

# Hyperelastic modelling of yarn structures for dynamic applications

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**Abstract.** Dry fabrics comprised of high performance polymeric fibers have been widely used as protection layers in structures submitted to high velocity impacts (HVI). Their outstanding impact energy dissipation ability combined with an high strength-to-weight ratio make them a preferable choice in different applications such as bullet vests or blade containment systems over standard materials. Among the different approaches adopted to study these structures numerical methods assume a central role.

Thanks to their reduced costs and the related possibility of evaluating the effects of single phenomena, they are often used to predict the structure ballistic limits or to study the physical events which occur during the penetration.

Among the different strategies adopted to model a fabric, mesoscopic models have been largely adopted by different authors. These models assume the yarns as a continuum body while the fabric geometry is explicitly described. Nowadays yarn material models are universally assumed to be linear elastic and orthotropic. This modelling approach mostly focuses on the longitudinal behaviour of the yarn, however fiber-scale analyses and experimental results shows the importance of three-dimensional stress state on the ballistic limit.

In order to obtain a three-dimensional description of the yarn strain state during the impact, a novel hyperelastic model for yarn structures here is developed. In a first step, fiber-level preliminary analyses have been performed to obtain the effective behaviour of these structure under the projectile collision. In the second step, the hyperelastic model has been implemented and identified thanks to microscopic elementary tests. Finally, a continuum model of the yarn have been performed. First results show the relevance of the hyperelastic model compared to the fiber-level observation and enhance the limit of the classical linear elastic material model.

## 1 Introduction

High performance aramid and polyethylene fibres such as Kevlar and Spectra are widely used for impact protection systems due to their high strength and strength-to-weight ratio. Usually woven in flexible fabrics, they are used as main material in soft body armors or secondary layers in ceramic protections.

Mechanical behaviour of these materials under high velocity impacts has been largely studied and determined using different methods. Classical solutions include analytical [1], semi-empirical [2] and numerical approaches [3].

Within this group, numerical approaches are one of the most promising since they usually relies on less hypotheses compared to the analytical ones and do not require large investments in materials and time as experimental campaigns. Different numerical models have been proposed for fabrics under high velocity impact and they are historically classified according to the minimum scale individually modeled in the textile.

Macroscopic models assume single and multilayered textiles as a continuum body [4]. This approach tends to be preferable in terms of computational costs but it lacks of a correct representation of the complex physics behind

the failure of the system which is intrinsically related to the weaving geometry and yarn to yarn interaction [3].

In mesoscopic models, textile yarns are individually modelled as a continuum body. Here the weave geometry is explicitly reproduced and phenomena as yarn pullout, yarns failure and friction dissipation due to yarn relative motion are naturally taken into account. Classical yarn models mostly focus on the correct representation of the filament longitudinal behaviour, then they are not able to correctly model yarn failure related to multiaxial loads [5].

Finally, microscopic or multifilament models descent into the scale of fibers. In this case, yarns which comprise the fabric are modeled as an assemble of real or equivalent fibers [6] in order to take into account fiber-fiber friction dissipation and yarn transverse behaviour. Obviously this type of models considers the majority of the physical aspects which contribute to define the structure ballistic properties, anyway they are related to a significant, usually unacceptable, computational cost even for a single textile layer model.

In order to improve the global response of the actual mesoscopic models and extend their ability in representing the yarn transverse response, a novel yarn continuum model based on an hyperelastic constitutive law here is employed. The ability of the proposed model have been

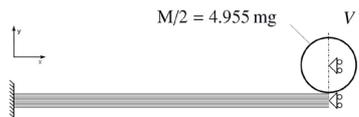
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assessed for the case of transverse impact on a single yarn. The results of the new mesoscopic approach are compared with those obtained by the same simulation performed at the microscale. In the first part of the paper the physical configuration of the impact test is presented. Secondly microscopic and mesoscopic models are described with their specific peculiarities and discretization. Finally, results of both models are discussed and compared.

