EPJ Web of Conferences **189**, 00013 (2018) https://doi.org/10.1051/epjconf/201818900013
LNES 2017

Energy from nuclear fission

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Summary. — The main features of nuclear fission as a physical phenomenon will be revisited, emphasizing its peculiarities with respect to other nuclear reactions. Some basic concepts underlying the operation of nuclear reactors and the main types of reactors will be illustrated, including fast reactors, showing the most important differences among them. The nuclear cycle and radioactive nuclear waste production will be also discussed, along with the perspectives offered by next-generation nuclear assemblies being proposed. The current situation of nuclear power in the world, its role in reducing carbon emission and the available resources will be briefly illustrated.

1. – Physics of fission

What is fission and why can it produce energy? The basic reason is the behaviour of the nuclear mass and nuclear binding energy for different nuclear species. As is known from relativity, the mass of a bound system is given by the sum of the masses of its components, plus the binding energy of the system. The binding energy is by definition a negative quantity, *i.e.*, the mass of the composite system is always less than the sum of the masses of its components. When applied to nuclear masses, this concept can be expressed through the equation

$$M(Z,A) = ZM_p + (A-Z)M_n + B(Z,A), \tag{1}$$

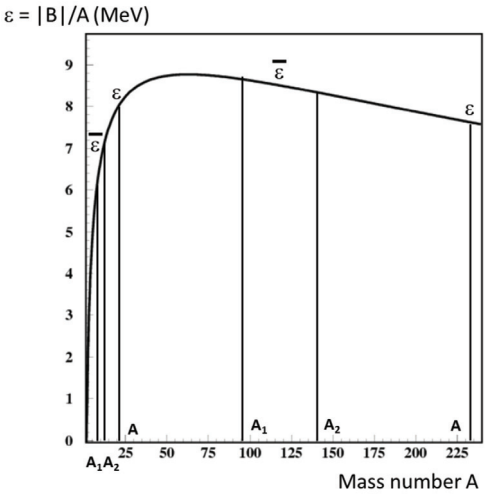

Fig. 1. – Absolute value of binding energy per nucleon, as a function of the mass number A.

where $M(Z, A)$ is the mass of a nucleus with Z protons and A protons and neutrons (*i.e.*, $A - Z$ neutrons), M_p is the proton mass, M_n is the neutron mass and $B(Z, A)$ is the binding energy of the system, with $B(Z, A) < 0$, *i.e.*, $M(Z, A) < ZM_p + (A - Z)M_n$. Z is the atomic number, while A is the mass number. The absolute value of the binding energy is the energy that must be supplied from outside to overcome the binding nuclear force and set all nucleons free. We can also define the binding energy per nucleon, *i.e.*, the average quantity of energy that must be supplied per each single nucleon when separating all nucleons from one another. As one can see from fig. 1, where the absolute value of the binding energy per nucleon is reported, $\epsilon = \frac{|B|}{A}$, such value increases from the lightest nucleus, the deuteron, to the region of iron, then it decreases towards the heavier nuclei. In other words, iron is more bound than the light nuclei, while, *e.g.*, uranium is less bound than iron.

1˙1. *Energy balance*. – For a nuclear reaction to give up more energy than it absorbs, it has to move in the direction of stronger binding. Indeed, if a system goes from lower binding energy to higher binding energy, the total mass will decrease in going from the initial to the final state, so that the difference in mass will correspond to energy being released by the system. Again looking at fig. 1, it is clear that two light nuclei undergoing nuclear fusion, *i.e.*, a nuclear system moving from the left-hand side of the graph in fig. 1 to the right, will move towards higher binding, thereby releasing nuclear energy, *i.e.*, giving rise to an *exoenergetic* nuclear reaction, namely a reaction which yields a net amount of energy. This is the mechanism at the basis of nuclear fusion [1], *e.g.*, in the reaction $^2\mathrm{H} + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + n$, where about 17.5 MeV are released in the process. On the other hand, heavy nuclei above iron can undergo fission, where the nuclear system splits into two nuclei, moving from the right-hand side of the graph in fig. 1 to the left, again

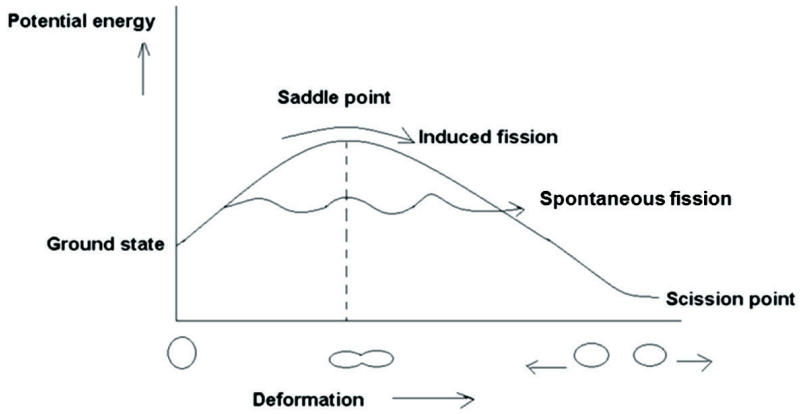

Fig. 2. – Nuclear potential energy as a function of the distance between the two fragments.

moving towards higher binding. This is called the process of nuclear fission and can also be exoenergetic. Let us consider in detail the energy balance of a fission process where a nucleus with Z protons and $(A-Z)$ neutrons splits into two nuclei,

$$(Z, A) \rightarrow (Z_1, A_1) + (Z_2, A_2), \tag{2}$$

where obviously $Z = Z_1 + Z_2$ and $A = A_1 + A_2$ and the masses of the nuclei are $M(Z, A)$, $M(Z_1, A_1)$ and $M(Z_2, A_2)$, respectively. The energy released in the reaction is called the Q-value and it is given by definition by

$$Q = M(Z, A) - M(Z_1, A_1) - M(Z_2, A_2). \tag{3}$$

By using eq. (1) we obtain

$$\begin{aligned} Q &= B(Z, A) - B(Z_1, A_1) - B(Z_2, A_2) = \\ &-\epsilon A + \epsilon_1 A_1 + \epsilon_2 A_2 = -\epsilon A + \bar{\epsilon} A = (\bar{\epsilon} - \epsilon) A, \end{aligned} \tag{4}$$

where

$$\bar{\epsilon} = \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{A_1 + A_2}\,. \tag{5}$$

Again looking at fig. 1, it is easy to see that if the initial nucleus is a heavy one (as indicated on the right of the picture), its fission into two medium-mass nuclei is such that $Q > 0$. By repeating a similar calculation for two light nuclei undergoing fusion, it is easy to see that also in that case one can get $Q > 0$.

The fact that fission of heavy nuclei can be energetically favored does not mean that fission will happen easily in a spontaneous manner. Indeed, by looking at fig. 2, imagine

the two daughter nuclei (called *fission fragments*) being bound together in a potential well: to be separated from each other, they will have to pass through the Coulomb repulsion barrier. This happens in the so-called *spontaneous fission* and the lifetime for the nucleus to decay via spontaneous fission will vary depending on the probability for the two nuclei to pass through the barrier via quantum tunneling. Clearly, if energy is supplied to the initial heavy nucleus from the outside, the system will jump to a more excited quantum state, *i.e.*, a higher energy state of the two fragments in the potential well. Therefore, the probability for the two fragments to penetrate the barrier will increase: this is what we call *induced fission* and can actually happen with appreciable probability for nuclei with $A \gtrsim 230$. Now, suppose that one more neutron is added to such a heavy nucleus (Z, A): in general, it will feature a significant binding energy (a few MeV) in the newly formed $(Z, A+1)$ nucleus. This means that when a neutron is captured by a heavy nucleus, a few MeV of energy will be released and made available for the two fragments inside the nucleus to approach the top of the barrier, thereby increasing the probability of the $(Z, A+1)$ nucleus to undergo fission by orders of magnitude. However, given the specific dependence of the binding energy on Z and A, typically for odd-A nuclei the binding energy of the extra neutron captured is higher than for even-A nuclei. The former nuclei are called *fissile* and they can undergo fission with high probability even when capturing a very slow neutron (even for a neutron in thermal equilibrium with the surrounding medium). The latter nuclei are called *fissionable* and they can undergo fission with significant probability only when capturing a neutron with kinetic energy of the order of 1 MeV or more. Two famous examples of the above are the fissile $^{235}_{92}$U and the fissionable $^{238}_{92}$U (as can be seen in the following).

