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Breit-Wheeler pair production from Worldline Instantons

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Abstract. We calculate the cross-section of Breit-Wheeler process $\gamma\gamma \to e^+e^-$ in external electric field below the perturbative threshold by the semiclassical method of worldline instantons.

1 Introduction

Breit-Wheeler process $\gamma\gamma \to e^+e^-$ [1] is a standard quantum electrodynamics (QED) process. In vacuum, due to energy conservation, it occurs if the center mass energy of two initial photons exceeds a certain threshold equal to double electron mass. In an external electric field energy conservation for particles violates, hence the Breit-Wheeler process may occur even below the vacuum threshold. We show that in this regime the process may be described by semiclassical methods similar to ones applied for a calculation of the Schwinger effect — pair production from the vacuum in constant electric field.

Semiclassical method of "worldline instantons" is a good approach to study some non-perturbative processes in quantum field theory like particle production in the external field. It was firstly used by [2] to calculate the Schwinger effect – a particle production from the vacuum in constant electric field, and was later generalized to time- and space- dependent external electric fields [3, 4]. It was shown that the method may be generalized to an external particle in the initial state: the processes of photon decay to an electron-positron pair in electric [5] and magnetic [6] field have been considered, as well as neutrino decay to electron and W-boson [7]. The straightforward generalization of the method is to consider two particles in the initial state, like for Breit-Wheeler process $\gamma\gamma \to e^+e^-$.

2 Pair production in external electromagnetic field

Let us start from general formalism for calculations in worldline approach. We consider scalar QED in external classical electromagnetic field A_μ, with the Euclidean action: $S_E = \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^2 + \left|D_\mu\phi\right|^2 + m^2|\phi|^2\right)$. General partition function $Z[A_\mu]$ is defined in a general way, where we omit quantum fluctuatioins near classical elecromagnetic field A_μ:

$$Z[A_\mu] = \int D\phi D\phi^* \, \mathrm{e}^{-S_E} = \mathrm{e}^{-W[A_\mu]}. \quad (1)$$

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Integrating over scalar electrons, one obtains the effective action $W[A_\mu]$ for electromagnetic field in Schwinger proper time representation [2, 8]:

$$W[A_\mu] = \int d^4x\left(-\frac{1}{4}F^2_{\mu\nu}\right) + \int_0^\infty \frac{ds}{s}e^{-sm^2}\,\mathrm{Tr}\left(e^{-(-D^2_\mu s)}\right) = \tag{2}$$
$$= \int d^4x\left(-\frac{1}{4}F^2_{\mu\nu}\right) + \int_0^\infty \frac{ds}{s}e^{-sm^2}\int_{p.b.c.} Dx_\mu \exp\left(-\int_0^s d\tau\left(\frac{\dot{x}^2_\mu}{4} + ieA_\mu\dot{x}_\mu\right)\right).$$

In the first line of (2) we introduce Schwinger proper time while in the second line we perform transformation into the path integral formalism. The notation *p.b.c.* means periodical boundary conditions for the path integral: $x_\mu(0) = x_\mu(s)$. Let us notice that the exponent of the second line of (2),

$$S_m[x_\mu] \equiv m^2 s + \int_0^1 d\tau\left(\frac{\dot{x}^2_\mu(\tau)}{4s} + ieA_\mu\dot{x}_\mu(\tau)\right), \tag{3}$$

coincides with the Euclidean action for a charged particle in electromagnetic field; the variable s is called einbein.

