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Phase Dynamics in Arrays of Coupled Vortex Spin-Torque Nano-Oscillators

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Abstract. In this work, the mode analysis technique of complex networks nonlinear self-oscillatory vortex-based spin-torque nano-oscillattors (STNOs) with nonidentity and nonisochrony is developed. We construct adjacency matrices of different type of networks and calculate the normal modes. After the calculation of normal modes we shift to truncated equations for slowly varying amplitudes and phases in the normal coordinates using generalized quasi-Hamiltonian approach. Finally, we present the phase dynamics based on the Kuramoto-approach and compare different networks to the ability of synchronization.

1 Introduction

A great attention has attracted by studies of the phase locking in the arrays of spin transfer nano-oscillators (STNOs), which benefit from the spin-transfer torque phenomenon [1] and giant or tunnel magnetoresistance effects to generate microwave voltage signal. These oscillators in nanoscale can be easily tuned by a dc-current or external magnetic field, and compatible with CMOS-architectures. The enumerated features allow for consideration of STNOs for voltage or current controlled oscillators for telecommunication systems, and elements for building biological inspired computing systems [1]. Bio-inspired computing requires a capability to fabricate dense networks of interacting oscillators and to control the degree of coupling between STNOs. There are several studies of STNOs phase locking achieved by various physical mechanisms: through electrical connection in series of oscillators [2-4], by spin-wave propagation [5,6], by antivortices [7,8], and by dipolar coupling [9,10]. The synchronization and topological excitations in arrays of STNOs through the Kuramoto model described in [11]. Nevertheless, the theoretical description of the synchronization dynamics of STNO is more complicated than traditional limit-cycles oscillators (van der Pole oscillators, Josephson junctions, rotating pendula) which have constant orbit radius, and can be described in single oscillator case by the Adler equation. Due to technologically fabricated nonidentity of STNOs in the network [12] they can be expressed by generalized Adler-like equations.

In real networks there are as many frequencies (we will call them "modes") as the number of oscillators. When more than two oscillators are mutually coupled the finding of each mode structure and determination of their stability (or stability of group of them) becomes the main problem that has to be clarified. When the dissipation parameter is small enough, one can use the linear normal mode formalism for analyzing of the dynamics of the network of oscillators. If this condition is not satisfied, one can use the nonlinear normal mode formalism, which is much harder than the linear one. In general, the majority of these networks has the small dissipation parameter, and therefore it is possible to use the linear normal mode formalism for these systems.

The purpose of this paper is an analyzing of the structure of linear normal mode and mutual synchronization in different types of networks of nonlinear oscillators, from simple to complex cases. We investigate how network topology affects the stability of modes for the vortex-based STNOs. We show the impact of degeneration and synchronization of oscillation modes to the problem of optimal bio-inspired network design and microwave synthesizers. We find that hierarchical networks are characterized by a smaller number of stable nontrivial modes than partially hierarchical, or randomly organized networks. Our analysis gives rise to an approach for specifying of topological transformations of networks that can enhance synchronization.

2 Quasi-Hamiltonian Approach

We need to describe the nonlinear properties of a system of mutually coupled nonidentical and nonisochronous oscillator of complex networks from a common point of view. The quasi-Hamiltonian approach is sufficiently convenient for this description, especially for writing truncated equations (using averaging method). The quasi-Hamiltonian equations with N degrees of freedom are characterized by coordinates $q_1,\dots,q_N$ and impulses $p_1,\dots,p_N$. They satisfy the following equations:

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$$\frac{\partial q_j}{\partial t} = \frac{\partial H}{\partial p_j} + G_{1j}; \frac{\partial p_j}{\partial t} = -\frac{\partial H}{\partial q_j} + G_{2j}, \quad (1)$$

where $G_{1;2j}$ are the dissipative functions, and H is the Hamiltonian of the system. If $G_{1;2j} = 0$ then (1) gives simple Hamiltonian form of equations. Now lets change over to canonical variables $Q_j = \alpha_j q_j$, $P_j = p_j/\alpha_j$, where α_j are dimensionless constants (Q_j and P_j have the same dimension). After introducing the complex variables:

$$b_j = \left(Q_j + iP_j\right)/\sqrt{2}; b_j^* = \left(Q_j - iP_j\right)/\sqrt{2}, \quad (2)$$

and using following expression for Hamiltonian (here $X_j = \left\{Q_j, P_j\right\}$):

$$\frac{\partial H}{\partial X_j} = \frac{\partial H}{\partial b_j} \cdot \frac{\partial b_j}{\partial X_j} + \frac{\partial H}{\partial b_j^*} \cdot \frac{\partial b_j^*}{\partial X_j}. \quad (3)$$

