

## A-dependence of $\Lambda\Lambda$ -bond and charge symmetry energies

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**Abstract.** The  $\Lambda\Lambda$ -bond energies ( $\Delta B_{\Lambda\Lambda}$ ) of double- $\Lambda$  hypernuclei provide a measure of the nature of the in-medium strength of the  $\Lambda\Lambda$  interaction. Likewise, the charge symmetry breaking in mirror nuclei with  $\Lambda$  and  $\Lambda\Lambda$  is expected to shed light on  $\Lambda N$  and  $\Lambda\Lambda N$  interactions. A generalized mass formula, constructed earlier with broken SU(6) symmetry, is optimized and employed to calculate the separation energies from light to heavy nuclei. The new experimental data on  $\Lambda\Lambda$ -separation energy of a few double- $\Lambda$  hypernuclei, and  $\Lambda$ -separation energy of several single- $\Lambda$  hypernuclei have put more stringent constraint on this mass formula. The  $\Delta B_{\Lambda\Lambda}$  values calculated with this optimized formula are in good agreement with the experimental data. This optimized mass formula can be used to predict  $\Lambda\Lambda$ -bond energy in neutron-rich environment, and to extract Coulomb-corrected symmetry energy from experimental data as well. It suggests existence of bound hypernuclei beyond the normal neutron-drip line.

### 1 Introduction

Hypernuclei are relevant to multiple areas of physics, one of which is the study of the interior of neutron stars with high density cores. A pathway to understanding the properties of hyperons and their activities in neutron stars is comprehension of hyperon-hyperon interactions in the nuclear medium. Hyperon potentials in dense matter control the composition of dense neutron-star matter and studies at normal nuclear density are needed for the construction of models of density-dependent interactions for use at higher densities [1]. In this work, we used a generalized mass formula to calculate the binding energy of single- $\Lambda$  and double- $\Lambda$  hypernuclei and their variation with the neutron numbers. The  $\Lambda\Lambda$ -bond energy has also been studied. The above quantities are defined as,

$$B_{\Lambda}({}^A_{\Lambda}Z) = M({}^{A-1}Z) + M(\Lambda) - M({}^A_{\Lambda}Z) \quad (1)$$

$$B_{\Lambda\Lambda}({}^A_{\Lambda\Lambda}Z) = M({}^{A-2}Z) + 2M(\Lambda) - M({}^A_{\Lambda\Lambda}Z) \quad (2)$$

$$\Delta B_{\Lambda\Lambda}({}^A_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}({}^A_{\Lambda\Lambda}Z) - 2B_{\Lambda}({}^A_{\Lambda}Z) \quad (3)$$

Here, the mass number ( $A$ ) of the hypernuclei is the total number of baryons i.e., sum of the neutrons, protons and hyperons in the nuclei. The  $B_{\Lambda}$  and  $B_{\Lambda\Lambda}$  are  $\Lambda$  and  $\Lambda\Lambda$ -binding energies in

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their respective hypernucleus,  ${}^A_{\Lambda}Z$  and  ${}^A_{\Lambda\Lambda}Z$ . We explored the possibility of the existence of bound hypernuclei beyond the normal neutron-drip line and found that the hyperons can help a nucleus to hold more neutrons.

The bond energies ( $\Delta B_{\Lambda\Lambda}$ ) are the measures of the energy released when the  $\Lambda\Lambda$ -bond is broken. These energies assist in understanding the nature of the in-medium strength of the  $\Lambda\Lambda$ -interaction. In our calculations the bond energy is found to have a pronounced A-dependence.

Another interesting topic is the observed large charge symmetry breaking in light nuclei [2, 3]. The pair of hypernuclei with the same total number of baryons ( $A = N + Z + \Lambda$ ), but the neutron (N) and proton (Z) numbers interchanged, are called mirror nuclei. A large difference in binding energies ( $B_{\Lambda}$ ) between  ${}^4_{\Lambda}\text{H}$  (N=2, Z=1,  $\Lambda=1$ ) and  ${}^4_{\Lambda}\text{He}$  (N=1, Z=2,  $\Lambda=1$ ) was observed in the experimental data and it was attributed to charge symmetry breaking (CSB) effect [4]. The CSB is calculated as,

$$\Delta B_{\Lambda} = B_{\Lambda}({}^{N+Z+\Lambda}_{\Lambda}Z) - B_{\Lambda}({}^{Z'+N+\Lambda}_{\Lambda}Z') \quad (4)$$

Recently Botta et al. [5] have summarized some of the CSB values that shows that the  $\Lambda$ -N interaction is not charge independent. We calculated binding energy differences of several mirror nuclei. The CSB is not predicted by our calculation, since our mass formula gives the Coulomb energy difference. Nevertheless, this mass formula can be used to extract the Coulomb-corrected CSB from the experimental data. The Coulomb difference is found to have a small A dependence.