The effective action (2) will be used for the calculations of the rate of various types of pair production in electromagnetic field. The simplest process of such calculations is the Schwinger effect — pair production from the vacuum in external electric field $\mathbf{E}$. The rate of the process is proportional to the imaginary part of the effective action (2):

$$\Gamma = \mathrm{Im}\ W[A_\mu]. \tag{4}$$

Both integrals over x_μ and s in (2) are taken in the saddle point approximation. The leading order classical solution is a circle [2] (see Fig.1, left panel)

$$x_0^{cl} = \frac{m}{eE}\sin(2\pi\tau), \qquad x_1^{cl} = \frac{m}{eE}\cos(2\pi\tau),$$
$$x_2^{cl} = x_3^{cl} = 0, \qquad s = \frac{2\pi}{eE}.$$

The action on the solution x_μ^{cl} is $S[x_\mu^{cl}] = \frac{\pi m^2}{eE}$. Integration over fluctuations near the classical

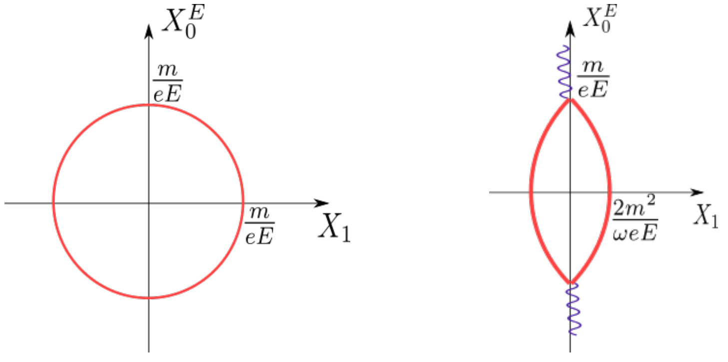

Figure 1. Worldline instanton — the classical solution describing pair production in external electric field. Left panel: pair production from the vacuum (the Schwinger effect). Right panel: photon decay to e^+e^- pair (photon frequency $\omega \gg 2m$).

solution gives pre-exponential factor the result for the width of pair production is:

$$\Gamma = \frac{(eE)^2}{(2\pi)^3}\, e^{-\frac{\pi m^2}{eE}} . \tag{5}$$

The width of the Schwinger effect is exponentially suppressed for any reasonable electric field. The critical field $E_{cr} \sim m^2/e$, for which the suppression exponent is of the order of unity, is unachievable in the laboratory. However, external photon(s) may catalyse the Schwinger process [9], significantly decreasing the suppression exponent.

3 Dealing with external photons

In order to consider generalizations of the Schwinger effect with several photons in the initial state, one should first specify the representation of the initial photons in the worldline formalism. Thus, the photon vertex for an external photon of momentum k_μ and polarization ε_μ in scalar QED is expressed as [10]:

$$V[k_\mu, \varepsilon_\mu(k)] = \int_0^1 d\tau\, \varepsilon_\mu(k)\dot{x}_\mu(\tau) e^{ik^\mu x^\mu(\tau)}. \tag{6}$$

The N-point photon amplitude, in turn, take the form [10]:

$$M[k_1, \varepsilon(k_1); ... ; k_N, \varepsilon(k_N)] = (-ie)^N \int_0^\infty \frac{ds}{s} e^{-m^2 s}(4\pi s)^{-2} \langle V[k_1, \varepsilon(k_1)] ... V[k_N, \varepsilon(k_N)]\rangle, \tag{7}$$

where the correlator $\langle V[k_1, \varepsilon(k_1)] ... V[k_N, \varepsilon(k_N)]\rangle$ means averaging over particle action (3),

$$\langle V[k_1, \varepsilon(k_1)] ... V[k_N, \varepsilon(k_N)]\rangle \equiv \int_{p.b.c} Dx_\mu\, [V[k_1, \varepsilon(k_1)] ... V[k_N, \varepsilon(k_N)]]\ \mathrm{e}^{-S_m[x_\mu]}. \tag{8}$$

The overall exponent for the N-point photon amplitude (7) includes the particle action $S_m[x_\mu]$ (3) as well as N new terms $ik^\mu_j x^\mu(\tau_j)$ ($j = 1..N$) each from the corresponding photon vertex (6). While the term $S_m[x_\mu]$ is an integral of particle Lagrangian over Euclidean proper time τ, the terms $ik^\mu_j x^\mu(\tau_j)$ plays the role of interactions in a fixed moments of proper time τ_j. As we will see later, these terms give boundary conditions for classical solutions.

