

Numerical Analysis and Experiment for Stress Wave Propagation in Two Connected Cylindrical Bodies with Different Cross-Sectional Area and Same Mechanical Impedance

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Abstract. In this study, the behaviour of elastic stress wave propagating two connected cylindrical bodies was examined using dynamic finite element method (FEM). They consist of two bodies with different cross-sectional area, different Young's modulus and identical mechanical impedance. It was found that when an incident wave passes through the boundary step between two different cross-sectional areas, a pair of reflected waves which has the same amplitude and opposite sign was observed, despite the same mechanical impedance. This phenomenon appears to be caused by the loading and unloading the boundary section due to the arrival and the passage of incident wave. It was also found that a connection manner to insert the smaller diameter cylinder into the other cylinder with a little length is quite effective for the reduction of the reflected wave, because of the superposition of waves from two edges and control of local deformation. This phenomenon was verified by a series of impact experiments using two cylindrical bodies connected by interference fit.

1 Introduction

Recently, lightweight structures constructed by the materials with high strength have been used not only for airplane and space crafts but also cars and railway vehicles. This is because they offer us the reduction of running cost and of energy consumption which may be eco-friendly. In designing lightweight structure, it is essential to use a safety factor as small as possible [1]. Therefore, the optimally designed structures are usually thin, slim. In such lightweight and complex structures subjected to impact loading, relatively large stress concentration due to the superposition of stress waves appears. Thus, various accidental fractures occur at unexpected locations. When a stress wave generally passes through a boundary between two bodies with different mechanical impedance, the transmitted and reflected waves are usually generated [2-3].

According to one-dimensional elastic stress wave theory [3], it is understood that the reflected wave does not appear when the mechanical impedance $A\rho C$ is identical, where A is cross-sectional area, ρ is density, C is wave velocity of two rods connected. Even this requirement is satisfied, however, any reflected waves actually appear because of the presence of uncontacted area in the boundary [4-5], the abrupt changes of their cross-sectional area, dispersion effect of stress wave, etc.

In order to clarify the reason why a pair of reflected waves was created and an effective method to reduce the magnitude of reflected waves, the behaviour of elastic stress wave propagating two connected cylindrical elastic bodies with different cross-sectional area and the same mechanical impedance was examined by using dynamic finite element method (FEM). Some experiments were also carried out for the verification of suitable connection without the generation of any reflected waves.

2 Numerical Analysis

2.1 Model and conditions for calculation

For the analysis, we used the FEM code, LS-DYNA ver. ls971s R6.0.0, and a stepped bar model which consists of two cylinders with different cross-sectional area, as shown in Fig.1. These bodies were divided by axisymmetric square element with a side length $a_0 = 0.5$ mm. The time increment Δt was set to satisfy Courant condition, i.e. $C(a_0/\Delta t.) < 1$. The input bar has a diameter $d_1 = 20$ mm (i.e. cross-sectional area, $A_1 = \pi d_1^2/4$) and a length $L_1 = 1000$ mm, and the output bar has $d_2 = 60$ mm (i.e. $A_2 = \pi d_2^2/4$) and $L_2 = 750$ mm. Input bar was assumed to be made of an ordinary steel, Young's

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modulus E_1 of 205 GPa and density ρ_1 of 7900 kg/m³ (i.e. speed of stress wave, $C_1 = \sqrt{E_1/\rho_1} = 5.09$ km/s). The output bar was made of an imaginary material with $E_2 = 22.8$ GPa and $\rho_2 = 878$ kg/m³ (i.e. $C_2 = \sqrt{E_2/\rho_2} = 5.09$ km/s). These values were chosen as satisfying two conditions, $\sigma_r = 0$ (i.e. $A_1\rho_1C_1 = A_2\rho_2C_2$) and $C_1 = C_2$, where σ_r is the reflected stress wave from the discontinuous cross-section between these elastic bodies, which is denoted by D-section later, and given by

$$\sigma_r = \frac{A_2\rho_2C_2 - A_1\rho_1C_1}{A_2\rho_2C_2 + A_1\rho_1C_1} \sigma_i \quad (1)$$

