat{\boldsymbol{e}}_\varphi) = 2. \qquad (9\text{-A})$$

The square of B and $\boldsymbol{A}$ is

$$\boldsymbol{A}^2 = B^2 = 1\,. \qquad (10\text{-A})$$

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