Magnetic fluid bridge between two cones and a cylinder in the magnetic field of a current-carrying wire

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Abstract. A magnetic fluid (MF) changes its surface shape in the magnetic field of a current-carrying wire while the current is changing. In the present paper, we propose to study a MF bridge which can open or close the channel formed by two cones and a cylinder by imposing the magnetic field of a wire. Numerical modelling of the bridge behavior for different values of MF volumes and currents in the wire is done for two cases: when the MF wets and does not wet surrounding solid boundaries. It is shown that the presence of limiting cones allows the MF to sustain the pressure drop which is much higher in case of non-wetting than in case of wetting. In case of wetting, the MF cannot sustain any pressure drop at low currents, but in case of non-wetting, the MF can do it even at zero current. It is found that in case of non-wetting, spasmodic and hysteresis phenomena are possible for some values of MF volumes and currents in the wire. The use of a MF in valves, dispensers and pumps is one of possible actuation methods.

1 Introduction

A magnetic fluid (MF) is a suspension of monodomain ferromagnetic particles (its size is about 10 nm), coated with a surfactant, in a non-magnetic carrier fluid. Such superparamagnetic fluid is attracted strongly by magnetic fields, for example, by the magnetic field of a straight current-carrying wire. The spasmodic spreading of a MF drop along a wire is studied theoretically [1-3] and observed in the experiment [3]. A MF drop on a wire with limiting conical surfaces is studied in [4]. A MF bridge between coaxial cylinders (there is a straight current-carrying wire on the axis of these cylinders) cannot sustain any pressure drop [1, 5]. In the present paper, we propose to use the magnetic field created by a wire to operate a MF bridge which can open and close the gap of special geometry (formed by coaxial conical and cylindrical surfaces). A numerical technique for investigating a MF bridge which can open such gap by the varying magnetic field of a current-carrying wire is proposed in [6]. Such MF bridge is limited by two cones, so the bridge can serve as a MF valve which sustains a pressure drop.

A MF is widely used in microfluidics as an actuation method in valves, dispensers and pumps. In [7] it is shown that a level of pressure in the range of tens of millibar can be expected from a MF. Different magnetic fields could be used to actuate the MF in such devices: for example, external permanent magnets [8-13], a coil [13-14], an oscillating magnetic line dipole [15], an electromagnet [16], and a straight current-carrying wire as in the present paper.

2 Problem statement

We consider a heavy, incompressible, homogenous, isothermal MF of the volume V between a cylindrical surface of the radius R_c and two limiting right circular truncated conical surfaces with different apex angles α_1 and α_2 (Fig. 1). All these surfaces are coaxial, and a straight wire of the radius r_0 carrying the current I is located on its axis. The cones intersect in a circle of the wire radius r_0. In this geometry, the MF bridge can sustain some pressure drop $\Delta p = p_1 - p_2$. The pressure p_1 is maintained above the MF and the pressure p_2 is maintained under the MF. The MF is immersed in a non-magnetic liquid (its magnetization $M_l = 0$) with the same density (the case of hydroweightlessness). If the MF does not wet solid boundaries then $90^\circ < \theta_1, \theta_2, \theta_3 \leq 180^\circ$, where θ_1 is the wetting angle of the upper conical surface, θ_2 – of the lower conical surface, θ_3 – of the outer cylinder. If the MF wets solid boundaries then $0^\circ \leq \theta_1, \theta_2, \theta_3 \leq 90^\circ$ (the case $\theta_i > \alpha_i$, $i = 1, 2$ is only considered).

The MF has an axially symmetric surface $z = h(r)$, $r^2 = x^2 + y^2$ (the axis z is directed along the axis of the wire). In this geometry, the magnetic field of the wire $\boldsymbol{H}$ is not deformed by the MF and $|\boldsymbol{H}| = H$, $H(r) = 2I/(cr)$, where c is the speed of light in vacuum [17]. We consider that for a MF with a low concentration of the same ferromagnetic particles, its magnetization M_f can be described by the Langevin law [18]: $M_f(\xi) = M_S L(\xi)$, $L(\xi) = \coth \xi - 1/\xi$, $\xi = mH/(kT)$, $m = M_S/n$. Here M_S is the saturation magnetization of a MF, m is the magnetic moment of one ferromagnetic particle, n is the number of

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ferromagnetic particles per unit volume of a MF, T is the MF temperature, k is the Boltzmann constant, ξ is the Langevin parameter which is directly proportional to the current I in the wire.