Since an impact of striker easily makes extra vibrations on the top of an incident stress wave, a history of particle velocity, shown in Fig. 2, was fed into the left-hand side of input bar. By this input method of an incident wave, relatively smooth incident pulse with the magnitude of stress of about 100 MPa was obtained. The generated axial stress waves are measured at the point P which is at the surface of input bar far from 500 mm at both edges and the point Q which is at the surface of output bar far from 250 mm at connecting edge (see Fig.1).

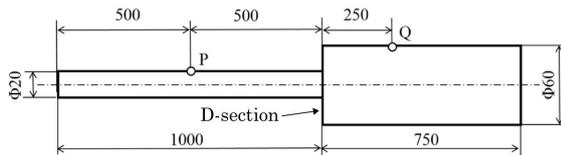


Fig. 1. Axisymmetric model consisting of two rods with different cross-sectional areas and the same mechanical impedance.

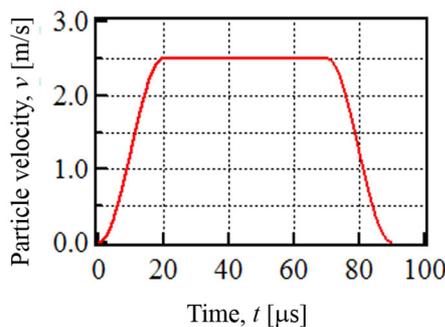


Fig. 2. Velocity history input into left edge of input bar which corresponds to stress wave of 100 MPa.

2.2 Reflected and transmitted stress waves

Figure 3 shows longitudinal normal stress waves σ obtained at both points P and Q, respectively on input and output bars. Relatively smooth rectangular incident and transmitted waves appear. The intensity of the transmitted wave σ_t is about -11 MPa which is almost 1/9 of the incident pulse σ_i (≈ 100 MPa), because of the difference of cross-sectional area. Curiously enough, the incident wave accompanied by a couple of pulses which has the same amplitude and opposite sign and their magnitude of about 34 MPa was measured in the input

bar, in spite of the same mechanical impedance of input and output bars.

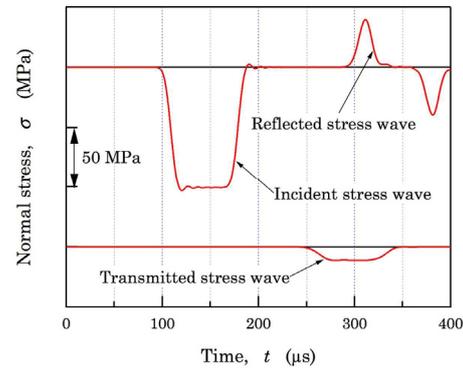


Fig. 3 Stress pulses propagating in input and output bars obtained at point P and Q.

In order to understand the identification of a pair of stresses, we drew a location-time curve of stress waves, as shown in Fig.4. Two oblique lines show, respectively, the locations of the ascent and the descent of an incident wave at any time t . By assuming that there are some reflected waves from the D-section, two broken lines were also added. These lines were drawn by taking the wave velocity $C = 5.09$ km/s and the length of input and output bars into account. The stress waves shown in Fig.3 were superimposed on the figure. From this figure, we can find that a pair of pulses comes into being when the stress rises up and falls down after the incident pulse reaches the D-section. Therefore, we can consider that it is a kind of reflected wave from the D-section.