We obtain equations of motion in the form

$$\frac{\partial b_j}{\partial t} = -i\frac{\partial H}{\partial b_j^*} + F_j; \frac{\partial b_j^*}{\partial t} = i\frac{\partial H}{\partial b_j} + F_j^*. \quad (4)$$

Here $F_j = \left(\alpha_j G_{1j} + iG_{2j}/\alpha_j\right)/\sqrt{2}$ and the second equation in (4) may be obtained from the first one by complex conjugation. There is a wide range of changes from canonical variables b_j, b_j^* to other variables. The possibility of choosing various canonical variables is an advantage of the Hamiltonian approach.

Assuming b_j, b_j^* to be small, let us expand the function $H\left(b_j, b_j^*\right)$ in a power series in b_j, b_j^*. We are not interested in the zero term $H(0,0)$, and the first order of H is equal to zero. Therefore, the expression of H begins with the second order terms in the form

$$H = H_2 + H_3 + H_4 + \ldots \quad (5)$$

In the general form of network geometry we can write

$$H_2 = \sum_{l,k=1}^{N} \left\{A_{lk} b_l b_k^* + 0.5\left(B_{lk}^* b_l b_k + B_{lk} b_l^* b_k^*\right)\right\}. \quad (6)$$

The Hamiltonian (6) may be diagonalized by linear uv-transformation

$$c_j = \sum_{m=1}^{N} \left\{u_{jm} b_m + v_{jm} b_m^*\right\}. \quad (7)$$

After special choosing of matrices u_{jm}; v_{jm} in (7) the Hamiltonian (6) takes the form

$$H_2 = \sum_{j=1}^{N} \omega_j c_j c_j^* + \sum_{k,m=1;k\neq m}^{N} \Omega_{km} c_k c_m. \quad (8)$$

Here $\left[\Omega_{km}\right]$ is the matrix of linear coupling between oscillators. This expression for H_2 we will use for calculating linear modes of oscillator networks. If the oscillator is nonisochronous (frequency is a function of the amplitude), we can obtain H_3 in a simple form

$$H_3 = \sum_{j=1}^{N} \mathrm{E}_j c_j c_j^* c_j. \quad (9)$$

After substituting (7)-(9) in (4) equations of motion for c_j we will have the following view

$$\frac{\partial c_j}{\partial t} = -i\omega_j c_j - i\sum_{k=1}^{N} \Omega_{jk} c_k - iN_j \left|c_j\right|^2 + \Upsilon_j, \quad (10)$$

where $\Upsilon_j = \sum_{m=1}^{N} \left[u_{jm} F_m + v_{jm} F_m^*\right]$. For several types of oscillators it is possible to write Υ_j in the simple form

$$\Upsilon_j(x) = \Upsilon_{j,-}(x) - \Upsilon_{j,+}(x). \quad (11)$$

Here $x = \left|c_j\right|^2$ - generalized power of complex variable c_j, $\Upsilon_{j,+}$ is a positive damping function (Hilbert damping in ferromagnetic or positive resistance in van der Pole scheme) and $\Upsilon_{j,-}$ is a negative damping function (spin-transfer torque effect in ferromagnetic or negative resistance in the van der Pole scheme). If the power x is small, we can expand the functions $\Upsilon_{-,+}$ in a power series. Now let us shift to the mode analysis of the fully identical nonlinear oscillators.

3 Mode Analysis & Truncated Equations

First, let us calculate normal modes λ_j of the complex networks. An unperturbed equation of (10) for $c_j^{\sim}$ is

$$\frac{\partial c_j^{\sim}}{\partial t} = -i\omega_j c_j^{\sim} - i\sum_{k=1}^{N} \Omega_{jk} c_k^{\sim}, \quad (12)$$

or for $\mathbf{c} = \left[c_1^{\sim}, \ldots, c_N^{\sim}\right]^T$ in the vector form

$$\frac{\partial \mathbf{c}}{\partial t} = -i \cdot \Xi \cdot \mathbf{c}, \quad (13)$$

with coupling matrix Ξ. These equations (12) or (13), we can transform into the canonical, using linear matrix transformation $\mathbf{c} = T \cdot \mathbf{a}$, $\mathbf{a} = \left[a_1, \ldots, a_N\right]^T$ into the

$$\frac{\partial \mathbf{a}}{\partial t} = -i \cdot \left(T^{(-1)} \cdot \Xi \cdot T\right) \cdot \mathbf{a}. \quad (14)$$