It follows from the Optical theorem that the cross-sections of pair production are proportional to the imaginary part of the corresponding photon amplitude (depending on the number of initial photons). Thus, the decay width of a photon with momentum k and frequency ω is proportional to the imaginary part of 2-point photon amplitude:

$$\Gamma_{\gamma\to e^+e^-} = \frac{1}{2\omega}\, \mathrm{Im}\, \mathcal{M}\left[k, \varepsilon(k); k, \varepsilon^*(k)\right]. \tag{9}$$

while the cross-section of Breit-Wheeler process — to the imaginary part of 4-point amplitude:

$$\sigma_{\gamma\gamma\to e^+e^-} = \frac{1}{2E_{c.m.}p_{c.m.}}\, \mathrm{Im}\, \mathcal{M}\left[k_1, \varepsilon(k_1); k_2, \varepsilon(k_2); k_1, \varepsilon^*(k_1); k_2, \varepsilon^*(k_2)\right], \tag{10}$$

where $E_{c.m.}$ and $p_{c.m.}$ are correspondingly energy and momentum of one initial photon in center-of-energy frame.

4 The photon decay $\gamma \to e^+e^-$ in external electric field.

Let us first review the width of photon decay in electric field. Let us consider a photon of energy ω propagating orthogonally to the electric field $\mathbf{E}$. Taking into account the expressions for the photon width (9) and 2-photon amplitude (7), one obtains

$$\Gamma_{\gamma\to e^+e^-} = \frac{e^2}{2\omega}\mathrm{Im}\int_0^\infty \frac{ds}{s}\int_{p.b.c} Dx_\mu \oint d\tau_1 \varepsilon_\mu(k)\dot{x}_\mu(\tau_1)\oint d\tau_2 \varepsilon^*_\mu(k)\dot{x}_\mu(\tau_2)e^{-S_m[x_\mu;\tau_1,\tau_2]}, \quad (11)$$

where the action $S_m[x_\mu;\tau_1,\tau_2]$ is given by

$$S_m[x_\mu;\tau_1,\tau_2] = m^2 s + \int_0^1 d\tau\left(\frac{\dot{x}_\mu^2}{4s} + ieA_\mu\dot{x}_\mu\right) - ik_\mu(x_\mu(\tau_1) - x_\mu(\tau_2)). \quad (12)$$

The last two terms in (12) came from two photon vertices (6). All integrals in (11) can be taken in the saddle point approximation. Saddle point equations outside the points τ_1, τ_2 are the same as for vacuum pair production; variation in the points τ_1, τ_2 gives boundary conditions (see for details [5, 6]). The solution x_μ^{cl} is a closed trajectory in 4-dimensional coordinate space which is two arcs of a circle, connected at points τ_1, τ_2 — see Fig.1., right panel. The action (12) on this solution gives, for arbitrary ω, (cl. [9]):

$$S[x^\mu_{cl}] = \frac{m^2}{eE}\left[\left(2 + \frac{\omega^2}{2m^2}\right)\arctan\left(\frac{2m}{\omega}\right) - \frac{\omega}{m}\right]. \quad (13)$$

At small ω the Schwinger result (5) is restored; for large ω ($\omega \gg 2m$) the suppression exponent is significantly weaker than the Schwinger exponent: $S[x^\mu_{cl}] = \frac{8m^3}{3\omega eE}$; the pre-exponential factor is beyond of the scope of this note.

