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Narrow-width tetraquarks in large-N_c QCD

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Abstract. The properties of possibly existing tetraquarks are studied in the large-N_c limit of QCD by means of four-point correlation functions of meson currents. The necessity of a detailed analysis of the singularities of Feynman diagrams, by means of the Landau equations, to recognize those diagrams that might contribute to the formation of tetraquark states, is emphasized. It is found, in general, that tetraquarks, if they exist, should have narrow widths of the order of N_c^{-2}.

1 QCD at large N_c

In the limit of large numbers of colour, N_c, with a simultaneous decrease of the coupling constant g as $N_c^{-1/2}$, QCD simplifies and provides many qualitative predictions for hadron physics [1, 2]. Thus, at leading order of N_c, QCD correlation functions of quark colour-neutral bilinear operators have only non-interacting ordinary mesons as intermediate states, made essentially of a pair of quark and antiquark fields, together with gluon fields [2]. This result, together with the fact that quark quadrilinear colour-neutral operators can always be decomposed, by means of Fierz transformations, into combinations involving colour-neutral quark bilinears, has been considered as a sign of the nonexistence of stable exotic mesons, like tetraquarks, made of two quark and two antiquark fields, together with gluons, surviving the above limit [3].

Weinberg has observed, however, that if tetraquarks exist as bound states in the large-N_c limit with finite masses, the crucial point is, even if they contribute to subleading diagrams, the qualitative property of their decay widths: are they broad or narrow? In the latter case, they might be observable. He has shown that, generally, they should be narrow, with decay widths of the order of N_c^{-1} [4]. In the same line of approach, Knecht and Peris have shown that, in a particular exotic channel, tetraquarks should even be narrower, with decay widths of the order of N_c^{-2} [5]. Cohen and Lebed, considering more general exotic channels, with an analysis based on the analyticity properties of two-meson scattering amplitudes, have found that the decay widths should be of the order of N_c^{-2} or smaller [6]. The possibility of smaller decay widths has been reported in [7].

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2 Line of approach

We study the properties of exotic and cryptoexotic tetraquarks through the analysis of meson-meson scattering amplitudes. Exotic tetraquarks are defined as containing four different quark flavours. Cryptoexotic tetraquarks contain three different quark flavours or less.

We consider four-point correlation functions of colour-singlet quark bilinears,

$$j_{ab} = \overline{q}_a q_b, \tag{1}$$

having a coupling with a meson M_{ab}, made of an antiquark a and quark b:

$$\langle 0|j_{ab}|M_{ab}\rangle = f_{M_{ab}}, \qquad f_M \sim N_c^{1/2}, \tag{2}$$

where the large-N_c behaviour of the coupling constant has also been outlined [2]. Spin and parity are not indicated in the above formulas and will be ignored in the subsequent analyses, since they are not relevant for the qualitative aspects of the problem.

We consider all possible s-channels where a tetraquark may be present. To be sure that a QCD Feynman diagram may contain, through a pole term, a tetraquark contribution, one has to check that it receives a four-quark (more precisely, two-quark and two-antiquark) contribution in its s-channel singularities, plus additional gluon singularities that do not modify the N_c-behaviour of the diagram.

If the tetraquark contains quarks and antiquarks with masses m_j, $j = a, b, c, d$, then the diagram should have a four-particle cut starting at $s = (m_a + m_b + m_c + m_d)^2$. Its existence is checked with the use of the Landau equations [8, 9].

Diagrams that do not have s-channel singularities, or have only two-particle singularities (quark-antiquark), cannot contribute to the formation of tetraquarks at their N_c-leading order. They should not be taken into account for the N_c-behaviour analysis of the tetraquark properties.

An account of the present work can be found in [10].

3 Exotic tetraquarks

We consider the case of four distinct quark flavours, denoted 1,2,3,4, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{34} = \overline{q}_3 q_4, \quad j_{14} = \overline{q}_1 q_4, \quad j_{32} = \overline{q}_3 q_2, \tag{3}$$

and the corresponding scattering processes:

$$M_{12} + M_{34} \;\to\; M_{12} + M_{34}, \qquad \text{direct channel I}, \tag{4}$$

$$M_{14} + M_{32} \;\to\; M_{14} + M_{32}, \qquad \text{direct channel II}, \tag{5}$$

$$M_{12} + M_{34} \;\to\; M_{14} + M_{32}, \qquad \text{recombination channel}, \tag{6}$$

called "direct channel I", "direct channel II" and "recombination channel", respectively.

