Rare CP-violated η and η' meson decays and neutron EDM.

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Abstract. The data for the upper limit on the electric dipole moment of the neutron (nEDM) can be explained by using different mechanisms beyond the Standard Model (SM). The nEDM can be generated by a CP-violating transition of η and η' mesons into pion pairs. We derive the upper limits for the rates of the CP-violating decays $\eta(\eta') \to 2\pi$ are by orders of magnitude more stringent than those from existing experiments so far.

1 Introduction

The investigation of the electromagnetic moments of hadrons, in particular, of proton and neutron, and leptons is one of the promising goals in our understanding of the hadron structure. The search for an internal EDM for a single particle has been ongoing for the past 60 years. The EDM is connected with the violation of CP symmetry. The existing upper experimental limits for the EDM of the electron or the neutron have values of order of $10^{-26} \div 10^{-27}\mathrm{e}\cdot\mathrm{cm}$, which are more than six orders of magnitude larger than SM predictions. The prediction of the nEDM in the SM is $|d_n^{CKM}| \approx (1\ldots6)\times10^{-32}\mathrm{e}\cdot\mathrm{cm}$ [1].

Within the strong-interaction sector, the neutron EDM is induced by the CP-violating θ-term of quantum chromodynamics (QCD)

$$\Delta\mathcal{L} = \theta\frac{g_s^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}, \tag{1}$$

where g_s is the QCD coupling constant, and $G^a_{\mu\nu}$ and $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}$ are the usual stress tensor of gluon fields and its dual. The θ-term preserves the renormalizability and gauge invariance of QCD, but breaks the P- and T-parity invariance. It plays an important role in QCD, e.g., for the QCD vacuum, the topological charge, and the solution of the $U(1)_A$ problem of the mass of the η' meson (see, e.g., Refs. [2, 3]). An explanation for the apparent smallness of the θ coupling (solution of the strong CP-violation problem) was proposed by

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Peccei and Quinn [4], who suggested θ to be a field $\theta(x)$, which is decomposed into an axial field $a(x)$ (axion) that preserves CP-conservation, and a small constant $\bar{\theta}$ that encodes the CP-violating effect. For a recent overview see, e.g., Ref. [5].

Future experiments at Mainz and at Jefferson Laboratory have the possibility to increase the statistics of measurements of rare decays of η and η' mesons. Direct measurements of rare decays can show how large the difference between the CP-violation process in the EDM and processes in rare decays are, or if they obey the same mechanism.

This calculation aims to show that the bound on the width of rare CP-violating decays of η and η' mesons into pairs of pions is several orders of magnitude lower than the existing experimental limits of branching fractions. The current experimental upper limits for the branching fractions of these processes are [8, 9]:

$$\mathrm{Br}(\eta \to \pi\pi) < \begin{cases} 1.3 \times 10^{-5}, & \pi^+\pi^- \\ 3.5 \times 10^{-4}, & \pi^0\pi^0 \end{cases} \tag{2}$$

$$\mathrm{Br}(\eta' \to \pi\pi) < \begin{cases} 1.8 \times 10^{-5}, & \pi^+\pi^- \\ 4 \times 10^{-4}, & \pi^0\pi^0 \end{cases} , \tag{3}$$

indicating that any signal observed within $\sim 13-14$ orders of magnitude between the existing experimental bounds and the strong CPV expectations could be interpreted as an effect of physics beyond the SM.

For the masses and full widths of the η and η' mesons we use the PDG values [8]: $m_\eta = 547.853 \pm 0.024$ MeV, $\Gamma_\eta^{\rm full} = 1.30 \pm 0.07$ keV and $m_{\eta'} = 957.78 \pm 0.06$ MeV, $\Gamma_{\eta'}^{\rm full} = 0.199 \pm 0.009$ MeV.