## 2 Formalism and Results

A hypernucleus is considered as a core of a normal nucleus plus hyperon(s). A generalized mass formula for non-strange normal nuclei (with N number of neutrons and  $Z_c$  number of protons) and strange hypernuclei (with  $n_Y$  number of hyperons, each of mass  $m_Y$ , charge  $q_Y$  and strangeness S) with total charge  $Z = Z_c + n_Y q_Y$ , was proposed earlier in a SU(6) symmetry breaking framework [6]. This mass formula is optimized with the newly available more accurate data on  $\Lambda\Lambda$ -hypernuclei summarized in Ref. [1]. It led to a minor modification of one of its parameters from 26.7 to 27.8 in the "strange" part of the formula. The binding energy in this revised formula is given by,

$$B(A, Z) = 15.777A - 18.34A^{2/3} - 0.71Z(Z-1)/A^{1/3} - 23.21(N - Z_c)^2 / [(1 + e^{-A/17})A] + n_Y [0.0335m_Y - 27.8 - 48.7|S|/A^{2/3}] + \delta \quad (5)$$

Where, the pairing term is,

$$\delta = +12A^{-1/2}(1 - e^{-A/30}) \quad (6)$$

for even proton-even neutron number,

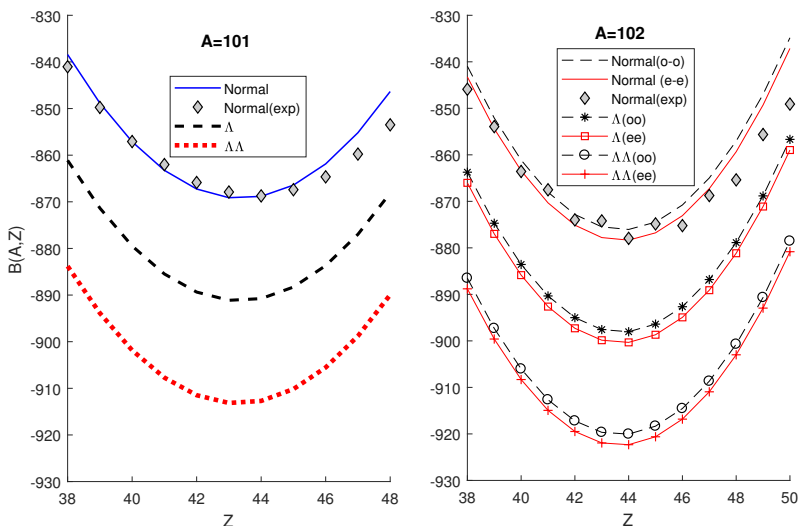
$$= -12A^{-1/2}(1 - e^{-A/30}) \quad (7)$$

for odd proton-odd neutron number,

$$= 0 \quad (8)$$

when  $N + Z_c$  is odd. The separation energies ( $S_Y$ ) for single- $\Lambda$  and double- $\Lambda$  hypernuclei is given by,

$$B_Y = B(A, Z)_{hyper} - B(A - n_Y, Z_c)_{core} \quad (9)$$



**Figure 1.** Plot of Binding Energies of normal, $\Lambda$ -, and  $\Lambda\Lambda$ -nuclei vs proton number  $Z$  for  $A=101$  and  $A=102$ .

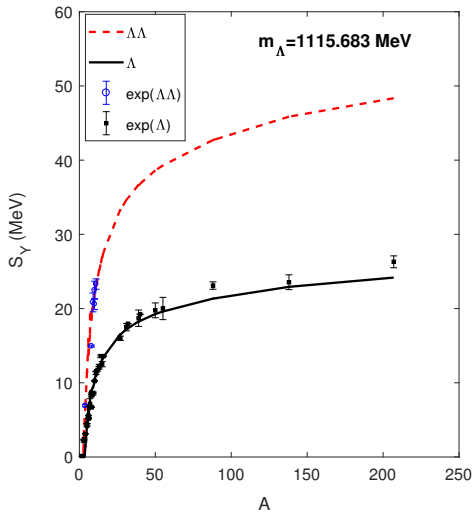
In the following we show application of this generalized mass formula for normal nuclei as well as hypernuclei. The calculated binding energies of both the  $\Lambda$  and  $\Lambda\Lambda$  hypernuclei are found to be in good agreement with the experimental data [1, 3, 5–9].

Fig.1 shows the mass parabola for  $A=101$  and  $A=102$ . The experimental data for normal nuclei [10] agree well with the prediction of the generalized mass formula. It can be clearly seen that  $\Lambda$ -hypernuclei are more bound than the normal nuclei, and  $\Lambda\Lambda$ -hypernuclei are more bound than both normal nuclei and  $\Lambda$ -hypernuclei.