Fission is not the only exoenergetic nuclear reaction induced by neutrons. Just to make two examples, the reaction $n + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + p$, when initiated by a very slow neutron (whose kinetic energy we can neglect), can release about 0.76 MeV in form of the kinetic energy of the two final nuclei. The reaction $n + {}^6\mathrm{Li} \rightarrow {}^3\mathrm{H} + {}^4\mathrm{He}$ can release about 4.78 MeV, again taken by the motion of the final nuclei. These features are used in ^{3}He tubes and in scintillation detectors with ^{6}Li coating, in particular to detect slow neutrons. Then what is so special about fission of heavy nuclei? First of all, it can happen with very high probability for fissile nuclei when exposed to slow neutrons. Secondly, the fission process is accompanied by the emission of a few *prompt neutrons*, *i.e.*, in addition to the splitting of the initial nucleus into two fragments, a few neutrons are emitted by the fragments in a very short time after the splitting. Such neutrons are fast, *i.e.*, they have high speed and high kinetic energy (about 2 MeV on average) and, as we will see shortly, if properly slowed down, they can be very effective in inducing another fission, and so on and so forth. Therefore, fission, under certain conditions, can give rise to a *chain reaction*, where the neutrons emitted by one fission can produce additional fissions, with a global very significant release of energy.

1˙2. *Energy released and reaction products.* – As one could in principle derive from eq. (4), when a uranium nucleus fissions into two nuclear fragments, about 0.1% of uranium mass appears as fission energy of ~ 200 MeV. Such an energy release is bigger

than any other exoenergetic nuclear reaction by more than an order of magnitude (see, for instance, the above example of neutron capture on ^{6}Li) and bigger than chemical reactions by several orders of magnitude (just for comparison, in the chemical reaction where a methane molecule is burnt, $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, about 9.2 eV of energy are released, more than 7 orders of magnitude less). Together with the fission fragments, an average of 2.5 prompt neutrons are emitted, with a mean kinetic energy per neutron of about 2 MeV (the total mean kinetic energy of all prompt neutrons being about 4.8 MeV). The average number of neutrons emitted is one of the many important parameters characterizing fission and it is called ν, whose value in general is of the order of 2–3, depending on the nucleus undergoing fission and on the incident neutron energy. Out of the 200 MeV released per fission, about 180 MeV appear at the time of fission as kinetic energy of the emerging two nuclei, plus kinetic energy of the prompt neutrons, plus energy in form of prompt gamma rays (photons) emitted when nuclear fragments are produced in a nuclear excited state that decays electromagnetically in a time so short that is considered negligible (say, typically less than a femtosecond, to give an idea, but sometimes much longer, depending on the nuclear state). Additional energy is released later because most nuclear fragments undergo β-decay (a process caused by the nuclear weak force, where Z of the nucleus increases by one unity, while A stays the same, and concurrently an electron and an antineutrino are emitted by the nucleus; β-decay is a very common process for the decay of unstable light- and medium-mass nuclei). Most of this energy is carried by the electron, the associated antineutrino and by the delayed gammas emitted when the beta-decay transition leads to an excited state of the final nucleus. However, since the antineutrinos will escape from the reactor with negligible interaction with the materials crossed, the amount of energy from β-decay actually contributing to the thermal budget of the reactor is about 13 MeV. When radioactive fragment build-up reaches a steady state, these 13 MeV (6.5% of fission energy) contribute to the instant reactor thermal power. At the exact moment when the reactor is shut down by stopping the chain reaction, the thermal power due to these radioactive decays, called *residual heat*, remains and decays slowly with time (see below).

1˙3. *Cross sections and flux*. – The cross-section is a physical quantity and an observable that characterizes a nuclear reaction (elastic, inelastic scattering, etc.). The cross section is connected to the range of the involved forces and represents the effective area of a nuclear target in a certain type of reaction or collision. Here we will only consider the so-called total cross section, defined as follows. Given a flux of particles incident on a single nucleus (target) per unit area $\mathrm{d}S$ per unit time

$$\frac{\mathrm{d}N_{in}}{\mathrm{d}S\,\mathrm{d}t}\,, \tag{6}$$

and given a reaction rate, which represents the number of interacting particles (scattered or absorbed projectiles) per unit time,

$$R = \frac{\mathrm{d}N_{reac}}{\mathrm{d}t}\,, \tag{7}$$

then the total reaction cross section σ is

$$\sigma = \frac{\frac{\mathrm{d}N_{reac}}{\mathrm{d}t}}{\frac{\mathrm{d}N_{in}}{\mathrm{d}S\,\mathrm{d}t}}\,. \tag{8}$$

As is evident from the above equation, σ has the physical dimensions of a surface, which is logical given its interpretation as an effective target area.

It is possible to show that, given a macroscopic target comprising several nuclei with mass density ρ and small thickness x, struck by a particle beam of intensity I (particles/s),

$$R = I\frac{\rho x}{A}N_A\sigma, \tag{9}$$

where A is the target atomic weight and N_A is the Avogadro number. Note that $\frac{\rho x}{A}$ is the number density of nuclei in the target (*i.e.*, number of nuclei per unit volume). For a target of arbitrary thickness, by dividing it in thin slices of thickness $\mathrm{d}x$, one obtains that

$$I(x) = I(0)\exp\left\{-\frac{\rho}{A}N_A\sigma x\right\}. \tag{10}$$

In nuclear physics, the cross section unit is the *barn*, corresponding to 10^{-24} cm^2.

The quantity

$$\Sigma = \frac{\rho}{A}N_A\sigma \tag{11}$$

is called the *macroscopic cross section* and represents the probability of interaction per unit length of material crossed (and, therefore, has dimensions of the inverse of a length). It is easy to show that $1/\Sigma$ is the mean free path of an incident particle in the material and Σv is the frequency at which a projectile with speed v interacts with the crossed material.

1˙4. *Types of nuclear reactions*. – The scattering process between two nuclei A and B can be *elastic*, when the reaction is $A+B \to A+B$ (just a collision without change in the form of the two colliding systems), *inelastic* in the case of reactions like $A+B \to A+B^*$ (where B^* means an excited state of nucleus B), $A+B \to A+C+D$ (where the nucleus B splits into C and D), $A+B \to C+D$ (where A and B, *e.g.*, exchange some protons or neutrons and, therefore, become different nuclei), etc.

The simplest type of nuclear reaction occurring in a nuclear reactor is the so-called *potential scattering*, where neutrons scatter elastically off the nuclear potential without ever penetrating the nucleus itself (similar to billiard ball collisions). By quantum mechanical arguments, it is possible to show that at low energies the cross section for such a reaction is essentially just the geometrical cross section of the nucleus. Such type of collision has a rather flat energy dependence from about 1 eV up to the MeV range.

Another very relevant reaction mechanism is *neutron capture*. Generally speaking, the addition of one more neutron to a nucleus liberates a certain quantity of energy coming from the binding of that neutron. This energy can make the nucleus jump to an excited state, from which it will typically either decay by gamma emission or by fission. The first process, followed by gamma emission, is called *radiative capture* and is clearly in competition with fission. By quantum-mechanical arguments, it is possible to show that at low energies the neutron capture cross-section will follow a $1/v$ law, v being the speed of the neutron. The same is true for fission, but only for fissile nuclei for which fission can occur even following the capture of a very slow neutron.