In the case of "direct" channels, the corresponding four-point correlation functions are

$$\Gamma_\mathrm{I}^{(\mathrm{dir})} = \langle j_{12} j_{34} j_{34}^\dagger j_{12}^\dagger \rangle \,, \qquad \Gamma_\mathrm{II}^{(\mathrm{dir})} = \langle j_{14} j_{32} j_{32}^\dagger j_{14}^\dagger \rangle \,. \tag{7}$$

The leading and subleading Feynman diagrams for $\Gamma_\mathrm{I}^{(\mathrm{dir})}$ are represented in Fig. 1. Similar diagrams also exist for $\Gamma_\mathrm{II}^{(\mathrm{dir})}$. It is understood that to each diagram there corresponds an infinite number of diagrams with insertions of gluon exchanges not changing the topology of the diagram and having the

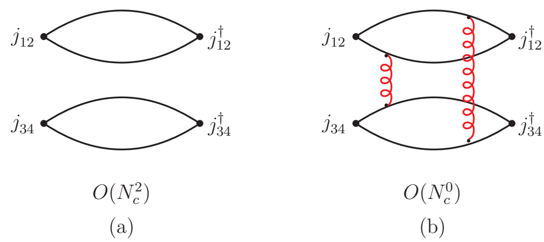

Figure 1. Leading and subleading diagrams in the direct channel I of (7). Full lines represent quarks, curly lines gluons.

same N_c-behaviour. It is the sum of such diagrams that may create singularities at the hadronic level, such as meson and/or tetraquark poles or two-meson cuts.

The leading behaviour of the direct-channel correlator functions is $O(N_\mathrm{c}^2)$, while that of the subleading diagrams is $O(N_\mathrm{c}^0)$. However, the leading diagrams (a) of Fig. 1 are disconnected and describe the propagation of two free ordinary mesons. It is only diagrams of the type (b) that may contribute to the scattering amplitude. On the other hand, analyzing, with the aid of the Landau equations, the structure of the singularities of diagrams (b) of Fig. 1, one finds that they have s-channel four-quark singularities, indicating that they may participate in the formation of tetraquark poles. One then deduces the behaviour of that part of the scattering amplitude that may come from a tetraquark intermediate state:

$$\Gamma_{\mathrm{I},T}^{(\mathrm{dir})} = O(N_c^0), \qquad \Gamma_{\mathrm{II},T}^{(\mathrm{dir})} = O(N_c^0). \tag{8}$$

For the "recombination" channel, the four-point correlation function is

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{34} j_{32}^\dagger j_{14}^\dagger \rangle. \tag{9}$$

The leading and subleading diagrams are represented in Fig. 2.

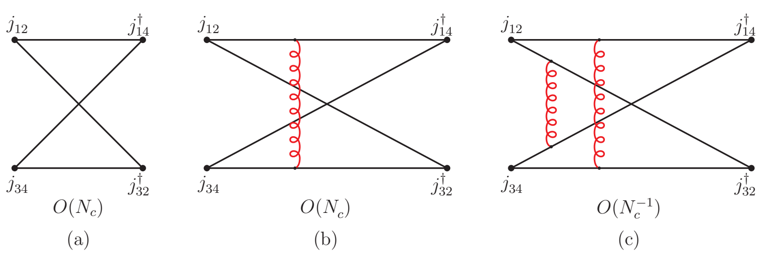

Figure 2. Leading and subleading diagrams of the recombination channel (9).