2 Neutron EDM

The decomposition of the electromagnetic vertex function of baryons in terms of relativistic form factors $F_I(Q^2)$, with $I = E, M, D, A$, is

$$\begin{aligned}\bar{N}(p_2)\Gamma(p_1,p_2)N(p_1) &= \bar{N}(p_2)\Big[\gamma^\mu F_E(Q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_M(Q^2) \\ &+ \frac{\sigma^{\mu\nu}\gamma_5 q_\nu}{2m}F_D(Q^2) + (\gamma^\mu q^2 - 2mq^\mu)\gamma_5 F_A(Q^2))\Big]N(p_1),\end{aligned} \tag{4}$$

where $Q^2 = (p_2 - p_1)^2$, m is the baryon mass, γ^μ, γ_5 are the Dirac matrices, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $F_E(Q^2), F_M(Q^2), F_D(Q^2)$ and $F_A(Q^2)$ are the electric, magnetic, dipole and anapole form factors of baryons. The electric dipole moment of the neutron is defined as $d_n^E = -F_D(0)/(2m_N)$, where m_N is the mass of the neutron.

As a starting point of our calculation of the nEDM, we write a phenomenological effective Lagrangian that generates the P- and T-violating processes $\eta \to 2\pi$:

$$\mathcal{L}^{CPV} = f_{\eta\pi\pi} m_\eta \eta \pi_a \pi_b \delta_{ab}\,, \tag{5}$$

where $f_{\eta\pi\pi}$ is the corresponding dimensionless coupling constant, m_η is the η-meson mass, and the indices a, b refer to isospin. A similar form for the interaction Lagrangian holds for η'. The difference between the η and the η' meson coupling is encoded in the CPV coupling, which is related to the masses m_η and m_η'.

On the other hand, the θ CP-violating term in QCD can be deduced using the QCD sum rules for the $\eta\pi\pi$ transition amplitude. The effective $\eta\pi\pi$ coupling in this scenario is given by [6, 7]:

$$f_{\eta\pi\pi}^\theta = -\frac{\theta\, m_\pi^2 m_u m_d}{\sqrt{3}F_\pi(m_u + m_d)^2 m_\eta}, \tag{6}$$

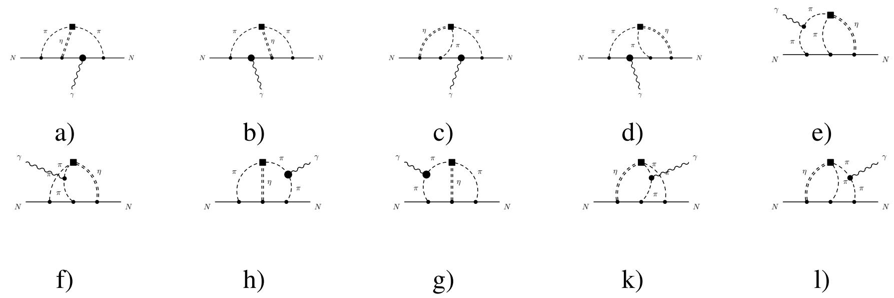

Figure 1. Diagrams describing the nEDM in the framework of PS interactions between mesons and baryons. The interaction with the external electromagnetic field occurs through the minimal electric coupling to charged baryon or meson fields. The solid square denotes the CP-violated $\eta\pi^+\pi^-$ vertex.

where θ is the QCD vacuum angle, $m_{u,d}$ are the u and d current quark masses, $F_\pi = 92.4$ MeV is the leptonic decay constant of the pion, and $m_\pi = 139.57$ MeV is the charged pion mass.

The bound for the decay constant in Eq. (5) is $f_{\eta^{(\prime)}\pi\pi} \sim (0.03 \div 0.05) \times \theta$ for η and η' meson, respectively, and $\theta < 6 \cdot 10^{-10}$ [7, 10]. This makes the experimental searches for the $\eta(\eta') \to 2\pi$ hopeless due to the branching ratios, which depend on the respective coupling constants squared. The second scenario corresponds to the situation, where the EDM and the CP-violating $\eta(\eta') \to \pi\pi$ decays are generated by two distinct mechanisms.

Given the interest in addressing these decay channels experimentally at Jefferson Lab [11], it is informative to inquire, how much room there is for new physics contributions that could lead to anomalously large $\eta\pi\pi$ coupling constants. The unknown new-physics mechanism would then generate a non-zero $f_{\eta\pi\pi}$, which through pseudoscalar meson couplings to the nucleon generates the EDM at two-loop level. The $1/(4\pi)^2$ factor appearing for each loop loosens the EDM constraint somewhat.