The capture process can proceed through a so-called resonance reaction, which occurs when the sum of CM energy of the neutron-nucleus system and the binding energy of the captured neutron matches one of the energy levels in the compound nucleus formed after capture. This phenomenon is indicated by sharp peaks in the capture cross section (see fig. 3). Neutrons which go around in a reactor core with an initial kinetic energy above the resonance region, can then "cross" a resonance when they scatter around and lose energy and can therefore be more strongly absorbed also by elements other than fuel.

Since the nuclear radius is roughly 10^{-12} cm, the geometrical cross sectional area of the nucleus is roughly $10^{-24}\,\mathrm{cm} = 1\,\mathrm{barn}$. However, the combination of quantum-mechanical and specific dynamical effects from the nuclear forces can make nuclear cross sections extremely variable over a few orders of magnitude. As an example, in fig. 3 are reported the cross sections for the processes of fission and radiative capture for the two uranium isotopes found in nature and used in nuclear reactors, ^{238}U and ^{235}U, as a function of the neutron energy [2]. It is clearly seen that in ^{235}U, for fast neutrons with kinetic energy around 100 keV (or 0.1 MeV), both fission and radiative capture show a cross section in the order of the barn, but for lower energies the radiative capture for both nuclei and the fission for ^{235}U grow towards much higher values, following the above-mentioned $1/v$ law.

Fission itself can produce fission fragments with very strong radiative capture cross sections: such specific fission products are called *neutron poisons*, *e.g.*, ^{135}Xe ($\sigma \simeq 2\times10^6$ barns for thermal —see below— neutrons, which is a *very* large cross section in nuclear physics), as they subtract neutrons from the chain reaction.

1˙5. *Neutron density and flux.* – We define as neutron density the expected number of neutrons with energy between E and $E + \mathrm{d}E$, in the volume $\mathrm{d}^3\vec{r}$ at the space position $\vec{r}$, at a time t at a certain position and at a certain time in a nuclear assembly. This quantity is indicated by $n(\vec{r}, E, t)$ and can be measured, for instance, in neutrons per cm^3 (or in cm^{-3}). The reaction density $R(\vec{r}, E, t)$ is defined as the number of reactions in the volume $\mathrm{d}^3\vec{r}$ about $\vec{r}$, at a time t, initiated by neutrons with energy between E and $E + \mathrm{d}E$ and is given by the equation

$$R(\vec{r}, E, t) = n(\vec{r}, E, t)\,\Sigma\, v, \tag{12}$$

where again v is the neutron speed. The reaction density is measured in reactions per cm^3 per second. We give a special name to the quantity $n(\vec{r}, E, t)\,v$: it is called the

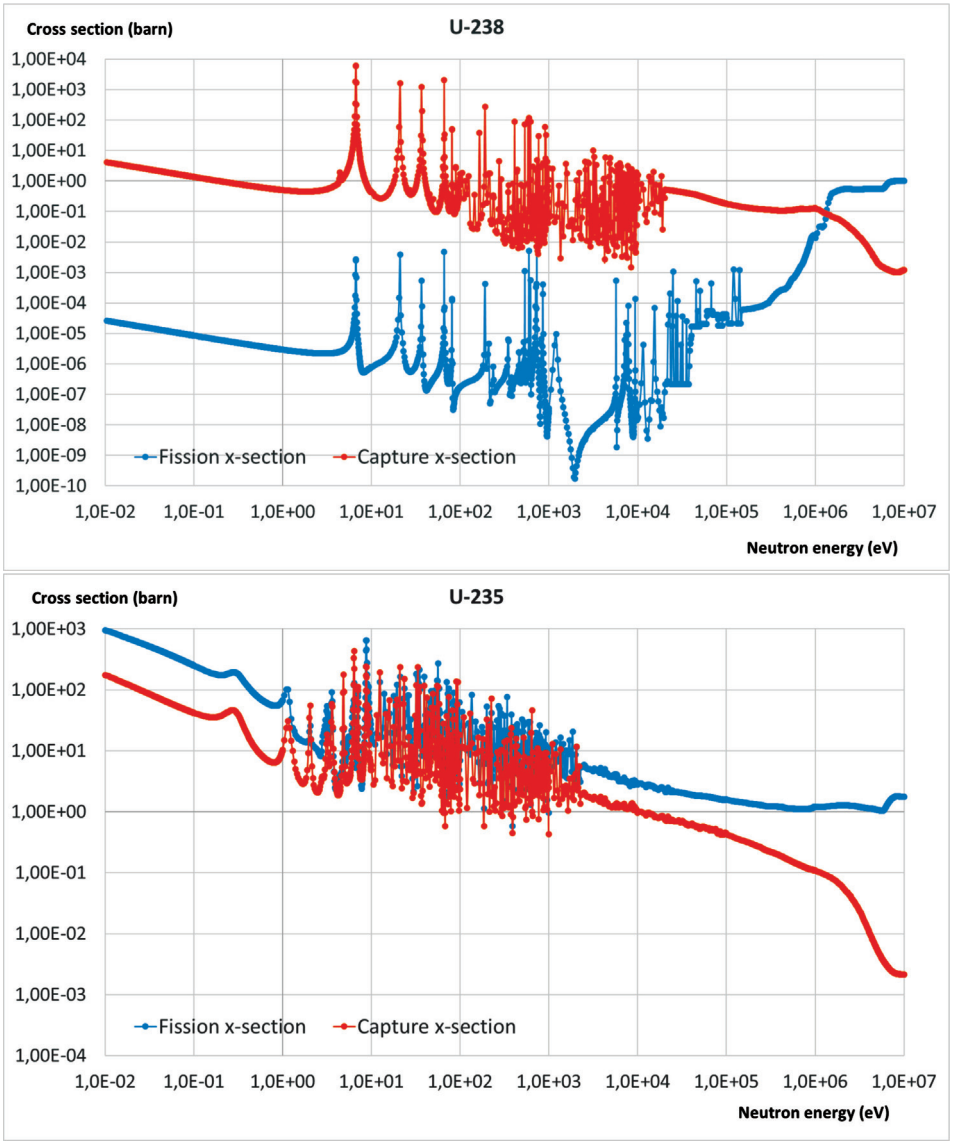

Fig. 3. – Fission (blue) and radiative capture (red) cross sections for the two isotopes of uranium.

neutron "flux" $\phi(\vec{r}, E, t)$ and is measured in neutrons per cm^2 per second. This quantity is essentially the number of neutrons arriving from whatever direction and crossing a unit surface in unit time (it differs from other definitions of flux in physics but we shall not discuss these differences here). Therefore, given a neutron flux ϕ and a macroscopic cross-section Σ, the reaction density, number of reactions per unit volume, R is simply given by $\Sigma\phi$. As an immediate example, consider a thermal reactor (see below) with 3 GW

thermal power = $3{\cdot}10^9$ J/s (it is indicated by the symbol 3 GWth and due to the efficiency of conversion to electric power, this corresponds to 1 GW electric power, indicated by 1 GWe). Assume each fission releases an order of 180 MeV energy = $3 \cdot 10^{-11}$ J at the moment of fission. Then it follows immediately that in the whole reactor the fission rate is about 10^{20} fissions/s, which corresponds to 2–$3 \cdot 10^{20}$ neutrons/s emitted and about $6 \cdot 10^{20}$ neutrinos/s from fragments' β-decay. By using the known value of the fission cross section for thermal (very slow) neutrons on ^{235}U, one can see that the flux inside the reactor must be in the order of $\phi \simeq 10^{13}$–10^{14} neutrons cm^{-2} s^{-1}.