2.3 Occurrence of reflected wave

Figure 5 shows the distribution of longitudinal nominal stress around the D-section at $t = 235$ μ s which corresponds to the situation after about 40 μ s since

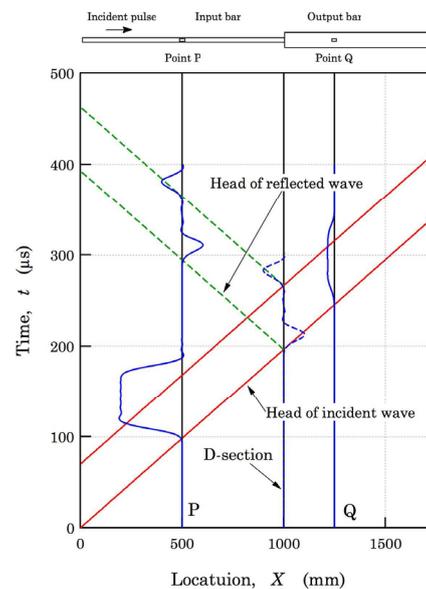


Fig. 4. Location-time curve of incident, reflected and transmitted pulses.

incident wave reaches the D-section. (see Fig.4). By taking the symmetry into account, only upper half from centre axis is shown. Then, the stress transient field like the propagation pattern of spherical wave from edge of input bar is seen. By moving a pair of reflected waves back to the location of the D-section along the reflected wave lines (see broken waves in Fig.4), it can be found that the reflected waves come to being during about 100 μs from about 200 μs to 300 μs , after the incident pulse reaches the D-section. In order to understand the change of stress distribution around the D-section, ten contour drawings with different times from $t = 192 \mu\text{s}$ to $t = 302 \mu\text{s}$ are shown in Fig.6. In these drawings, the displacement of only longitudinal direction was magnified 100 times to observe clearly the relation between the deformation and the stress distribution around D-section.

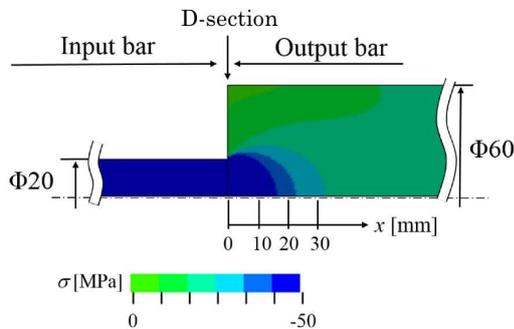


Fig. 5. Longitudinal normal stress distribution around D-section at $t = 235 \mu\text{s}$.

The first four figures from $t = 192 \mu\text{s}$ to $223 \mu\text{s}$ corresponds to the generation of tensile reflected wave. A compressive wave comes from input bar and spread into output bar. Then, D-section is bent by the invasion of input bar into output bar, i.e. the boundary between input and output bars is curved toward inside of output bar. This situation is the situation that the edge of input bar is pulled into output bar. This may be why the tensile reflected wave came to being. While, the last four figures, from $t = 271 \mu\text{s}$ to $302 \mu\text{s}$ corresponds to the generation of compressive reflected wave. The edge of input bar is pushed back by the edge of output bar because the D-section curved recovers to be a flat section. Thus, we can say that a couple of reflected waves is caused by the reaction of D-section when an incident compressive stress wave reaches and passes through D-section.

2.4 Effect of insert type connecting manner on reduction of reflected wave

Since a pair of reflected waves always appears following incident wave, it is quite important to find the effective method to make the reflected waves small for the minimum probability of unexpected stress concentrations. By introducing typical six kinds of connecting part into a space between input and output bars, we investigated the effect of the connecting part on

the reflected wave [5], and consequently, it was found that the best connecting manner can be brought by the insert type connection. i.e. the input bar with smaller diameter is inserted into the output bar with larger diameter, as shown in Fig. 7.