For eigenmodes λ_j we need

$$T^{(-1)} \cdot \Xi \cdot T = diag\left[\lambda_1, \ldots, \lambda_N\right]. \quad (15)$$

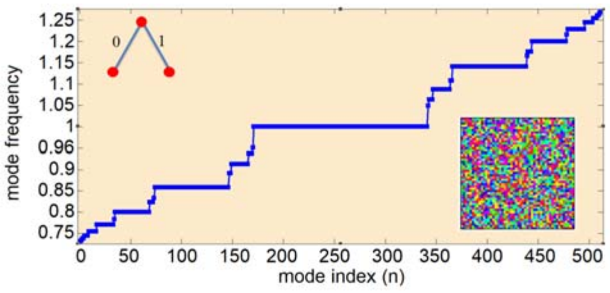
Form (14), (15) we can write

$$c_j^{\sim} = \sum_{m=1}^{N} t_{jm} a_m. \quad (16)$$

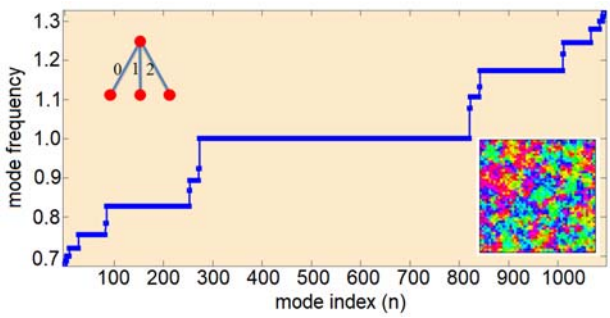
Now the unperturbed solution a_j could be expressed as follows (here U_j, ϕ_j are constants)

$$a_j(t) = U_j(t) e^{-i\lambda_j t - i\phi(t)}. \quad (17)$$

For the small coefficients $\Upsilon_{-,+}$, if we regard U_j, ϕ_j as slowly varying functions of time, then the dynamics of $U_j = U_j(t)$ and $\phi_j = \phi_j(t)$ can be represented by averaged equations or truncated equations.



(a)



(b)

Fig. 1. Mode structures of the 2-adic (a) and 3-adic (b) networks of STNOs and phase clusters.

For the 2-adic and 3-adic tree-like networks (see Fig.1) we have the fractal structure of the normal modes spectrum, which looks like devils staircase fractal. The randomly and hierarchically distributed networks shown in Fig.2. We analyzed the typical examples of the networks with N = 25 number of oscillators.

The structure of linear normal mode of the hierarchical network shown in Fig.2a. Here we can see the nonsymmetrical structure of the dependence and one wide section of the degenerate normal modes, which is equal to 9. Other modes are nondegenerate (nonequal to each other). From the other hand if we move into the hierarchically distributed network (Fig.2b) with the number of modes is equal to the previous one 25, we will see the different picture. Here we have the four blocks of degenerate modes with L=5 for three blocks and L=10 for one block, where L is the number of degenerate modes. Therefore, in general the number of degenerate modes for the hierarchically distributed network is larger than for the randomly organized.

For the task of power summation and mutually phase locking, it is better to use the hierarchically organized network where the number of degenerate modes is high. From the other hand, for the task of multimode communication it is better to use randomly distributed networks, because the number of degenerate modes is low. These effects confirmed by the nonlinear mode analysis of the microwave oscillators with these two topological structures of networks

4 Phase Dynamics & Kuramoto approach

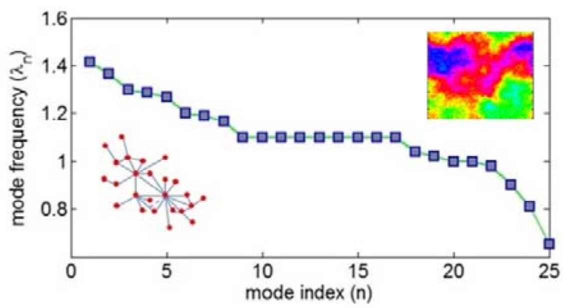
In the particular case, it is possible to use phase-approximation (Kuramoto-approach), when the amplitudes of oscillators signals are established faster than phases. Following this approximation for phases ϕ_j for each vortex STNO is

$$\frac{d\phi_j}{dt} = \omega_j + \sum_{k=1,k\neq j}^{N} \chi_{kj} \cdot \sin\left(\phi_k - \phi_j\right), \qquad (18)$$

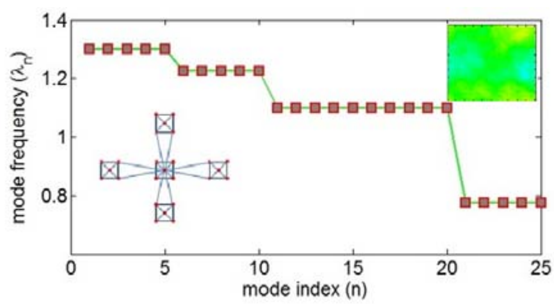
where ω_j are eigenfrequencies of oscillator ϕ_j, χ_{jk} are parameters of connection between ϕ_j and ϕ_k, which are decreased with increasing of a distance between oscillators, that is $\chi_{jk} \sim 1/d_{jk}$, where d_{jk} is the distance between vortex STNO.