1˙6. *Fuel.* – We already encountered the definition of *fissile* (*e.g.*, ^{233}U, ^{235}U, ^{239}Pu) and *fissionable* (*e.g.*, ^{238}U) nuclei. There are also nuclei that can produce a fissile isotope via neutron radiative capture and subsequent β-decay. These nuclei are called *fertile*, meaning that they can be used to produce fuel and essentially comprise ^{238}U, ^{232}Th. As an example, ^{238}U can produce a fissile nucleus via the chain of reactions

$$n + {}^{238}\mathrm{U} \to {}^{239}\mathrm{U} + \gamma \to {}^{239}\mathrm{Np} + e^- + \bar{\nu} \to {}^{239}\mathrm{Pu} + e^- + \bar{\nu}. \tag{13}$$

Natural uranium is composed by 0.7% ^{235}U and 99.3% ^{238}U. Given the cross sections shown in fig. 3, it is easy to understand that in a reactor where most neutrons are in thermal equilibrium with the surrounding materials, *e.g.*, they have energies < 1 eV, ^{235}U is the uranium isotope that actually plays the role of fuel by providing the dominant contribution to the fission rate.

Let us go back to our ideal 1 GWe reactor burning only ^{235}U. We already calculated that about 10^{20} fissions/s occur in the reactor, which means that 10^{20} (^{235}U nuclei)/s disappear (actually somewhat more because of radiative capture). Therefore roughly 40 mg/s of ^{235}U are "burnt" in the reactor. For 1 year of operation at 80% load factor (the actual working time), this means a consumption of about 1 ton of ^{235}U: in volume of pure metallic ^{235}U, this would be a cube of about 36 cm side. For comparison, the same amount of electric power is obtained by burning 1.6 Mtoe, million tons of oil equivalent (considering a slightly higher conversion efficiency of oil-fired power plants), or about 2 billion cubic meters of natural gas, or 2.2 million tons of coal. In a real fission reactor ^{235}U consumption is partly compensated by plutonium (^{239}Pu) breeding according to eq. (13), as ^{239}Pu is also a fissile nucleus. In practice, the initial load of a typical 1 GWe PWR (see below) comprises about 27 tons of enriched uranium, of which about 950 kg are ^{235}U. After 1 year 280 kg of ^{235}U remain: about 560 kg of ^{235}U and 380 kg of other isotopes, mostly ^{239}Pu have been burnt, which is approximately the 1 ton we have been estimating above.

1˙7. *Fast and slow neutrons.* – We have seen that the cross sections for nuclear reactions induced by neutrons depend a lot on the energies involved, therefore it is important to introduce some classifications of the neutrons according to their energies. It is customary to adopt the following classification: *slow* neutrons are those with kinetic energy $T_n < 1$ eV, in particular thermal neutrons at room temperature have T_n around 0.025 eV or 25 meV (the value of kT, where k is the Boltzmann constant and T is the temperature

of the system); *epithermal* neutrons are those with $1\,\text{eV} < T_n < 100\,\text{keV}$ (0.1 MeV); finally, *fast* neutrons have $0.1\,\text{MeV} < T_n < 20\,\text{MeV}$. Obviously neutrons in general can have energies above 20 MeV but this is an extreme limit in reactor physics (*e.g.*, neutrons from fusion of a deuteron and a tritium nucleus have 14 MeV fixed energy).

It is easy to show in non-relativistic kinematics that after a scattering off a nucleus with mass number A, *on average* the kinetic energy of the neutron changes according to the ratio

$$\frac{T_n'}{T_n} \simeq \frac{1+A^2}{(1+A)^2} \tag{14}$$

(where T_n and T_n' are the kinetic energies before and after the scattering, respectively). For a heavy nucleus $A \gg 1$, one has that $T_n' \simeq T_n$ or, in other words, the neutron has to undergo many collisions in order to significantly lose energy. Consider instead the case $A = 1$ (this is relevant for any medium containing hydrogen, where protons, hydrogen nuclei, will be among the targets). In this case $T_n' = \frac{T_n}{2}$, *i.e.*, on average a neutron will lose half of its energy at each collision and, therefore, few collisions are sufficient to rapidly decrease its energy. For collisions on a light nucleus like carbon ($A = 12$), the situation will be similar to that of hydrogen, although the average loss of energy per collision will be lower. Light materials containing hydrogen (water, paraffin, plastic materials in general, etc.) or containing light nuclei (graphite, etc.) are called *moderators*, for their effectiveness in slowing down neutrons. In a thermal reactor, the fuel is immersed in water, which is used both as coolant and as moderator: the fast neutrons from fission will be slowed down very quickly via collisions with protons and oxygen nuclei in water and, therefore, they will have a very high fission cross section for ^{235}U (see fig. 3).

2. – Chain reaction and simple reactor kinetics

For the reactor to work effectively and safely, the chain reaction must not diverge and must not die away. This means that, amongst the 2–3 neutrons emitted on average in the fission process, precisely one has to induce another fission event, no more and no less. The remaining fission neutrons will then either be absorbed by radiative capture or will leak out from the system. The character of the chain reaction depends essentially on the reactor composition and on its geometry and is defined through the quantity called k, also called multiplication coefficient,

$$k = \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}}\,. \tag{15}$$

The condition $k = 1$ corresponds to a *critical* reactor, *i.e.*, a reactor actually working and producing thermal power from fission. For $k > 1$ we have a supercritical reactor (fission reactions tend to run away) and for $k < 1$ we have a subcritical reactor (fission

reactions tend to die away). In a "simple minded" reactor time evolution, called *kinetics*, it is relatively easy to show that the number of neutrons in the system evolves as

$$n(t) = n(0) \exp\left(\frac{k-1}{\tau}\, t\right), \tag{16}$$

where $n(0)$ is the initial number of neutrons and τ represents the average lifetime of a neutron in the system before it disappears due to fission, capture or escape (leakage). Here we clearly see that $k = 1$ corresponds to a steady state, while $k > 1$ implies that the system is diverging and $k < 1$ corresponds to a decreasing population. Based on this simple kinetics, we see that the time constant governing the response of the reactor would be $\frac{\tau}{k-1}$. Since the typical neutron lifetime in a thermal power reactor is of the order of 10^{-4} s, we see that even for $k = 1.001$, the neutron population and, therefore, the reactor power would increase by a factor 2.7 in 0.1 s, which would not be a manageable situation from a practical point of view. But there is one missing piece in the picture of fission discussed above. Actually, there is a small fraction (less than a percent) of neutrons originating from the fission process that are not emitted immediately, but some time later. These neutrons are emitted by highly excited fission fragments on a time scale from milliseconds to seconds and for this reason are called *delayed neutrons*. In simple terms, a nuclear reactor is made critical thanks to such small fraction of delayed neutrons and, therefore, they are the ones that dominate the reactor response time making it much longer, thereby allowing reactor control by neutron-absorbing control rods.

2˙1. *A bit more on neutron multiplication and transport.* – By considering in more detail what processes can intervene along the route of the next-generation neutrons emitted after a single fission, one can come up with the following definition of the multiplication coefficient k:

$$k = \eta f P_{NL}, \tag{17}$$

where f is the conditional probability that, if a neutron is absorbed, it will be absorbed in the fuel instead of structural elements; η is the average number of neutrons produced per neutron absorbed in the fuel, in other words the average number of neutrons per fission ν multiplied by the probability that the absorption process leads to fission instead of radiative capture $\eta = \nu \frac{\sigma_f^F}{\sigma_a^F}$, where σ_a^F is the absorption cross section in the fuel (absorption = fission + radiative capture) and σ_f^F is the fission cross-section in the fuel. Finally, P_{NL} is the probability that a neutron does not escape from the reactor (no-leakage). For an infinite reactor clearly $P_{NL} = 1$ and we get $k_\infty = \eta f$. If we consider that fissions can occur both from slow and fast neutrons, there is a correction factor $\epsilon > 1$ which simply tells that fissions can be more than just those produced by slow neutrons (as we saw before, in a thermal reactor the latter will be the vast majority). If we also take into account that neutron moderation to small speeds implies a number of collisions, we see that the neutron may not survive such collisions and be absorbed, so we

will add a factor p representing the fraction of fission neutrons that survive moderation without being absorbed. For an infinite reactor where $P_{NL} = 1$, we arrive at the *4 factor formula*:

$$k_\infty = p\epsilon\eta f. \tag{18}$$

If we instead consider a finite reactor and for the sake of precision we consider the probability of no-leakage for thermal (slow) and fast neutrons separately, *i.e.* if we write $P_{NL} = P_{TNL}P_{FNL}$ (T for thermal and F for fast), we arrive at a formula for the *effective* multiplication coefficient, the *6 factor formula*:

$$k_{eff} = p\epsilon\eta f P_{TNL}P_{FNL}. \tag{19}$$

The name "effective" means that we are not considering an infinite, homogeneous medium but a more realistic situation where obviously $k_{eff} < k_\infty$, *i.e.*, in a real reactor one has to consider that the unavoidable leakage due to neutrons escaping from the reactor will make the multiplication in the chain reaction smaller than in an ideal, infinite medium. In practice, to make P_{TNL} and P_{FNL} as close as possible to one and increase k_{eff}, a *reflector* is put around the reactor core containing the fuel, *i.e.*, a thick blanket of material (graphite, Cu, Pb depending on the reactor type) with the purpose of scattering back (reflecting) the neutrons into the core. It is interesting to look at the behavior of the quantity η as a function of neutron energy, shown in fig. 4. It is clear that for thermal neutrons, this number is around 2, which means that effectively there are two neutrons available for further processes, one of which can trigger another fission, while the other can be lost in captures or leak out of the system. In case of capture on a fertile nucleus like ^{238}U, plutonium breeding can occur, but overall this will happen with relatively low probability. On the contrary, for high kinetic energies about 10–100 keV (fast neutrons), η becomes larger than 2 and so more neutrons will be available for breeding new fuel. Therefore, a so-called *fast reactor*, *i.e.*, one in which neutrons are not moderated (see subsection "Nuclear reactor types"), may produce more plutonium than it consumes, in which case we will have a *fast breeder*.

2˙2. *Subcritical assemblies*. – From eq. (15), we see that the multiplication coefficient k must be equal to 1 to attain criticality, *i.e.*, to make the reactor work and produce power by a self-sustained chain reaction. What happens if k is less than unity? Clearly, if you start from criticality, then reduce the multiplication by, for instance, inserting control rods significantly, the chain reaction will die away exponentially fast, as indicated by eq. (16). However, the chain reaction can still be kept going if an *external* neutron source is active within the system. It is possible to solve the appropriate equations with such an external source and find a stationary state where the neutron flux and, therefore, the system's power are constant. If we indicate the external source as S_0 (neutrons/cm^3/s supplied by the external source to the reactor core), for an infinite, homogeneous reactor

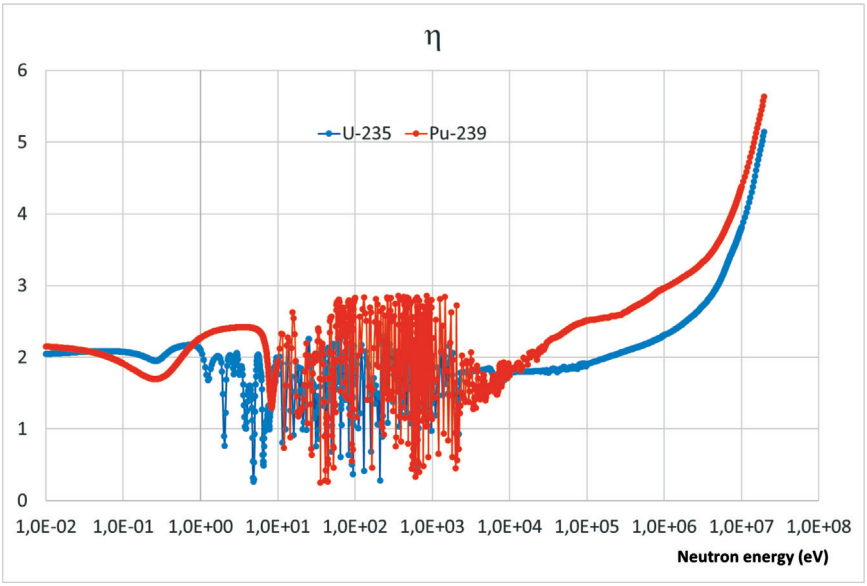

Fig. 4. – The parameter η as a function of neutron kinetic energy for the two main fissile nuclei.

the solution is

$$\phi = \text{const} = \frac{S_0}{k_\infty - 1}\Sigma_a, \tag{20}$$

where Σ_a is the macroscopic absorption cross section. We see that the neutron flux and, therefore, the power are proportional to the intensity of the source and are obviously increasing when the multiplication in the chain reaction k_∞ approaches unity. An external neutron source can be obtained by accelerating electrons or ions (protons, deuterons) in a suitable accelerator, then smashing the accelerated particles onto an appropriate, totally absorbing target. When using high-energy protons with kinetic energies in the order of hundreds of MeV to GeVs on a heavy nuclear target based, for instance, on mercury, tungsten, lead, several neutrons per incoming proton are obtained in the nuclear process called *spallation*. At lower energies, other types of targets can be used but spallation by high-energy ions is the most effective process for producing neutrons. A subcritical reactor core will therefore be comprising a suitable accelerator with a beam penetrating into the reactor core and stopped by a target placed in the middle of the core. This is called an *Accelerator Driven System, ADS* [3]. The interest in this type of reactor will be briefly discussed later.

3. – Main features of a nuclear fission reactor

We saw that neutrons slow down when undergoing collisions with nuclei (in particular with light nuclei), therefore their energies in the core can go from about 10 MeV (usually

the maximum energy of fission neutrons), down to as low as 10^{-3} eV. We also saw that neutron cross sections have a strong dependence on neutron energy, in particular absorption cross sections such as capture or fission decrease very strongly with energy for particular species like ^{235}U. When using fuels containing these particular species, it is easier to maintain a fission chain reaction using slow neutrons. Hence most nuclear reactors until now (the so-called Generation I to III+) use low-mass-number materials such as water or graphite to slow down or moderate the fast-fission neutrons, thereby slowing them down to energies comparable to the thermal energies of the nuclei in the reactor core. Therefore, by definition a *thermal reactor* is one where the average neutron energy is comparable to thermal energies (order of several tens of meV). Because of the very-high-fission cross section at thermal energies, this type of reactor requires the minimum amount of fissile material for fueling. As an example, a Light Water Reactor (LWR) can start with 3% ^{235}U + 97% ^{238}U (a mixture enriched with respect to natural uranium). Burn-up (consumption) of ^{235}U is partly compensated by breeding of ^{239}Pu via neutron radiative capture and subsequent β-decays and after 1 year operation, the core may contain something like 1% ^{235}U + 1% ^{239}Pu. On the contrary, in a *fast reactor* the average neutron energies in the core are typically above 100 keV. As previously discussed, this can be accomplished if the core contains mostly high-mass-number materials. This means that for core cooling the choice will be a liquid metal like sodium, lead or a lead-bismuth mixture, or alternatively the coolant should be a low-density material like helium gas. On the other hand, σ_f is smaller for fast neutrons, which means that a much higher percentage of fissile material is needed to sustain the chain reaction. Later on we will consider some possible advantages of fast reactors over thermal reactors. In summary, the typical scheme of a thermal and a fast reactor are illustrated in fig. 5.

3˙1. *Decay heat*. – Decay heat is the heat produced in the reactor core as a result of radioactive decay: the energy of the alpha, beta or gamma radiation is converted into atomic motion, but heat production comes mostly from β- and γ-decay of fission products. Decay of the short-lived radioisotopes created by fission continues for some time after shutdown. A practical approximation is given by the formula

$$\frac{P}{P_0} = 6.6 \cdot 10^{-2} \left[\frac{1}{(\tau - \tau_s)^{0.2}} - \frac{1}{\tau^{0.2}} \right], \tag{21}$$

where P is the decay power, P_0 is the reactor power before shutdown, τ is the time since reactor startup and τ_s is the time of reactor shutdown measured from the time of startup (in seconds). At shutdown, the heat power is about 6.5% of the previous core power ($\simeq$ 200 MWth for a 1 GWe reactor), which is sufficient to melt the core. About 1 hour after shutdown, the decay heat will be about 1.5% of the previous core power, after 1 day it will fall to 0.4% and after a week it will be only 0.2%. For this reason, spent fuel rods are kept for a long time in special cooling water pools, before being further processed. Removal of decay heat is therefore a very important aspect in designing a reactor and implementing safety measures, as the Fukushima accident has dramatically stressed.