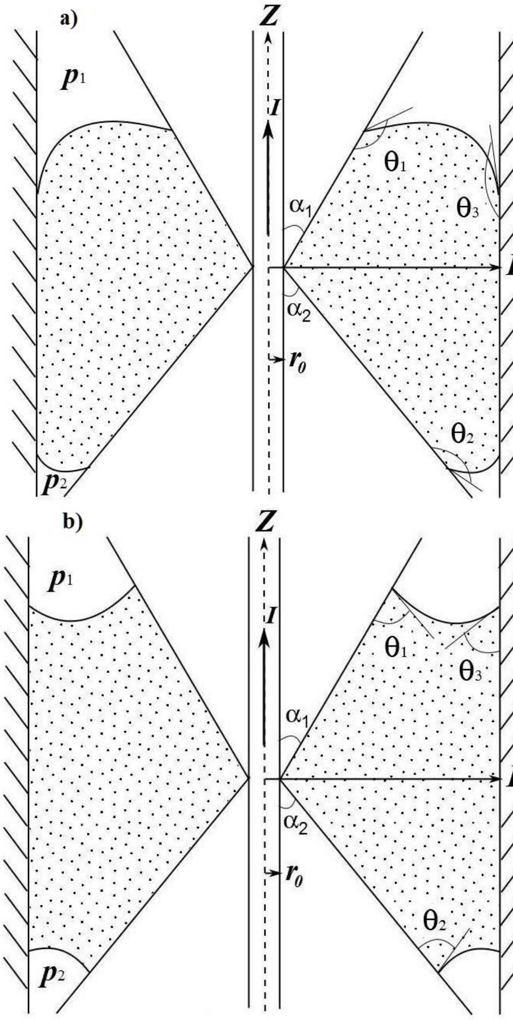

Fig. 1. MF bridge in case of (a) non-wetting and (b) wetting between two cones and a cylinder.

The hydrostatic equation and the boundary condition on the MF surface $h(r)$ [19] are used to get the general analytical solution for any axisymmetric shape of the MF surface $h(r)$ in the magnetic field created by a straight current-carrying wire [1-2, 4-6]. Let us introduce the following dimensionless parameters: $z^* = z/r_0$, $V^* = V/r_0^3$, $W = r_0\Delta p/(2\sigma)$, $\xi_0 = mH_0/(kT)$, $H_0 = 2I/(cr_0)$. Later, the signs "*" are omitted and parameters are considered as non-dimensional, unless otherwise specifically agreed.

Without loss of generality, we will further assume that $p_1 \geq p_2$. It should be noted that for $p_1 > p_2$, the MF bridge can take two different positions: to contact simultaneously the upper and the lower conical surfaces (Fig. 1) or to contact only the lower conical surface.

3 Numerical simulation

To simulate numerically the static shapes of MF surface, we fix the following problem parameters: $r_0 = 0.05$ cm, $R_c = 0.5$ cm, $T = 300$ K, $n = 1.9\cdot10^{17}$ cm^{-3}, $\alpha_1 = \alpha_2 = 5°$, $M_S = 56.6$ G, the coefficient of surface tension $\sigma = 20$ dyn/cm, $\theta_1 = \theta_2 = \theta_3 = 175°$ (non-wetting) and $\theta_1 = \theta_2 = \theta_3 = 70°$ (wetting).