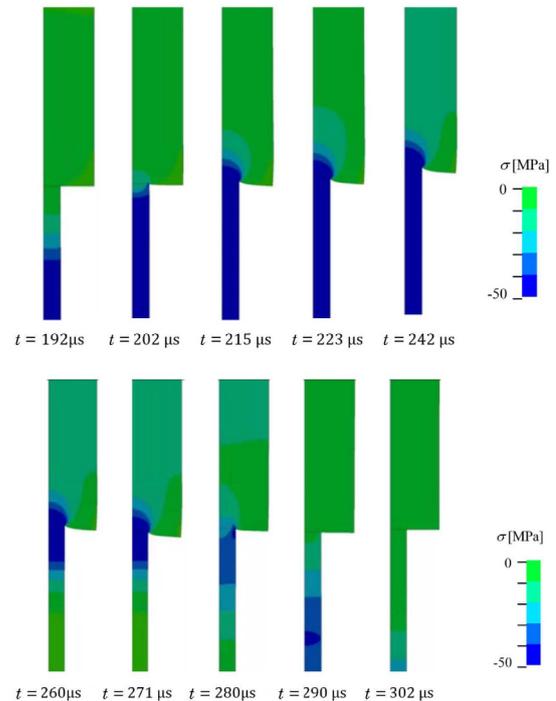


Fig. 6. Stress distribution around D-section and deformation of D-section during propagation of compressive stress wave.

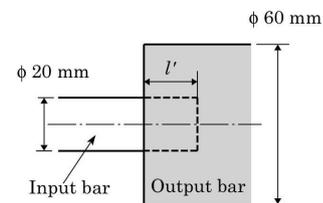


Fig. 7. Insert type connection between two rods with different cross-sectional areas.

In order to clarify the reason why the insert type connection is suitable for the reduction of reflected waves, it was investigated the effect of the insert length of input bar l' on the reflected waves (see Fig.7). The reflected waves coming from connections with the various insert lengths, $l' = 0, 10, 20, 30, 50$ and 100 mm , are shown in Fig.8. The reflected waves of $l' = 0 \text{ mm}$ are the same as those shown in Fig.1, of course. The magnitude of reflected wave decreases with the increase of the insert length from 0 to 20 mm. When the insert length becomes greater than 20 mm, however, the first one of a pair of reflected waves has opposite sign to that of the reflected wave of $l' = 0 \text{ mm}$ and its magnitude starts to increase with the increase of l' again. From this result, we can consider the insert length l^* making the total magnitude of a pair of reflected waves smallest, which corresponds to the sign change of the first reflected wave may. In the case shown in Fig.8, the

critical insert length is $l^* = 20$ mm because the amplitude of reflected waves becomes to be smallest.

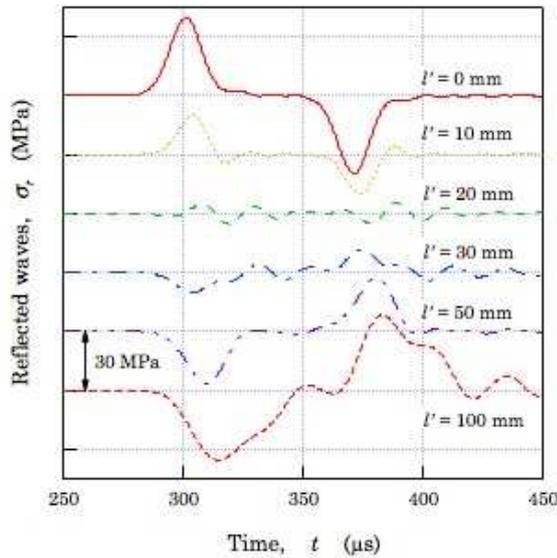


Fig. 8. Reflected waves from insert type connection with various insert length.