2.1 Part of calculation

The calculation of the nEDM was done here using a pseudoscalar (PS) approach with a non-derivative coupling between pion and nucleon. The vertex of interaction of the PS field with the baryons has a simple structure γ_5 and no momentum dependence, unlike the pseudovector

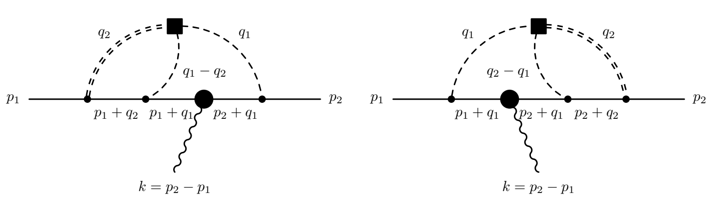

Figure 2. Detailed diagrams c) and d) from Fig.1 where the external field couples to the proton charge.

(PV) approach. This property of the interaction vertex in the PS model facilitates the calculations of the nEDM from the diagrams from Fig.1. The two-loop calculation does not have divergences here.

We show the detailed two-loop calculation of the neutron EDM for diagrams c) and d) from Fig. 1, which are shown in Fig.2. For the calculation of the other diagrams we used the same approach. The first loop amplitude, which does not interact with the external electromagnetic field, is written as

$$\frac{g_A g_A^\eta m_N^2}{F_\pi F_\eta} m_\eta f_{\eta\pi\pi} \int \frac{d^4 q_2}{(2\pi)^4} \frac{\bar{N}(p_1+q_1)\gamma_5(\hat{p_1}+\hat{q_2}+m_N)\gamma_5 N(p_1)}{((p_1+q_2)^2-m_N^2)(q_2^2-m_\eta^2)((q_1-q_2)^2-m_\pi^2)}, \tag{7}$$

where m_N , m_π and m_η are neutron, pion and η or η' mesons masses, $N(p_i)$ are baryon spinors. The baryon in the loop is off shell. We use the Feynman parametrization and for convenience we shift $q_2 \to q_2 - p_1 x_1 + q_1 x_2$. After integration over q_2 and if we omit different constant factors we obtain

$$-i\frac{1}{(4\pi)^2}\int dx_1 \int dx_2 \frac{\bar{N}(m_N x_1 - \hat{q}_1 x_2)N(p_1)}{(m_N^2 x_1^2 + m_\pi^2 x_2 + m_\eta^2(1-x_1-x_2) - 2p_1 q_1 x_1 x_2 + q_1^2 x_2(x_2-1))}. \tag{8}$$

The next step is to calculate the loop which is generated by the interaction with the external electromagnetic field

$$\frac{4 g_A g_A^\eta m_N^2}{F_\pi F_\eta} m_\eta f_{\eta\pi\pi} \int \frac{d^4 q_1}{(2\pi)^4} \frac{\bar{N}(p_2)\gamma_5(\hat{p_2}+\hat{q_1}+m_N)\gamma_\mu(\hat{p_1}+\hat{q_1}+m_N)(x_1 m_N - x_2\hat{q}_2)N(p_1)}{((p_1+q_1)^2-m_N^2)((p_2+q_1)^2-m_N^2)(q_1^2-m_\pi^2)(\beta - 2p_1 q_1 a + q_1^2)} h, \tag{9}$$

where the factor $\frac{1}{(4\pi)^2}\int dx_1 \int dx_2$ was omitted and $q_1' = q_1 + p_1 y_1 + p_2 y_2 - p_1 y_3 a$. We used the substitution

$$a = x_1 x_2 h; \qquad \beta = m_N^2 x_1^2 + m_\pi^2 x_2 + m_\eta^2 (1 - x_1 - x_2) h; \qquad h = \frac{1}{x_2(x_2-1)}.$$

After integration over q_1', and leaving implicit the integrals over Feynman parameters $\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^1 dy_1 \int_0^{1-y_1} dy_2 \int_0^{1-y_1-y_2} dy_3$, we obtain the following structure, which contributes to the neutron EDM:

$$\frac{1}{(4\pi)^4}\Big[2m_N^2(2x_2y_1y_2 - x_2y_1y_2^2 + 2x_2y_1^2 - 2x_2y_1^2y_2 - x_2y_1^3 - 2x_2ay_1y_3 + 2x_2ay_1y_2y_3 + 2x_2ay_1^2y_3$$
$$- x_2a^2y_1y_3^2 + x_1y_2^2 + 2x_1y_1 - x_1y_1^2 - 2x_1ay_2y_3 + x_1a^2y_3^2)T_1 - 2x_2(1-3y_1)T_2\Big] \frac{4g_A^2 g_A^\eta m_N^3}{F_\pi^2 F_\eta} m_\eta f_{\eta\pi\pi} \tag{10}$$