3.1 Break-up of the magnetic fluid bridge

At each value of the current ξ_0 it is possible to calculate the MF surface shapes with the fixed volume V before reaching the value of current $\xi_0 = \xi_{break}$ or $\xi_0 = \xi_{cr}$. At some break value of current $\xi_0 = \xi_{break}$, the upper MF surface contacts the lower one, the MF volume becomes minimal to bridge the gap between conical and cylindrical surfaces ($V = V_0$) and the MF bridge breaks up and opens the channel predictably (at the same time, in case of non-wetting the MF surfaces touch simultaneously the cylinder at the same point). However, at some critical value of current $\xi_0 = \xi_{cr}$, the analytical solution [1-2, 4-6], which describes the static shape of MF surface, may stop existing earlier than the upper MF surface contacts the lower one. In this case, the MF surface has a vertical tangent at some point $r = r_{cr}$ and the MF bridge breaks up and opens the channel unpredictably.

3.1.1 Case of non-wetting

We consider the dependence of the value z_1 (z-coordinate of the upper contact line of three media: MF, non-magnetic liquid and upper/lower cone) on the current ξ_0 (Fig. 2) for different values of the MF volume V in case of non-wetting at the pressure drop $W = 1.27$.

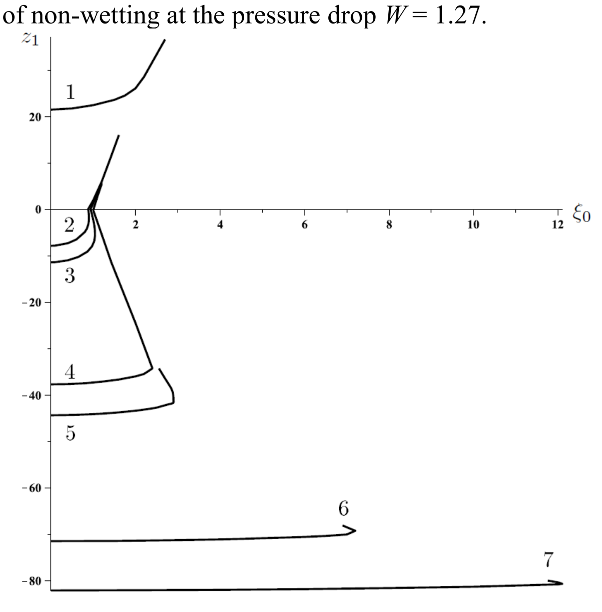

Fig. 2. Dependences $z_1 = z_1(\xi_0)$ for 1) $V = 27525$; 2) $V = 17770$; 3) $V = 16772$; 4) $V = 9580$; 5) $V = 7925$; 6) $V = 2460$; 7) $V = 1076$ in case of non-wetting at $W = 1.27$.

From Fig. 2 we can see that up to three different values of z_1 are possible for the fixed current ξ_0 and volume V (the higher MF equilibria for the curves 5, …, 7 and the middle one for the curves 3, 4 are unstable), so spasmodic and hysteresis phenomena are possible for some values of the MF volume V and of the current ξ_0 in the wire. The curves 1, …, 7 in Fig. 2

($0 < V \leq 27525$) come abruptly to an end when the MF bridge breaks up and opens the channel predictably at the current $\xi_0 = \xi_{break}$ (for the curves 4, 5 it happens twice at a lower and at a higher break current). The curves for $27525 < V < 40720$ (not shown in Fig. 2) are monotonically increasing functions like the curve 1 and it comes abruptly to an end when the MF bridge breaks up and opens the channel unpredictably at the current $\xi_0 = \xi_{cr}$. The volume $V = 40720$ corresponds to the maximal MF volume, which can be placed in the gap between conical and cylindrical surfaces.

3.1.2 Case of wetting

We consider the dependence of the value z_1 on the current ξ_0 (Fig. 3) for different values of the MF volume V in case of wetting at the pressure drop $W = 0.001$.

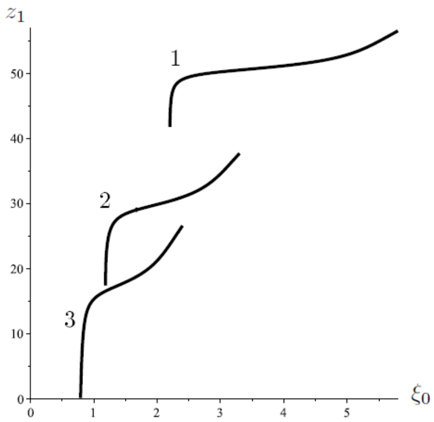

Fig. 3. Dependences $z_1 = z_1(\xi_0)$ for 1) $V = 27525$; 2) $V = 16772$; 3) $V = 9580$ in case of wetting at $W = 0.001$.