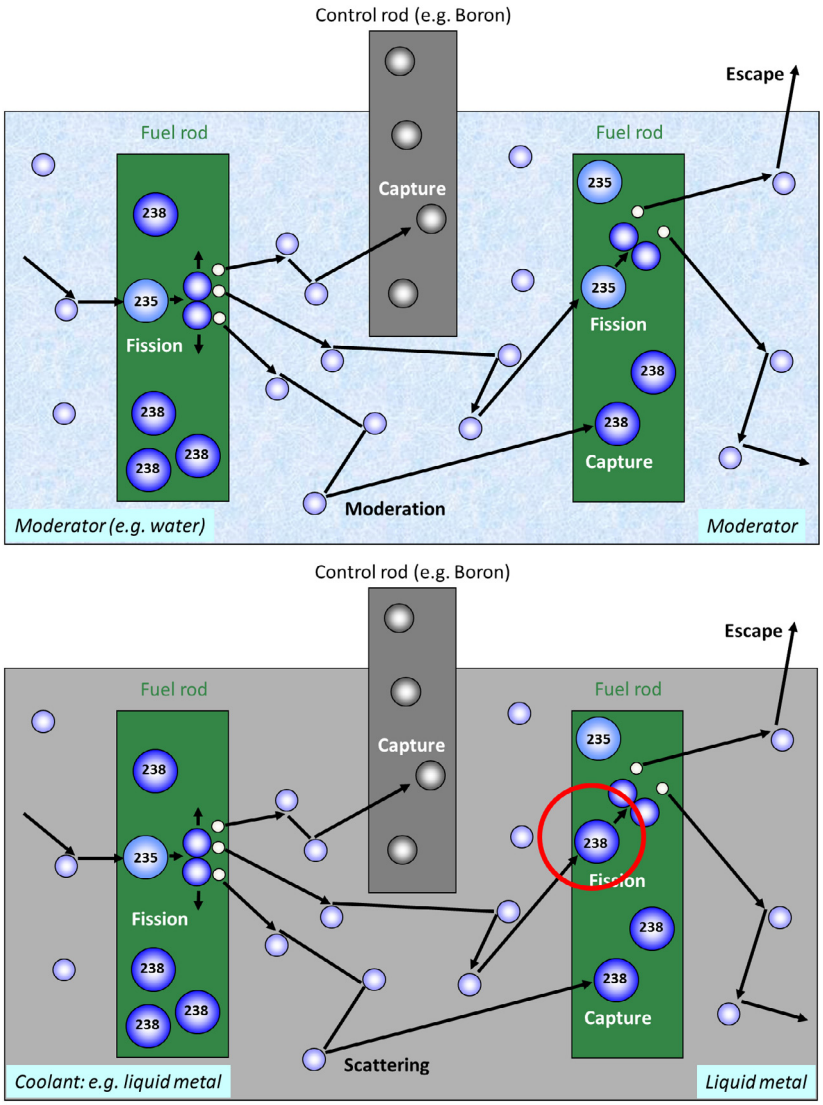

Fig. 5. – Conceptual schemes of thermal (top) and fast (bottom) reactor.

3˙2. *Nuclear reactor types*. – Most current reactors are based on fissile fuels, ^{235}U and ^{239}Pu, with fission induced by slow neutrons at thermal energies. Ordinary water serves as both coolant and moderating material in the reactor. There are two major types of Light Water Reactors (LWR): Pressurized Water Reactors (PWR) and Boiling Water Reactors (BWR). In a PWR, the primary coolant is water maintained under very high pressure so that the coolant reaches a high temperature without steam formation within the reactor. Heat transported out of the reactor core by the primary coolant is then transferred to a secondary loop connected to a steam generator. In a BWR, the

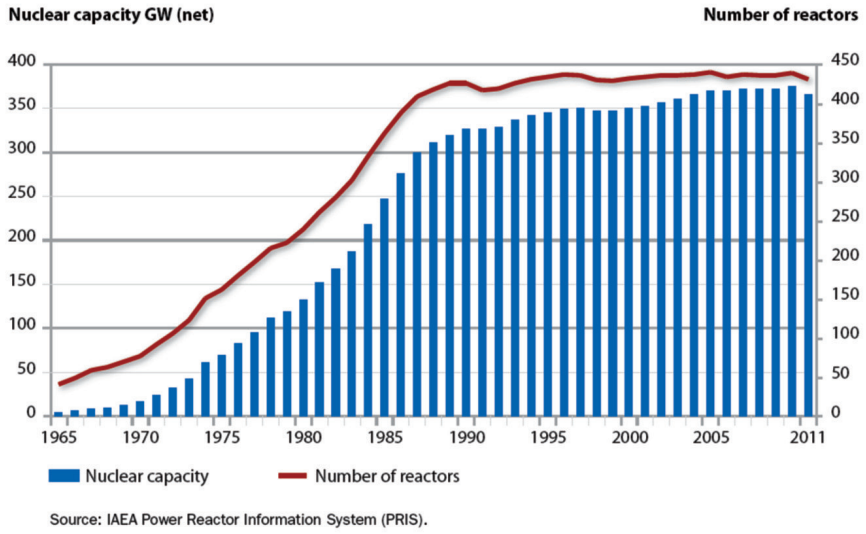

Fig. 6. – Worldwide nuclear generating capacity and number of operating reactors (1965–2011) [4], © OECD 2012.

primary coolant water is maintained at lower pressure, so that boiling and steam formation occur within the reactor core itself, which serves directly as the steam generator, while secondary loop and heat exchanger are not present. In both PWR and BWR, the nuclear reactor itself and the primary coolant are contained in a large steel pressure vessel designed to accomodate the high pressures and temperatures. The heavy-water (D_2O) reactor takes advantage of the lower neutron capture cross section in deuteron with respect to hydrogen. This allows to have more neutrons around in the core and to use low-enrichment uranium fuels (including natural uranium). This concept has been developed for instance in Canada in the CANDU (CANadian Deuterium Uranium) series of power reactors. This type of reactors is indicated by the acronym PHWR (Pressurized Heavy Water Reactor). Yet another design is based on gas as coolant. For instance the early MAGNOX reactors developed in the UK used low-pressure CO_2 as cooling gas. In general this reactor class is indicated by the acronym GCR (Gas Cooled Reactor). A more recent design is the High-Temperature Gas-cooled Reactor (HTGR, USA), that uses high-pressure helium as coolant. Gas-cooled reactors generally use graphite as moderator material to slow down neutrons to thermal energies. Finally, the Fast Breeder Reactor (FBR) does not use a moderator to work with fast neutrons (see sect. **2**) and breeds more fuel than it consumes. The coolant is typically a liquid metal.