where factor 4 goes from isospin

$$T_1 = \frac{h}{\Delta^2}; \qquad T_2 = \frac{h}{\Delta}; \tag{11}$$

and

$$\Delta = \beta y_3 + m_\pi^2(1 - y_1 - y_2 - y_3) + m_N^2(y_1 + y_2 - a y_3)^2. \tag{12}$$

In the calculation of the EDM, we used the Gordon identity:

$$i\sigma^{muv}k_v = -(p_1+p_2)^\mu + 2m\gamma^\mu \qquad i\sigma^{\mu\nu}(p_1+p_2)^\nu = -k^\nu$$
$$i\sigma^{\mu\nu}k_\nu\gamma^5 = -(p_1+p_2)^\mu\gamma^5 \qquad i\sigma^{\mu\nu}(p_1+p_2)^\nu\gamma^5 = -k^\mu\gamma^5 + 2m\gamma^\mu\gamma^5. \tag{13}$$

3 Discussion

Summing all contributions to the nEDM from Fig. 1 and using the parameters from model [13], we obtain the value

$$d_n^E \simeq (2.2 f_{\eta\pi\pi}(m_\eta^2) + 0.83 f_{\eta'\pi\pi}(m_\eta^2)) \times 10^{-13} \mathrm{e} \cdot \mathrm{cm}. \tag{14}$$

Here we use the $f_{\eta\pi\pi}(m_\eta^2)$ and $f_{\eta'\pi\pi}(m_\eta^2)$ CP-violating constants, where we took into account the pion rescattering effect described by the Omnes-Muskhilishvili (OM) function. It is obtained experimentally from the phase in each partial wave [12, 13]

$$\Omega_{IJ}(s) = exp\left[\frac{s}{\pi}\int\limits_{4m_\pi^2}^{\infty}\frac{ds'\delta_{IJ}(s')}{s'(s'-s-i\epsilon)}\right], \tag{15}$$

with δ_{IJ} the respective phase shift. The OM function above is normalized to 1 at $s = 0$.

In our particular case, there is only the s-wave $I = J = 0$ corresponding to the quantum numbers of the η's. We use the fit to $\pi\pi$ scattering data by Pelaez and Yndurain [12] who provided an analytical form for $\delta_{00}(s)$ in the region $4m_\pi^2 \leq s \leq 1.42^2$ GeV^2. The effect of rescattering is then assumed as

$$f_{\eta\pi\pi}(s) = f_{\eta\pi\pi}(0)\Omega(s). \tag{16}$$

Evaluating the OM function at $s = m_\eta^2(m_{\eta'}^2)$, we obtain

$$|f_{\eta\pi\pi}(m_\eta^2)| = 1.81|f_{\eta\pi\pi}(0)|, \qquad |f_{\eta'\pi\pi}(m_{\eta'}^2)| = 4.46|f_{\eta'\pi\pi}(0)|. \tag{17}$$

Thus, from the equation for the EDM, the bounds for the (branching ratios of) rare decays $\Gamma_{\eta\pi\pi}$ and $\Gamma_{\eta'\pi\pi}$ are strongly suppressed when compared to the existing data [9]

$$\Gamma(\eta \to \pi\pi) < 0.16 \times 10^{-24}\,\mathrm{keV}, \tag{18}$$

$$\Gamma(\eta' \to \pi\pi) < 2.24 \times 10^{-24}\,\mathrm{keV}. \tag{19}$$

Comparing the values obtained for $f_{\eta\pi\pi}$ and $f_{\eta'\pi\pi}$ with Refs. [6, 7] we can deduce an upper limit for the $\bar{\theta}$ parameter in the Peccei-Quinn mechanism:

$$\bar{\theta} < 1.2 \cdot 10^{-11}\,. \tag{20}$$

Direct measurements of rare η and η' meson decays or higher orders of magnitude than the prediction from the EDM of the neutron can be seen as a sign for the existence of mechanisms beyond the SM, which give contributions to these rare decays.

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