In spite of appearences, diagrams (a) and (b) of Fig. 2 do not have s-channel singularities. Their singularities appear in the u- and t-channels and correspond there to one-meson intermediate-state contributions. Therefore, they cannot contribute to the formation of tetraquark states. Only diagram (c) has s-channel (four-quark) singularities and thus may receive contributions from tetraquark states. The contribution of a tetraquark to the correlation function is then

$$\Gamma_T^{(\mathrm{recomb})} = O(N_c^{-1}). \tag{10}$$

The fact that the direct and recombination amplitudes have different behaviours in N_c [Eqs. (8) and (10)] , implies that two different tetraquarks, T_A and T_B, each having different couplings to the meson pairs, are needed to accommodate both types of behaviour.

Factorizing in the correlation functions the external meson propagators and the related couplings with the currents [Eqs. (1) and (2)], one obtains for the tetraquark–two-meson transition amplitudes the following behaviours:

$$A(T_A \to M_{12}M_{34}) = O(N_c^{-1}), \qquad A(T_A \to M_{14}M_{32}) = O(N_c^{-2}), \tag{11}$$

$$A(T_B \to M_{12}M_{34}) = O(N_c^{-2}), \qquad A(T_B \to M_{14}M_{32}) = O(N_c^{-1}). \tag{12}$$

The total widths of the tetraquarks are

$$\Gamma(T_A) = O(N_c^{-2}), \qquad \Gamma(T_B) = O(N_c^{-2}). \tag{13}$$

The meson-meson scattering amplitudes at the tetraquark poles (leading contributions) are represented in Fig. 3.

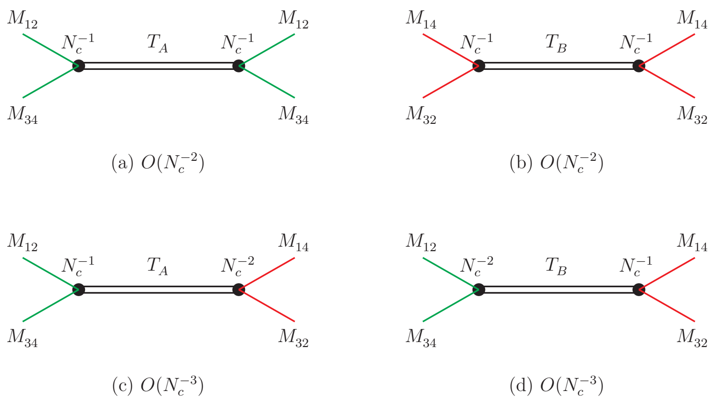

Figure 3. Couplings of the tetraquarks to two-meson states in meson-meson scattering.

4 Cryptoexotic tetraquarks

We now consider the case of three distinct quark flavours, denoted 1,2,3, with meson currents

$$j_{12} = \overline{q}_1 q_2, \quad j_{23} = \overline{q}_2 q_3, \quad j_{22} = \overline{q}_2 q_2. \tag{14}$$

The following scattering processes are considered:

$$M_{12} + M_{23} \;\to\; M_{12} + M_{23}, \quad \text{direct channel I}, \tag{15}$$

$$M_{13} + M_{22} \;\to\; M_{13} + M_{22}, \quad \text{direct channel II}, \tag{16}$$

$$M_{12} + M_{23} \;\to\; M_{13} + M_{22}, \quad \text{recombination channel}. \tag{17}$$

The direct channel four-point functions are

$$\Gamma_{\rm I}^{(\rm dir)} = \langle j_{12} j_{23} j_{23}^\dagger j_{12}^\dagger \rangle, \qquad \Gamma_{\rm II}^{(\rm dir)} = \langle j_{13} j_{22} j_{22}^\dagger j_{13}^\dagger \rangle. \tag{18}$$

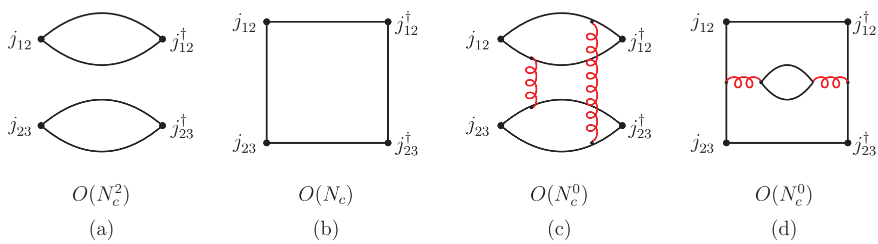

Figure 4. Leading and subleading diagrams in the direct channel I of (18).