4. – Fission energy in perspective

4˙1. *Worldwide figures.* – As illustrated in fig. 6, from the 1950s, when the first commercial reactors started to appear, until 1985 the number of operating reactors as

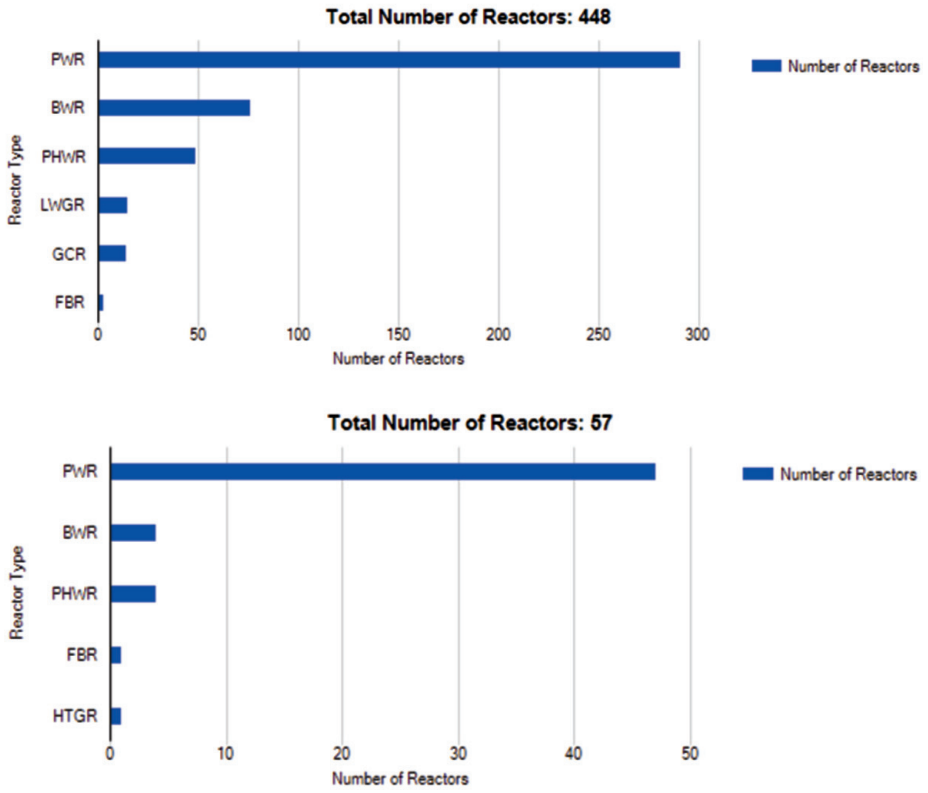

Fig. 7. – Number of operational and long-term shutdown reactors by type (top, as of October 2017). Reactors under construction by type (bottom, as of October 2017) [5], © 2017 International Atomic Energy Agency (IAEA).

well as the nuclear net generating capacity (*i.e.*, the available power) have been increasing about linearly with time, then the number of reactor has remained approximately stable, while the generating capacity has been still increasing somewhat due to the improved efficiency of existing plants and some replacement of older installations [4]. As of October 2017, there are 449 reactors in operation or in long-term shutdown in the world, for a total net installed capacity of about 392 GWe, and 57 under construction, for a total capacity of about 60.5 GWe [5]. In fig. 7, the top and bottom panels show the distribution of reactor types in operation and under construction in the world, respectively. Figure 8 shows the share of nuclear power in electricity production as of October 2017 [5] country by country, while fig. 9 shows the share of nuclear power in the world electricity generation [6].

4˙2. *Nuclear fuel cycle, natural resources, cost of electricity and carbon emission scenarios*. – How much uranium is consumed by the nuclear reactor fleet worldwide? And how long will uranium resources last? There are clearly important questions for project-

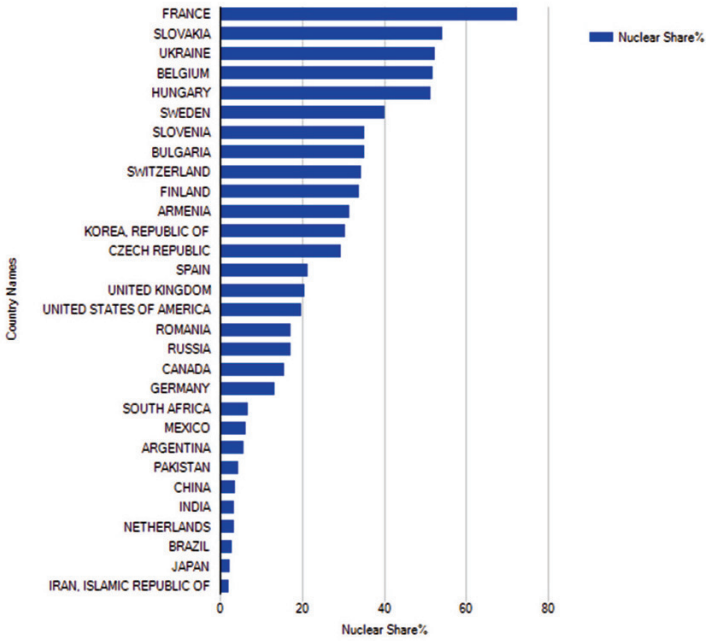

Fig. 8. – Nuclear share of electricity generation in each country (as of October 2017). Note: the share for Taiwan, China was 13.7% of the total [5], ©2017 International Atomic Energy Agency (IAEA).

ing the use of nuclear power from fission to the future. The whole series of activities starting from uranium mining down to disposal of radioactive waste is called the *nuclear fuel cycle*. In the case of the so-called *open fuel cycle* the spent fuel is extracted from the reactor, cooled in specific pools, then transferred to a storage infrastructure. In the case of the so-called *closed fuel cycle*, the spent fuel undergoes a reprocessing procedure, where plutonium is extracted from the fuel rods and used to produce new fuel containing both uranium and plutonium. World annual uranium requirements amounted to 61600 tU (tons uranium) in 2012 [7]. The so-called identified resources appear to be sufficient for over 120 years, based on the 2012 uranium consumption [7]. Consideration of the entire amount of conventional resources would increase the time scale to well over 300 years, but this would require significant efforts in securing all resources for an effective use. Moving to advanced technology reactors and recycling fuel could increase the long-term availability of nuclear energy from hundreds to thousands of years. In addition, thorium, which is more abundant than uranium in the Earth's crust, is another potential source of alternative fuel.

When considering the so-called Levelised Cost Of Electricity, levelised over the estimated plant lifetime, for various world areas, assuming specific discount rates and cost of carbon emissions, especially for a low discount rate scenario, the cost of electricity from nuclear power appears to be still competitive with other sources [8].

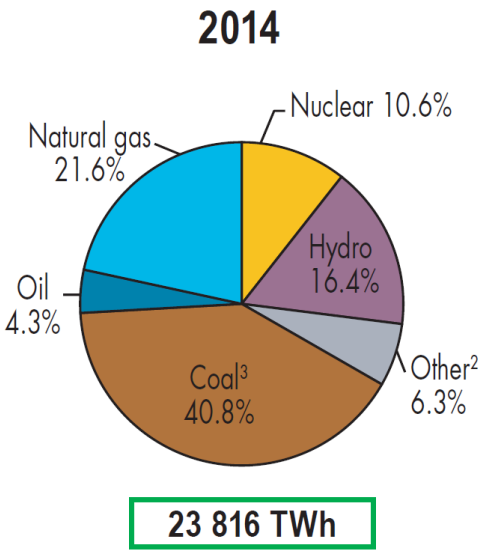

Fig. 9. – World electricity generation (2014) [6], © OECD/IEA 2016.

As far as carbon and other greenhouse gas emissions are concerned, for nuclear energy from fission an accurate estimate of the CO_2 equivalent emissions should include all activities in the whole plant lifetime, from fuel mining/extraction to the final decommissioning of the plant. Such a detailed study has been performed, for instance, in [9], showing that a nuclear plant produces an estimated 66 gCO_2 equiv/kWh, typical renewable energy installations range between 9 and 32, fossile fuels are all above 400. Considering the capability to deploy a large quantity of electric power per plant (typically an actual power of around 1 GWe with high availability) and the relatively low associate emission level per kWh, it is clear why nuclear energy is still considered one of the players in meeting greenhouse gas reduction targets. Indeed, both in the so-called New Policies Scenario and in the more aggressive "450" Scenario (not exceeding 450 ppm of CO_2 equivalent in the atmosphere) [6], nuclear power production is expected to grow.