## 2 Test Set up

The impact scenario consists in a single 25.4 mm length Kevlar KM2 600 yarn clamped at the extremities (Fig. 1) and impacted transversely in the centre by a cylindrical projectile. The 400 filaments which compose the yarn are assumed to be straight and circular with a constant diameter equal to  $12\ \mu\text{m}$ . A cylindrical projectile with a the mass of 9.91 mg and a diameter  $\phi$  of 2.2 mm is located in the centre of the yarn with contact condition at the initial time.

The initial speed have been set equal to  $120\ \text{m s}^{-1}$ . Due to the nature of the problem, symmetry conditions are applied for both the analyses.



**Fig. 1.** Transverse impact scenario.

### 2.1 Fiber Material Properties

Kevlar KM2 fibres exhibit a purely elastic strain-rate insensitive behaviour in the longitudinal direction and non-linear elastoplastic behaviour in their transverse direction [7, 8]. Longitudinal Young Modulus  $E$  and density  $\rho$  have been assumed equal to 84.62 GPa and  $1.44\ \text{g cm}^{-3}$  respectively [7] while transverse behaviour of the single fiber has been neglected. This last assumption is reasonably justified by the fact that yarn transverse kinematic is mostly related to fibers rearrangement within the cross section rather than single fibers transverse deformation.

Maximal strain in the longitudinal direction is assumed as failure criterion for the single fiber as for the yarn, with a strain limit  $\epsilon^{lim}$  equal to 0.0452. This choice is motivated by the fact that few experimental information are available nowadays for the multiaxial failure of polymeric fibres and yarns.

## 3 Microscopic model using Discrete Element Approach

In the present application, a revisited version of the discrete element method inspired by the work of Wang [9] is applied to model yarn fibres and explore the microscopic phenomena which have place during the impact. All the

400 fibres which compose the yarn are individually modeled as a series of equally spaced DEs relatively linked by truss elements, Fig. 2. These particles carry out the fiber inertial properties and the numerical treatment of contact. Here the fibres will be assumed rigid in the transverse direction, while the longitudinal elastic properties are provided by the truss constitutive behaviour.

Spherical DEs with a constant diameter of  $12\ \mu\text{m}$  are used to discretize the fibres. Relative distance among joined discrete elements has been set to  $12\ \mu\text{m}$  in the initial configuration, to provide fibres continuity.

Since all the fibres are equally discretized, the global mass of the yarn has been homogeneously distributed within the particles.



**Fig. 2.** Fibre model using DEs.

## 4 Mesoscopic model using Finite elements

In opposition to the microscopic model, here the yarn is modeled as a continuum body.

It is assumed to be straight with an elliptical cross section which does not vary along its length. The major and minor semi-axes of the section are taken equal to 0.0575 mm and 0.267 mm which results in a yarn volume fraction  $\nu = 0.88$ . A discretization of twelve elements per cross section has been assumed which results in 1200 four nodes linear solid elements for the whole yarn. A density of  $1.267\ \text{g cm}^{-3}$  has been computed for the equivalent continuum multiplying the original Kevlar density for the yarn volume fraction.

### 4.1 Hyperleastic constitutive law

A revisited version of the material model proposed by Charmetant [10] has been employed for the yarn. It is based on an additive decomposition of the strain energy density function Eq 1. Each one of the three terms is purely dependent by a single physical invariant  $I_e$ ,  $I_{tc}$  or  $I_{td}$  which is intrinsically related to a specific deformation mode: longitudinal elongation, transversal compaction and transversal distortion respectively.

$$W_i = f(I_e) + g(I_{tc}) + h(I_{td}) \quad (1)$$

$$f(I_e) = 0.5k_e I_e^2 \quad (2)$$

$$g(I_{tc}) = k_{tc} |I_{tc}|^p \quad (3)$$

$$h(I_{td}) = 0.5k_{td} I_{td}^2 \quad (4)$$

**Table 1.** Hyperelastic law material parameters.

$k_e$ [MPa]	$k_{tc}$ [MPa]	$p$	$k_{rd}$ [MPa]
80077.19	1055.82	2.205	649.75

Material parameters  $k_e$ ,  $k_{tc}$ ,  $p$ ,  $k_{rd}$  are reported in Table 1 and have been identified using an original multiscale methodology. Stress tensors components are then classically computed by internal energy derivation.