5 Breit-Wheeler process $\gamma\gamma \to e^+e^-$ in external electric field

The similar procedure may be applied for Breit-Wheeler process. Substituting the amplitude (7) for $N = 4$ into the cross-section (10), we obtain

$$\sigma_{\gamma\gamma\to e^+e^-}(k_1,k_2) \propto \mathrm{Im}\int_0^\infty \frac{ds}{s}e^{-m^2 s}\int_{p.b.c} Dx_\mu \prod_{j=1}^{2}\left(\oint d\tau_j \dot{x}_\mu(\tau_j)\varepsilon_\mu(k_j)\oint d\tau_{j+2}\dot{x}_\nu(\tau_{j+2})\varepsilon^*_\nu(k_j)\right)\cdot$$
$$\cdot\exp\left(-\int_0^s d\tau\left(\frac{\dot{x}_\mu^2}{4} + ieA_\mu\dot{x}_\mu\right) - ik_1^\mu x_\mu(\tau_1) - ik_2^\mu x_\mu(\tau_2) + ik_1^\mu x_\mu(\tau_3) + ik_2^\mu x_\mu(\tau_4)\right). \quad (14)$$

As in the previous case, we solve path integral in the saddle point approximation. We consider an ansatz for the saddle point solution x_μ^{cl} as a symmetric configuration in coordinate space (see Fig.(2)) where points $\tau_1 ... \tau_4$ are located symmetricaly around coordinate axes (see Fig.2, left panel). Particullary, points τ_1, τ_4 and τ_2, τ_3 may coincide (Fig.2, right panel).

Precise but complicated calculations of saddle point equations with boundary conditions (details to be appeared in a different paper [11]) show that at the saddle point configuration the upper and lower arcs (Fig.2, left panel) shrink into points (as at Fig.2, right panel). In other words, the instanton configuration consisting from two arcs minimizes the action. There is an explanation of this from a tunneling point of view: at the 4-arcs configuration the distance between all points scales with the electric field. However, this scaling is obviously unphysical

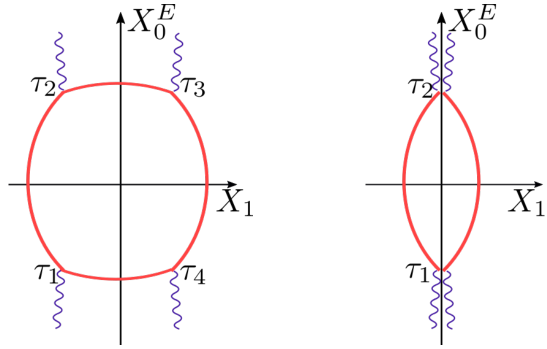

Figure 2. Worldline instanton describing Breit-Wheeler pair production. Left panel: general symmetric ansatz. Right panel: real form of the instanton.

for the distance between two initial photons; the real interaction disctance is beyond the main semiclassical approximation.

Let us calculate the Breit-Wheeler cross-section in the center-of energy frame. Consider two photons with momenta $k_1^\mu = (\omega, 0, \omega, 0)$ and $k_2^\mu = (\omega, 0, -\omega, 0)$ propagating orthogonally to the electric field $\mathbf{E} = (E, 0, 0)$. As the upper and lower arcs shrink into points, only two boundary conditions remain, so the task reduces to the calculation of electron-positron pair production in electric field, catalyzed by external 4-momentum $(k_1 + k_2)^\mu = (2\omega, 0, 0, 0)$ [1]. Naively, it may be treated as a decay of "virtual" photon with energy 2ω and zero spatial momentum. The result for the action on the classical solution is:

$$S[x_\mu^{cl}] = \frac{m^2}{eE}\left[\pi - 2\arcsin\frac{\omega}{m} - 2\frac{\omega}{m}\sqrt{1-\left(\frac{\omega}{m}\right)^2}\right]. \qquad (15)$$

The result coincides with [13]. The dependence of photon frequency for the suppression exponent (15), in comparison with the suppression exponent for the photon decay (12), is shown at Fig.3. We see that the Schwinger effect catalyzed by two photons is significantly less suppressed than catalyzed by a single photon. At the perturbative threshold $\omega/m = 1$ the suppression exponent (15) goes to zero.