Analysis of received system of phase equations shows that besides the steady-state regime of equal phases there are also complicated chaotic modes and chimera states, where one part of oscillators is phase-synchronizned and another part is in chaotic regime. It is necessary that non-identify of vortex STNO sizes would be least (units of *nm* when diameter of the sample is hungreds of *nm*) for capacity addition.

In the Fig.3 the phase planes of three Kuramoto-oscillators for completely phase locking, partial phase locking of 1 and 2, of 1 and 3 and asynchronous mode shown. Using same approach we calculated the phase distribution of different networks which are shown in Fig.1,2. It is easy to see that the partially hierarchical network (Fig.2b) is characterized by the lower number of phase clusters.

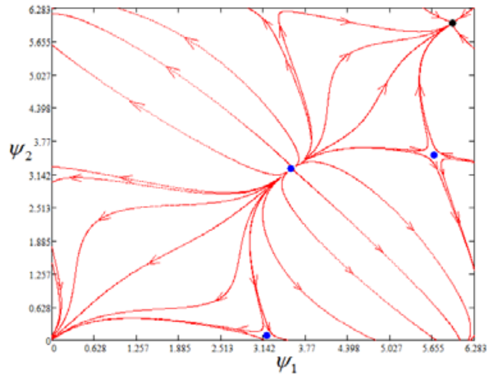


(a)

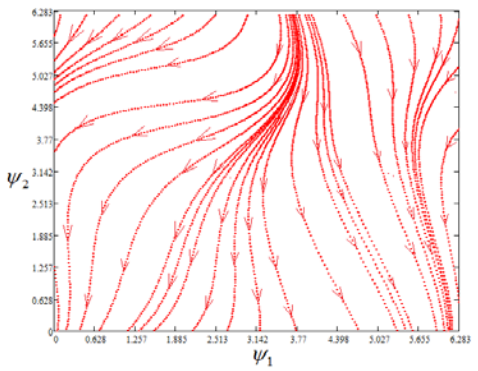


(b)

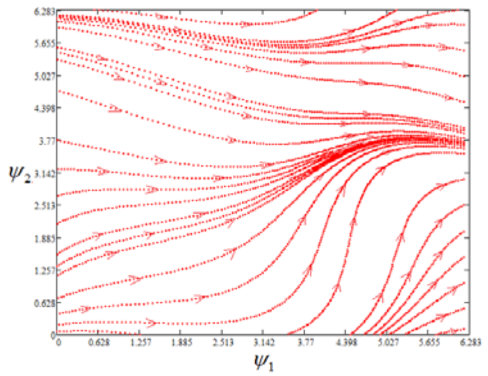
Fig. 2. Mode structures of the randomly (a) and hierarchically (b) distributed networks of STNOs and phase clusters..



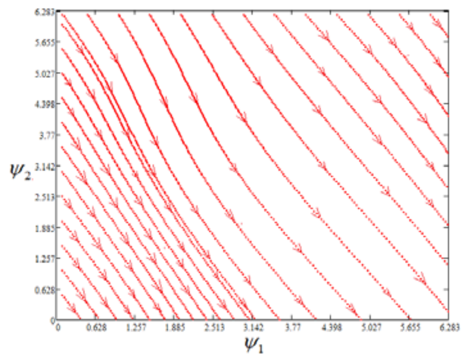
a)



(b)



(c)



(d)

Fig. 3. Phase planes of three Kuramoto-oscillators for completely phase locking (a), partial phase locking of 1 and 2 (b), of 1 and 3 (c) and asynchronous mode (d).

5 Conclusion

In this work we presented mathematical model that allows us to calculate linear normal modes by using quasi-Hamiltonian approach. We demonstrated the fractal structure of the normal modes spectrum and wrote truncated equations for slowly varying amplitudes and phases in the normal coordinates. The stability analysis of the normal modes submitted. The relationship between a structure of a network that consists of Kuramoto-oscillators, and the corresponding number of phase clusters is shown.

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