The leading and subleading diagrams of $\Gamma_{\mathrm{I}}^{(\mathrm{dir})}$ are represented in Fig. 4. Diagram (b) receives contributions from one-meson intermediate states (M_{13}). Diagram (c) may receive contributions from tetraquark intermediate states. Diagram (d) describes, apart from the radiative correction phenomenon, a possible mixing of the meson M_{13} with a tetraquark state. One deduces from diagram (c) the contribution of a candidate tetraquark state to the correlation function:

$$\Gamma_{\mathrm{I},T}^{(\mathrm{dir})} = O(N_c^0). \tag{19}$$

The leading and subleading diagrams of $\Gamma_{\mathrm{II}}^{(\mathrm{dir})}$ are similar to those of the exotic case [Eq. (7)] and are represented in Fig. 5. Diagram (b) may receive contributions from tetraquark intermediate states.

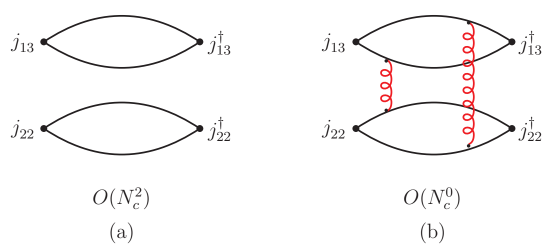

Figure 5. Leading and subleading diagrams of the direct channel II of (18).

One deduces the related contribution to the corresponding correlation function:

$$\Gamma_{\mathrm{II},T}^{(\mathrm{dir})} = O(N_c^0). \tag{20}$$

The recombination-channel four-point correlation function is

$$\Gamma^{(\mathrm{recomb})} = \langle j_{12} j_{23} j_{13}^\dagger j_{22}^\dagger \rangle. \tag{21}$$

Its leading and subleading diagrams are represented in Fig. 6.

As in the exotic case, diagrams (a) and (b) do not have s-channel singularities and cannot contribute to the formation of tetraquark poles. Diagrams (c) and (d) do have s-channel four-quark singularities and thus may receive contributions from tetraquark intermediate states. One then deduces the related contribution to the correlator function:

$$\Gamma_T^{(\mathrm{recomb})} = O(N_c^0). \tag{22}$$

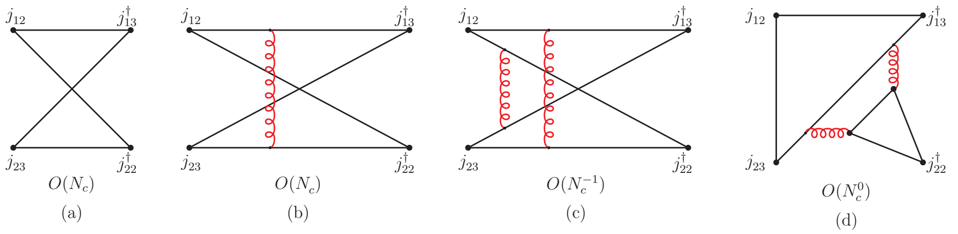

Figure 6. Leading and subleading diagrams of the recombination channel of (21).

In the present case, direct and recombination diagrams have the same N_c-behaviour. A single tetraquark T may accommodate all channels. The tetraquark–two-meson transition amplitudes are

$$A(T \to M_{12}M_{23}) = O(N_c^{-1}), \qquad A(T \to M_{13}M_{22}) = O(N_c^{-1}). \tag{23}$$

The total width of the tetraquark is

$$\Gamma(T) = O(N_c^{-2}). \tag{24}$$

The meson-meson scattering amplitudes at the tetraquark pole are represented in Fig. 7.