Let us consider less symmetric configuration, head-on collision of two photons with different frequencies: $k_1^\mu = (\omega, 0, \omega, 0)$ and $k_2^\mu = (\omega', 0, -\omega', 0)$. Again, in the leading semiclassical approximation, the process is equivalent to pair production, catalyzed by external 4-momentum $(k_1 + k_2)^\mu = (\omega + \omega', 0, \omega - \omega', 0)$. The action on the classical solution reads:

$$S[x_\mu^{cl}] = \frac{m^2}{eE}\left[\frac{\eta}{2} - \frac{(\omega+\omega')^2}{2m^2}\tan\frac{\eta}{4} + \frac{\eta}{8}\frac{(\omega-\omega')^2}{m^2}\right], \qquad (16)$$

where $\tan^2\frac{\eta}{4} = 4\frac{m^2-\omega\omega'}{(\omega+\omega')^2}$. The results are shown at Fig.4. We see that even soft photon ω' may significantly increase the probability of the Schwinger process.

6 Conclusion

The exponential part of the cross-section of Breit-Wheeler process in external electric field has been computed via worldline instanton method in a semiclassical regime of exponential suppression. Two photons in the initial state sufficiently increase the probability of the

[1]This fact has been also recently shown in [13].

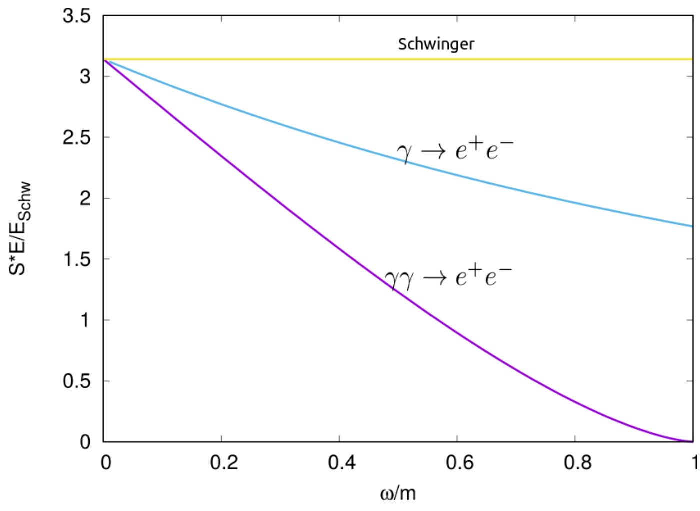

Figure 3. Scaled suppression exponent for photon decay (12) and Breit-Wheeler pair production (15) depending on the photon energy (purple and blue curves correspondingly). Yellow line shows suppression exponent for the Schwinger effect. $E_{Schw} = m^2/e$.

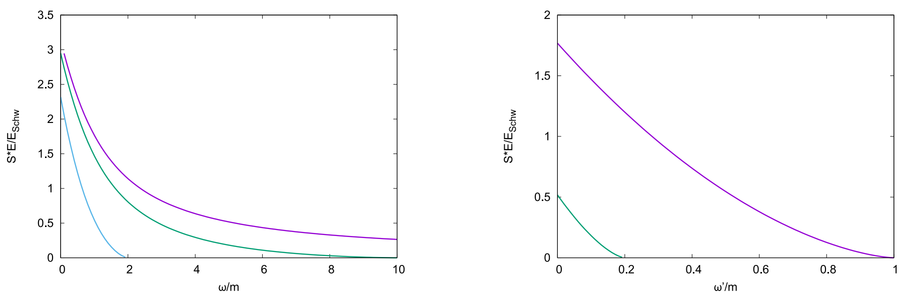

Figure 4. Left panel: Suppression exponent dependence on ω/m at different ω': $\omega' = 0$ (purple curve), $\omega' = 0.1m$ (green curve) and $\omega' = 0.5m$ (blue curve). Right panel: Suppression exponent dependence on ω'/m at different ω: $\omega = m$ (purple curve) and $\omega = 5m$ (green curve).

Schwinger process, better than a single initial photon. The calclulation support the idea that two dynamical componets assist the Schwinger process better than one [12, 13]. The following experimental test may be proposed: a head-on collision of two photons slightly below the perturbative Breit-Wheeler threshold in strong external field (from optical laser, for example). However, for quantative predictions a calculation of pre-exponential factor is necessary.

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