In order to understand this phenomenon, we thought two boundaries in another connection model with $l' = 500$ mm, as shown in Fig. 9(a). One is the left edge of output bar, denoted by boundary 1, and the other is the right edge of input bar, boundary 2. By using this model, we can separately obtain two reflected waves at point P' from boundary 1 and boundary 2 (see Fig. 9(b)). If the insert length l' is shorter than 500 mm, the second reflected wave may be observed earlier, like broken curve, for example. The sliding time can be easily calculated from the difference of l' and wave speed C , of course. Figure 10 shows the comparison between the original reflected waves in the case of $l' = 10, 20, 30, 50$ and 100 mm and the superimposed waves obtained from two separated reflected waves as shown in Fig. 11. From these figures, it is found that the reflected wave is produced by the super-position of waves generated from two boundaries, if the insert length l' is large enough, i.e. $l' \geq 50$ mm, at least. In the case of shorter insert length, however, the original reflected waves do not necessarily correspond to the superimposed reflected waves, especially smaller than l^* . When l' is small, therefore, the similar situation of stress to that observed in the connection of $l' = 0$ mm, i.e. pull-in and push-out phenomena of output bar, may predominantly affect.

2.5 Suitable insert length for reduction of reflected wave

In order to understand the relation between the magnitude of reflected waves and the insert length of various stepped structures, a series of analyses concerning the same input bar with diameter of 20 mm and output bars with various diameters ($D = 22, 40, 60, 80, 100$ mm) were carried out.

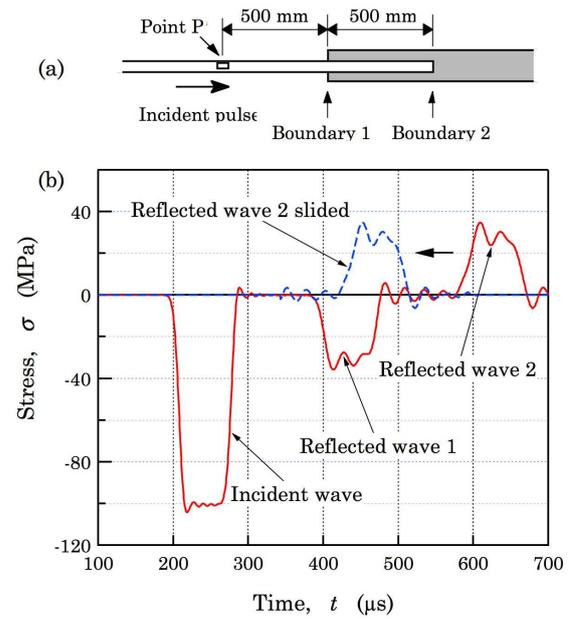


Fig. 9. Reflected waves separately obtained from long insert length model.

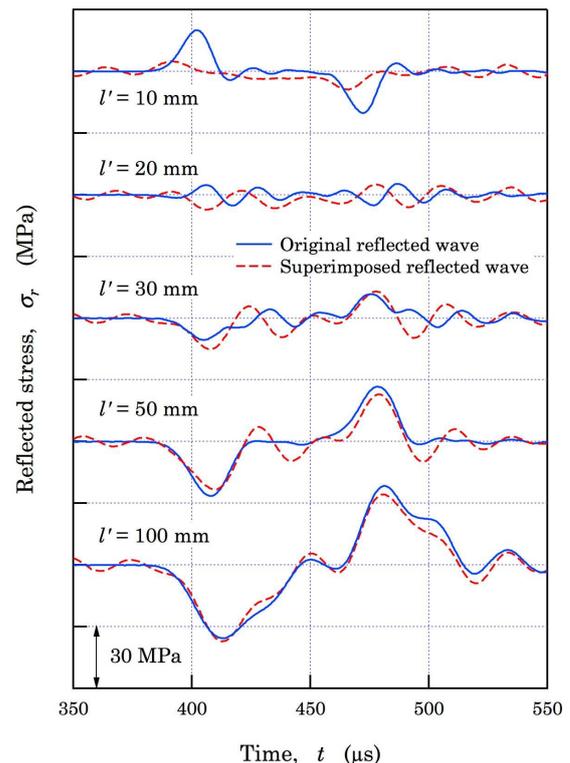


Fig. 10. Comparison of superimposed reflected waves with original reflected waves observed in connections with various insert length.