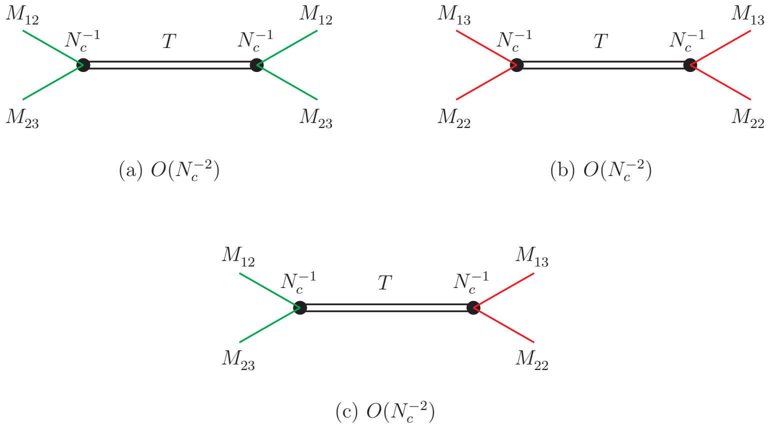

Figure 7. Couplings of the tetraquark to two-meson states in meson-meson scattering.

Possible mixings of tetraquarks with one-meson states, when allowed by the existing quantum numbers, are found to be of the order of $N_c^{-1/2}$ and do not alter, at leading order, the results previously found.

The case of cryptoexotic channels with two quark flavours can be treated in a similar way as for three. One finds here additional diagrams to those in the case of three flavours; they do not modify, however, the main qualitative features of the tetraquarks obtained above.

5 Open-type channel

In the case of three distinct quark flavours, denoted 1,2,3, one may also have the situation where the quark flavour 2, say, appears in two quark fields, rather than in a quark and an antiquark field. The meson currents are now

$$j_{12} = \overline{q}_1 q_2, \quad j_{32} = \overline{q}_3 q_2. \tag{25}$$

The following scattering process is then considered:

$$M_{12} + M_{32} \ \to \ M_{12} + M_{32}. \tag{26}$$

Here, the direct and recombination channels are identical. The corresponding four-point correlation function is

$$\Gamma = \langle j_{12} j_{32} j_{32}^\dagger j_{12}^\dagger \rangle. \tag{27}$$

The leading and subleading diagrams are represented in Fig. 8.

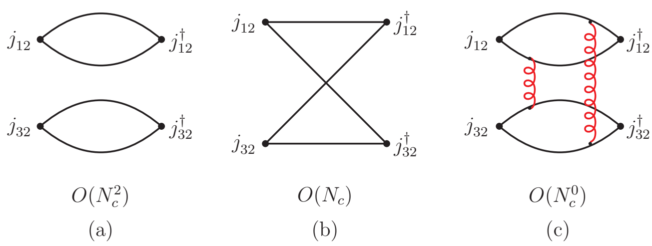

Figure 8. Leading and subleading diagrams of the correlation function (27).

Only diagram (c) contains s-channel four-quark singularities and may receive contributions from tetraquark intermediate states. The tetraquark–two-meson transition amplitude and the tetraquark total width have, respectively, the following behaviours:

$$A(T \to M_{12} M_{32}) = O(N_c^{-1}), \qquad \Gamma(T) = O(N_c^{-2}). \tag{28}$$

The meson-meson scattering amplitude at the tetraquark pole is represented in Fig. 9.

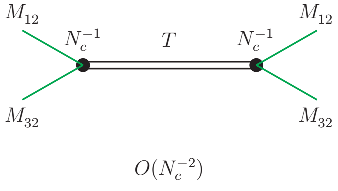

Figure 9. Coupling of the tetraquark to two-meson states in meson-meson scattering.

6 Conclusion

The analysis of the *s*-channel singularities of Feynman diagrams is crucial for the detection of the possible presence of tetraquark intermediate states in correlation functions of meson currents. If, due to the operating confining forces, tetraquarks exist as stable bound states of two quarks and two antiquarks in the large-N_c limit, with finite masses, then they should have narrow decay widths, of the order of N_c^{-2}, much smaller than those of the ordinary mesons, which are of order N_c^{-1}.

For the fully exotic channel, with four different quark flavours, two different tetraquarks are needed to accommodate the theoretical constraints of the large-N_c limit. In this case, each tetraquark has one predominant decay channel.

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