## 5 Results

Fig. 3 and 4 reports the yarn deformation at  $0\mu\text{s}$ (a),  $10\mu\text{s}$ (b),  $25\mu\text{s}$ (c) and  $40\mu\text{s}$ (d) for the microscopic and mesoscopic model respectively.

Transverse and spreading waves are observed in both the analyses. The transverse wave consist in the propagation of a vertical displacement field out of the impact zone. This wave firstly moves leftwards to the clamped edge in the period between  $0\mu\text{s}$  and  $20\mu\text{s}$ , it is reflected and then moves rightwards to the impact point,  $20\mu\text{s}$ - $30\mu\text{s}$ . When it reaches the impact point, yarn failure is observed in the microscopic model.

The spreading wave is related to the yarn section rearrangement.

When the yarn gets in contact with the projectile, the different fibres spread under the charge and yarn section is arranged into a new configuration. This rearrangement of the section travels in the form of a wave with the same propagation direction and the same speed of the transverse one, Fig.5.

The propagation of the spreading wave is naturally modeled at the microscale, since fibers rearrangement is explicitly taken into account, however is not so obvious to observe it in a continuous media. Here for the first time, according to the authors knowledge, this wave is observed into a mesoscopic model. This important novelty is related to the yarn hyperelastic constitutive law which does not require the classical hypothesis of null Poisson ratios currently assumed in linear elastic anisotropic models.

Fig. 6 reports a comparison among the projectile velocity history in the two models.

A very good correlation is obtained up to the instant in which microscopic model fails. This clearly indicate the ability of the homogeneous model in reproducing the mechanical response of the fibrous media. From the physical point of view, the projectile velocity trend is intrinsically related to the way in which its energy is stored into the impacted structure, A second observation concerns the failure criteria. In this impact scenario, failure is only ob-

served in the DEM model while continuous yarn keeps on absorbing energy even after the second reflection of the transverse wave. This clearly indicate that a pure longitudinal failure criteria is not sufficient to establish the equivalence among the two models.

## 6 Conclusion

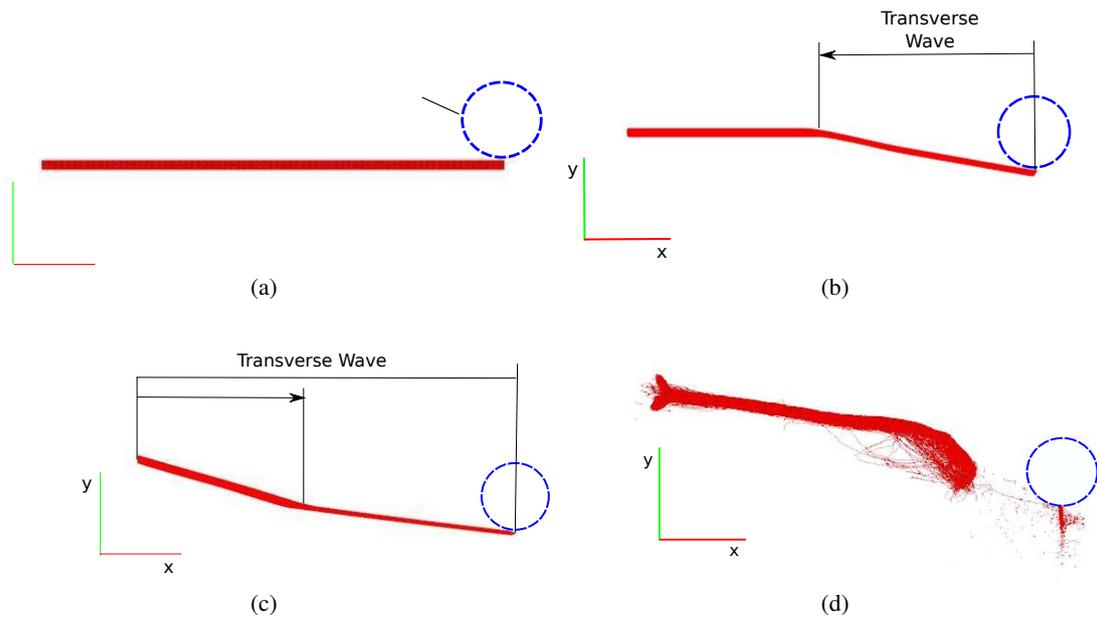
In the current work a new mesoscopic model for yarn structures have been presented and validated under the case of transverse impact. The validation has been performed comparing the results with those obtained by the modelisation of the same test at the microscopic scale.