For these values of the problem parameters, the MF bridge can take only one position: to contact simultaneously the upper and the lower conical surface (Fig. 1). The curves in Fig. 3 come abruptly to an end on the right when the MF bridge breaks up and opens the channel either predictably at the current $\xi_0 = \xi_{break}$ for $V \leq 1367$ (the curves are not shown in Fig. 3) or unpredictably at the current $\xi_0 = \xi_{cr}$ for $1367 < V < 40720$ (the curves 1, …, 3 in Fig. 3). The same curves come abruptly to an end on the left at the minimal current, for which the pressure drop $W = 0.001$ can be sustained by the MF bridge of the fixed MF volume V. Fig. 3 shows that in case of wetting MF bridge cannot sustain the low pressure drop $W = 0.001$ at low currents in the wire, but in case of non-wetting the MF bridge can do it even at zero current.

3.2 Rebuilding of the magnetic fluid bridge

If the MF bridge breaks up and opens the channel predictably at the current $\xi_0 = \xi_{break}$, then in case of non-wetting the whole MF volume V turns into the MF drop on the cones of the thickness r_d and in case of wetting the whole MF volume V divides into two volumes: one volume on the cones and one on the cylinder. The case when the MF bridge breaks up and opens the channel unpredictably at the current $\xi_0 = \xi_{cr}$ is not considered.

3.2.1 Case of non-wetting

We consider the dependence of the minimal MF volume V_0, which can bridge the gap between conical and cylindrical surfaces (when the MF drop thickness r_d is equal to the cylinder radius R_c), on the current ξ_0 at the pressure drop $W = 0$ (Fig. 4).

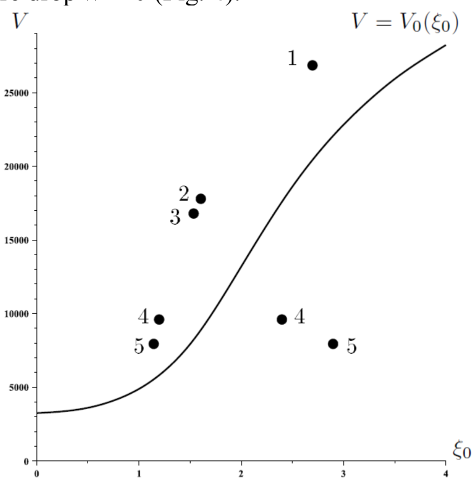

Fig. 4. The solid curve denotes the dependence $V = V_0(\xi_0)$ in case of non-wetting at $W = 0$.

Fig. 4 shows that the MF bridge cannot be rebuilt at any values of the MF volume, but only at $V \geq V_0(\xi_0 = 0) = 3428$. The points "•" of indices $i = 1, \ldots, 5$ in Fig. 4 correspond to the breaks-up of the MF bridge at the current $\xi_0 = \xi_{break}$ at the pressure drop $W = 1.27$ (see the curves in Fig. 2 of indices i which come abruptly to an end). The points "•" of indices $i = 4, 5$ are given twice in Fig. 4, because the "left" points correspond to the break-up of the MF bridge at a lower break current and the "right" points correspond to the break-up of the MF bridge at a higher break current. The points "•" of indices $i = 6, 7$ should be placed under the graph $V = V_0(\xi_0)$, but it is not depicted in Fig. 4, because it corresponds to $\xi_0 > 4$.