Young's modulus and density of these output bars are chosen as they satisfy the condition that the mechanical impedance $A\rho C$ of output bar is consistent with that of input bar, i.e. $\sigma_r = 0$ in Eq. (1). To estimate the magnitude of reflected waves, we adopted the reflected wave ratio W^* , defined by Eq. (2) as follows;

$$W = ACS, \quad W^* = \frac{W_r}{W_i} \quad (2)$$

where A is a cross-sectional area of a bar, C is a wave velocity, S is the absolute value of area surrounded by the stress wave and the base line of $\sigma = 0$ in the figure like Fig.3. Figure 11 shows the change of W^* against the insert length l' with the diameter of output bar as a parameter.

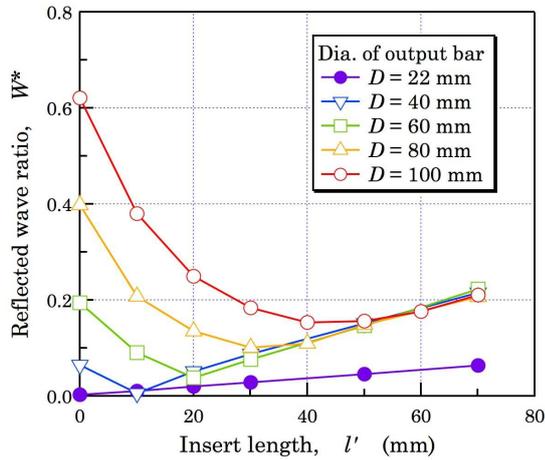


Fig. 11. Reflected wave ratio of connections with various insert length into various diameter of output bar.

The result of output bar with $D = 60$ mm in diameter is previous data shown in Fig.8 and the most suitable insert length l^* is 20 mm in this case, because the W^* of $l' = 20$ mm is smallest. The results of other output bars show the similar tendency and the most suitable insert length l^* increases with the increase of D . When D is smaller than 40 mm, the W^* monotonically increases with the increase of l' . In the case that the difference in diameters of input and output bars is small, the reflected waves become small and the energy of incident wave almost passes through the boundary.

3 Verification by Experiment

3.1 Experimental setup

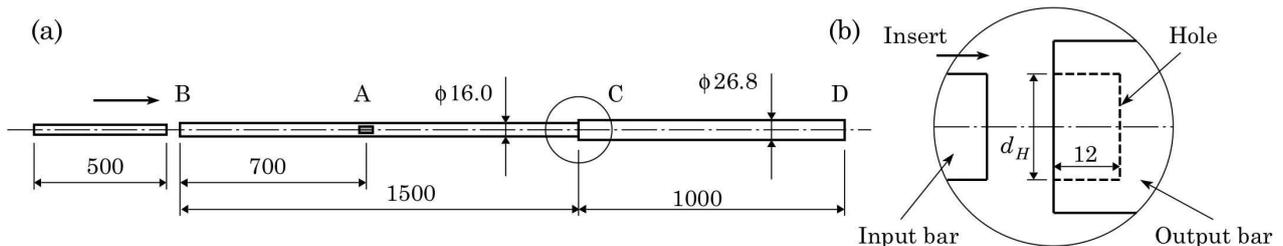


Fig. 12. Experimental setup for verification of analyses: (a) location of stress bars and (b) details of connecting.

In order to verify the behaviour of a pair of reflected waves from a stepped section between two elastic bodies obtained from FEM analyses, a number of experiments were carried out. Figure 12 shows the experimental setup using two different cylindrical bars of SUS304 ($\rho = 7930 \text{ kg/m}^3$, $E = 193 \text{ GPa}$), and aluminium alloy A2017-T5 ($\rho = 2840 \text{ kg/m}^3$, $E = 73 \text{ GPa}$). An incident and reflected stress waves were measured at a point A which is on input bar. The input bar has a diameter of 16.0 mm. While, the diameter of output bar was chosen to be $D = 26.8$ mm to satisfy the condition of mechanical impedance matching. The input and output bars were connected each other by using the insert type connection to make reflected waves as small as possible. We prepared two output bars. One has clearance fit which is brought by the bar with a hole of $d_H = 16.1$ mm in diameter and $l' = 12$ mm in depth on one edge surface of output bar, as shown in Fig.12(b). The other one has interference fit which has a hole with slightly small diameter of $d_H = 15.9$ mm to make different connecting conditions.