All the principal kinematic aspects of the impact observed in the microscopic analysis, including spreading wave, have been captured by the mesoscopic model. The two models are even equivalent from the quantitative point of view up to the failure of the microscopic one. If the coherence of the pre-failure phase reassures on the constitutive law adopted, the the difference after the failure phase pose an accent on the failure criteria here adopted.

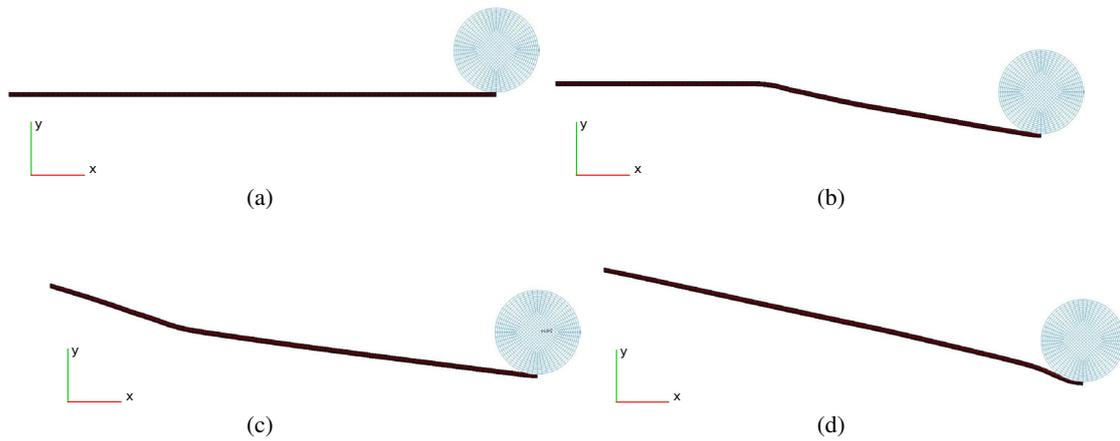
This critical point will be addressed in the future works and finally this constitutive behaviour will be adopted in the yarns at the fabric level.

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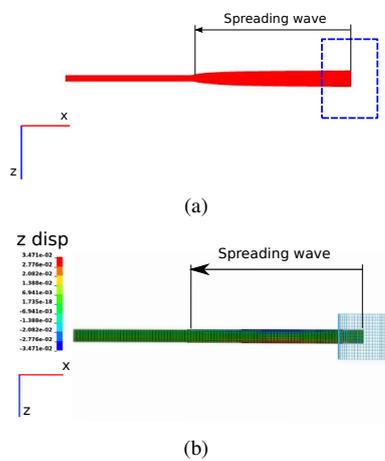
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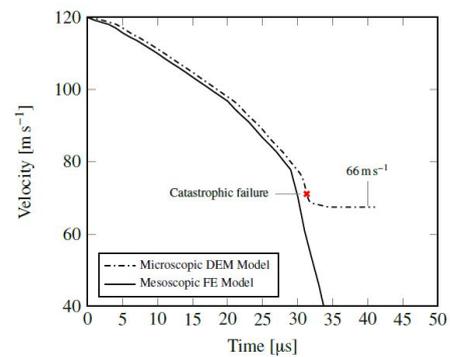
**Fig. 3.** Impacted Yarn using DEM.



**Fig. 4.** Impacted Yarn using FEM.



**Fig. 5.** Spreading wave propagation ( $10 \mu\text{s}$ ).



**Fig. 6.** Projectile velocity comparison for a transverse impact at  $120 \text{ m s}^{-1}$ .