4˙3. *Safety*. – Safety is a very important aspect in nuclear power production. The core of a nuclear reactor produces radiation both while in operation and after shutdown, because of the high content of radioactive materials in the fuel and because of the neutron irradiation of materials comprised by the core and surrounding the core, which produces radioactive nuclei starting from stable ones. Therefore, the safety culture that permeates modern industry is an even more essential aspect of nuclear industry. Attention to safety is present from the design stage down to operation, shutdown and decommissioning of the plants and particular care is devoted to the analysis of possible accidental scenarios. A fundamental principle is the *defence-in-depth*, which means that a series of barriers, starting from the solid fuel form itself up to the reactor containment building, is put

in place to contain radioactivity as much as possible at all times and in all instances. Obviously, in this respect, the role of independent regulatory authorities is crucial in overseeing all stages of a plant lifetime.

4˙4. *Radioactive waste.* – Among many important safety aspects of fission reactors, a particularly relevant one is the production of radioactive waste. There are two mechanisms for the production of radioactive species in addition to those already present in the fuel (*e.g.*, the two uranium isotopes are radioactive in the first place): the first mechanism is fission itself, which is producing many radioactive fission products (or fragments), *i.e.*, unstable nuclei that decay by β-decay at a later time, with the β-decay being followed very often by emission of gamma rays. The radioactive half-lives of fission products for β-decay is enormously variable, ranging from fractions of a second to million years. Fission products are continuously produced in the fuel material inside a reactor and can be found in the spent fuel once it is removed from the reactor at the end of a power production cycle. Fission products with a half-life until several days are responsible for the large amount of decay heat that is produced within the core immediately after reactor shutdown, *i.e.*, after stopping the chain reaction. However, their radioactivity dies away in a few days or weeks, which means that in case of accidental release of radioactive products they pose a threat only for such a limited amount of time and that when the spent fuel is stored in the cooling pools they do not produce anymore heat after a short amount of time. Other Short-Lived Fission Products (SLFP) with half-lives up to a few years can instead be a safety concern also after an accordingly longer period of time, as are Medium-Lived Fission Products with half-lives up to about 100 years (*e.g.*, ^{137}Cs and ^{90}Sr, with half-life of about 30 years). Such isotopes contribute to heat production in the spent fuel and can be a safety concern if accidentally released into the environment, for decades. Finally, Long-Lived Fission Products (LLFP) have half-lives over about 200000 years (no fission product has half-life between about 100 years and 200000 years). There is a second mechanism producing radioactive isotopes. Radiative capture, which leads to the production of plutonium, also produces a series of heavier (transuranic) nuclides, americium, curium, etc., with increasing atomic number $Z > 92$. Capture on ^{235}U leads also to the formation of ^{237}Np. Neptunium, americium, curium and other similar isotopes are called *Minor Actinides, MA*. Many of these MA decay predominantly by α-decay and have half-lives ranging from a few hundreds of years up to above 2 million years (^{237}Np).

The typical inventory from a 1 GWe PWR fuelled with about 27 tons of U (3.5% ^{235}U) at discharge (when the spent fuel is removed from the core) is approximately 280 kg of ^{235}U, 266 kg of Pu of which 56 kg of fissile ^{239}Pu, 20 kg of MA, 13 kg of ^{90}Sr and 30 kg of ^{137}Cs (both with 30 years half-life), and 63 kg of LLFP. The very long decay time of both LLFP and MA is the reason why disposal of spent fuel and similar waste requires very long-term storage, for which one possible solution envisaged is storage deep underground in the so-called geological repositories. Some countries adopt the so-called reprocessing of the spent fuel, where the plutonium is extracted from the fuel rods and used to produce new fuel containing both uranium and plutonium.

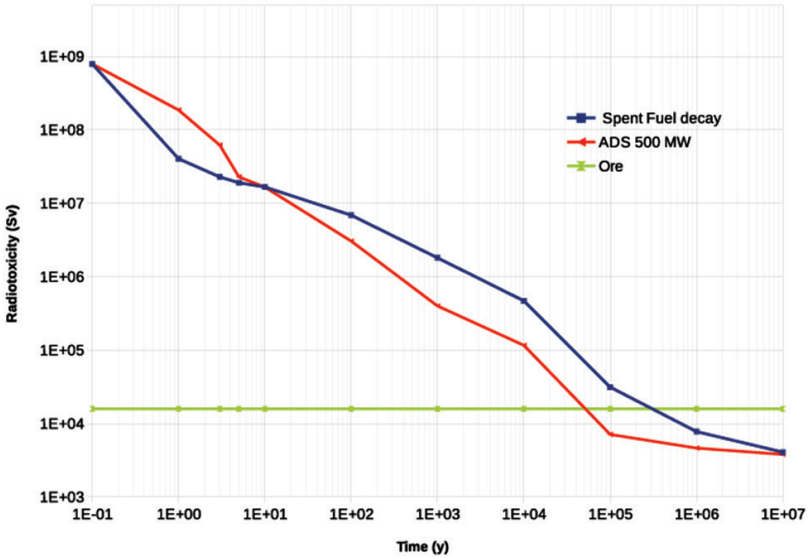

Fig. 10. – Radiotoxicity in two different fuel cycle scenarios. Blue curve: natural decay in the spent fuel; red curve: uranium and fission products separated and stored, plutonium and MA mixed in new fuel and irradiated for 3 years in a 500 MWth ADS.

4˙5. *Fast reactors, subcritical reactors and fuel breeding.* – In the effort to improve the safety, security and efficiency of nuclear plants, new concepts of reactors have been developed with goals including the minimization of the production of MA, a better and more efficient use of the fuel, a better thermodynamic efficiency, the possible production of hydrogen at high temperatures and finally improved safety features to minimize the risk of accidents. All these various concepts are considered within the so-called Generation-IV reactors, which are the subject of an international initiative [10]. As an example, we saw that in a fast reactor, by using, *e.g.*, a liquid metal or gas as coolant, the energy spectrum of the neutrons is harder. In such a reactor core, it becomes possible to burn not only fissile elements like ^{235}U and ^{239}Pu, but also fissionable elements like ^{238}U for which fission occurs significantly only above a certain energy threshold around 0.5 MeV, as can be seen in fig. 3. Although given the much lower fission cross-section typically the fuel has to be richer in fissile content, this means that also ^{238}U can be to some extent considered to be a fuel as well, which has obvious implications in terms of how long uranium resources will last. Another important aspect in fast-neutron systems is that, for energies, say, above 0.5 MeV, fission becomes dominant over capture, as again seen in fig. 3. This means that the MA production, which occurs namely via capture, is suppressed. On the other hand, in a fast reactor the MA produced can be partly destroyed in the reactor itself by fission. An important physics aspect when considering fission of MA is that the amount of delayed neutrons produced upon their fission is significantly less than for ^{235}U and ^{239}Pu. This means that, for instance, fueling a reactor purely with MA is not possible, as one would lose the crucial safety feature of the

reactor response times being dominated by the relatively long delayed neutron emission times. However, for ADS based on a subcritical reactor core that requires an external neutron source, the role of delayed neutrons is not so crucial and, therefore, there is the possibility to mix a larger quantity of MA into the fuel, thereby offering the possibility to use the apparatus to perform transmutation of the MA into fission products and reduce the radiotoxicity of the final materials in the fuel. This is called *P&T, partitioning and transmutation* approach, meaning that first some nuclides are separated from the spent fuel (partitioning) and then they are irradiated in a special core to be either destroyed or transformed into stable elements (transmutation). Figure 10 shows the radiotoxicity of the material originated from the spent fuel, *i.e.*, its potential danger to human health, estimated according to two scenarios of fuel cycle in a very simple model [11]. In the open cycle without reprocessing, the radiotoxicity of the spent fuel obtained from 1 ton of natural uranium (ore) reaches the level of the ore in about 300000 years. In the closed fuel cycle where MA are separated and burnt together with plutonium for three years in a high-power ADS (here uranium and FP are left aside), the radiotoxicity could be reduced to the ore level in an order of magnitude less years. More accurate models of the closed cycle show that by P&T a reduction to the ore level in a few hundred years can be achieved [12].

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