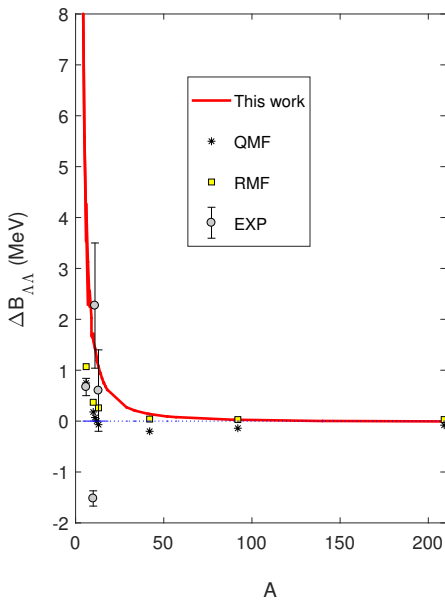
In Fig.2, a comparison of the experimental data on  $B_\Lambda$  and  $B_{\Lambda\Lambda}$  for different nuclei with the results of this work is presented. It shows that the calculations for the the  $\Lambda$ - and  $\Lambda\Lambda$ -hypernuclei are in good agreement with the experimental data [1, 3, 9].

In Fig.3, the  $\Lambda\Lambda$ -Bond energy is plotted against the Mass Number  $A$ . The Bond Energies are found to be in good agreement with the experimental data and predictions of Quark Mean Field (QMF) and Relativistic Mean Field (RMF) calculations [9], except at small mass number where the experimental data have large error bars. Also the data point at  $A=10$  is deeply negative which could be an extraction error of the data.

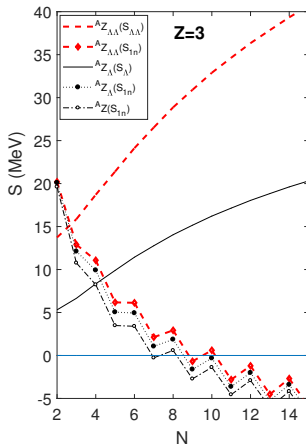
Fig.4 displays the one-neutron separation energies for normal nuclei with  $Z=3$ , and neutron-,  $\Lambda$ - and  $\Lambda\Lambda$ -separation energies for the corresponding  $\Lambda$ - and  $\Lambda\Lambda$ -hypernuclei. The n-drip point is reached when neutron-separation energies ( $S_n$ ) become negative for two successive neutron-rich nuclei. For normal nuclei,  $N=8$  is the neutron-drip point, whereas this mass formula predicts that for  $\Lambda$ - and  $\Lambda\Lambda$ -hypernuclei the  $N=10$  could be the last bound. Thus,  $\Lambda$ - and  $\Lambda\Lambda$ -hypernuclei can exist beyond the normal neutron drip point. Similar exotic nuclei are suggested for other elements also.



**Figure 2.** Plot of  $\Lambda$  and  $\Lambda\Lambda$ -separation Energy ( $S_\gamma$ ) versus mass Number ( $A$ ) for single- $\Lambda$  and double- $\Lambda$  hypernuclei, calculated using this mass formula and comparison with the available experimental data.



**Figure 3.** Plot of calculated  $\Lambda\Lambda$  bond energies ( $\Delta B_{\Lambda\Lambda}$ ) versus the mass number  $A$ , and comparison with the experimental data, QMF and RMF predictions[9].



**Figure 4.** Plot of  $\Lambda$ ,  $\Lambda\Lambda$ , and one neutron separation energy versus neutron numbers for normal nuclei,  $\Lambda$ - and  $\Lambda\Lambda$ -hypernuclei with proton number  $Z=3$ .

### 3 Summary

In this work,  $\Lambda$ -,  $\Lambda\Lambda$ -separation and  $\Lambda\Lambda$ -bond energies are calculated for a wide range of hypernuclei using a generalized mass formula. The results are in good agreement with the experimental data. This mass formula suggests that some hypernuclei can exist beyond the neutron-drip line of normal nuclei. Finding such neutron-rich hypernuclei experimentally (or, even proving their absence) can shed light on the unknown intricacies of the  $\Lambda N$  interactions in a neutron-rich environment. The  $\Lambda\Lambda$ -bond energy is found to decrease with increasing neutron number. This information is useful in model calculations for neutron stars. For mirror nuclei this mass formula provides the Coulomb energy differences that show a mild  $A$ -dependence. One can extract the Coulomb-corrected CSB effect from the experimental data using this mass formula. More experimental data, especially for neutron-rich light hypernuclei [11], are needed and this mass formula can provide a guideline for future experiments on hypernuclei.

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