If the point "•" is placed above the graph $V = V_0(\xi_0)$ (for example, the "left" points of indices $i = 4, 5$ in Fig. 4), then the thickness r_d of the drop, formed on the cones because of the MF bridge break-up at $\xi_0 = \xi_{break}$, is greater than the radius R_c and the MF bridge is rebuilt. If the point "•" is placed under the graph $V = V_0(\xi_0)$, then the thickness r_d of such drop is less than the radius R_c and at the current $\xi_0 = \xi_{break}$ the MF bridge is not rebuilt. For example, points of indices $i = 6, 7$ are under the graph $V = V_0(\xi_0)$ at $\xi_0 > 4$, but the corresponding volumes $V = 2460$ and $V = 1076$ are not enough to bridge the gap at the pressure drop $W = 0$ even if the current ξ_0 decreases to zero. However, for big enough volumes $V > V_0(\xi_0 = 0) = 3428$, the MF bridge can be rebuilt through the decreasing of the current ξ_0. For example, the "right" points of indices $i = 4, 5$ in Fig. 4, which are placed under the graph $V = V_0(\xi_0)$, correspond to the volumes $V = 9580$ and $V = 7925$. If the current ξ_0 decreases from the value $\xi_0 = \xi_{break}$ to some value on the graph $V = V_0(\xi_0)$ corresponding to the volumes $V = 9580$ and $V = 7925$, the drop thickness r_d increases to the value of the radius R_c and the MF bridge is rebuilt.

3.2.2 Case of wetting

We consider the dependence of the minimal volume V_0, which can bridge the gap between cones and a cylinder (when the upper MF surface contacts the lower one), on the current ξ_0 at the pressure drop $W = 0$ (Fig. 5).

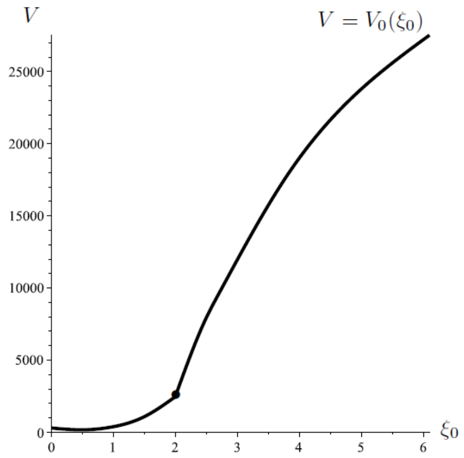

Fig. 5. Dependence $V = V_0(\xi_0)$ in case of wetting at $W = 0$.

In Fig. 5 the MF bridge of the volume $172 \le V \le 2460$ breaks up predictably at the current $0 \le \xi_0 = \xi_{break} \le 2$ (the part of the graph $V = V_0(\xi_0)$ to the left of the point "•"). The bridge of the volume $2460 < V < 40720$ breaks up unpredictably at $\xi_0 = \xi_{cr} > 2$ (the part of the graph $V = V_0(\xi_0)$ to the right of the point "•"). The graph $V = V_0(\xi_0)$ in Fig. 5 has a minimum $V = 172$. It means that for some range of volumes V and currents ξ_0, the bridge break-up can occur for both increase and decrease of the current. The volumes $172 \le V \le 2460$ are not enough to bridge the gap again even if the current ξ_0 decreases to zero. However, the bridge of big enough volume $2460 < V < 40720$ may be rebuilt after its break-up at the critical current.

4 Conclusion

The work demonstrates that the presence of limiting cones near a wire allows the MF bridge to sustain the pressure drop which is much higher in case of non-wetting than in case of wetting. In case of non-wetting, spasmodic and hysteresis phenomena may be presented for some fluid volumes and currents in a wire. In case of wetting, such phenomena are not found. In case of wetting, the bridge cannot sustain any pressure drop at low currents, but in case of non-wetting, the MF bridge can do it even at zero current.

The break-up and rebuilding of the MF bridge are investigated. The bridge breaks up either unpredictably at the critical current, for which the static shape of the MF surface stops existing and the MF surface has a vertical tangent, or predictably at the break current, for which the MF volume is minimal to bridge the gap between cones and a cylinder. In case of non-wetting, depending on the MF volume and the break value of current, the MF bridge can either be rebuilt or not. The bridge can be rebuilt only for big enough volumes. In case of wetting, the MF bridge of small volume cannot be rebuilt after its break-up at the break current, but the MF bridge of big enough volume may be rebuilt after its break-up at the critical current.

The results of the conducted research can be useful for the construction of different devices with the MF position controlled by the magnetic field, such as valves, dispensers and pumps.

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