3.2 Results of Experiment

In the case of clearance fit, the stress record measured by a strain gauge A is shown by a broken line in Fig.13. The rectangular incident wave is a compressive stress wave denoted by (1). The second tensile wave created by the arrival of incident wave front at connection C and third compressive wave caused by the unloading at C are a pair of reflected waves from connection C as mentioned in previous sections, denoted by (2) and (3), respectively. After testing, the input bar was separated from the output bar because of clearance fit and relatively large incident pulse. The difference of wave shape between 2-nd and 3-rd waves may be caused by the separation. For the same reason, no reflected wave from the edge D appeared in the record. Although the fourth wave, denoted by (4), has the opposite sign to wave (2), it has the same shape as that of wave (2). There is just same relation between wave (3) and wave (6). Therefore, wave (4) and (6) can be considered to be the reflected waves of wave (2) and (3) from edge B. Since the magnitude of the largest reflected wave (wave (2) in this case) is about 60 % of the incident wave, the target to reduce the reflected wave as small as possible is not accomplished, yet.

In the case of interference fit, however, there are no large reflected waves as shown by a solid line in Fig.13. Since the connection between input and output bars did not released even after testing, a reflected wave from the right edge D of output bar, denoted by (5), appears. A pair of reflected waves considered to come from the connection C, i.e. waves (2) and (3), are extremely small comparing with the rectangular incident wave, about 10 % of the incident wave. This means that the insert type connection is quite effective to reduce the magnitude of the reflected waves as the prediction due to FEM analyses.

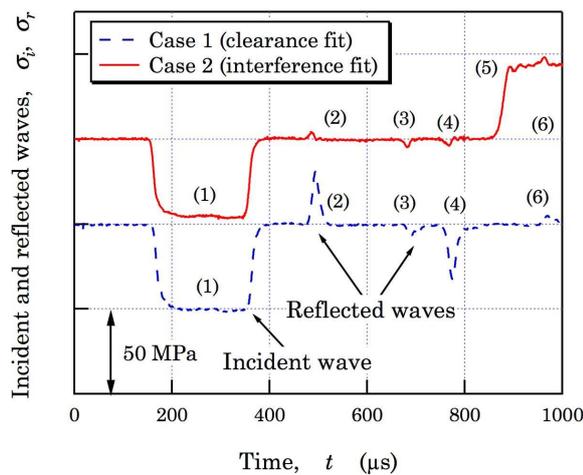


Fig. 13. Incident and reflected waves observed in experiments using input and output bars connected by clearance fit and interference fit.

4 Conclusions

In this study, the behaviour of elastic stress wave propagating two connected cylindrical elastic bodies with different cross-sectional area and Young’s modulus was investigated by using dynamic FEM analyses. Some experiments using input and output bars connected by clearance fit and interference fit were also carried out for the verification of FEM results. The results obtained are summarized as follows:

- (1) When a stress wave passes through a boundary connecting two elastic bodies with different cross-sectional area, a couple of reflected waves necessarily appear because of the reaction of discontinuous boundary due to the loading and unloading of incident wave passing, even mechanical impedance of two bodies is identical.
- (2) For the reduction of reflected waves from discontinuous boundary, the insert type connection, i.e. insert the cylinder with smaller diameter into the other cylinder with a suitable length, is quite effective.
- (3) The effectiveness of insert type connection was verified by some dynamic experiments using interference fit for the connection between two rods.

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