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Towards a non-perturbative approach to the Hierarchy problem

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Abstract. In this report, based on [1], we suggest that the hierarchy between the Electroweak (EW) and the Planck scales can emerge due to a non-perturbative effect that relates low-energy and strong-gravity physics. In this mechanism, the EW scale results from an exponential suppression of the Planck scale by an instanton. We illustrate the mechanism in a toy-model example and discuss what features of a theory in the strong-gravity domain would favor its successful implementation.

1 Introduction and setup

At the classical level, the Higgs boson mass m_H, which sets the Electroweak (EW) interaction scale, is the single dimensional parameter in the Standard Model (SM). Assuming that no heavy mass thresholds come with the physics beyond the SM, the only other scale which appears inevitably at high energies is provided by the Planck mass M_P. Comparing the two quantities, one obtains that

$$m_H/M_P \sim 10^{-17} \ . \quad (1)$$

It is natural to expect that a fundamental theory unifying the SM and gravity would suggest a dynamical mechanism by which the exponentially small number in eq. (1) is generated, rather than contain this number as an input parameter. This is one aspect of the problem of hierarchy of scales. Adopting quantum field theory framework brings new dimensions to it. They concern with the stability of the Higgs mass against radiative corrections engendered by the presence of heavy particles, see [2, 3] for reviews.[1] These quantum complications can be avoided by considering theories containing no such particles and by regularizing properly quadratic divergences to m_H [4–7].

In the limit $m_H = 0$, the classical Lagrangian of the SM acquires global conformal invariance (CI). It is tempting to suggest that the EW symmetry is unbroken classically and that the Higgs mass appears due to the violation of CI. One possibility of this is provided by the Coleman-Weinberg mechanism [8–11]. However, is becomes in contradiction with the observed values of the Higgs and the top quark masses [12, 13]. Another option would be to

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[1]It is assumed that these particles are coupled to the Higgs field.

suggest that the EW scale is generated in a way that is not captured by perturbation theory. To motivate this line of reasoning, note that eq. (1) can be conveniently written in the form

$$m_H \sim M_P e^{-B} \ , \tag{2}$$

where $B \approx 37$. It gives a hint that the non-zero value of m_H could result from an exponentially strong suppression of the Planck mass due to some instanton effect. In turn, the Planck mass may stand explicitly in the gravitational Lagrangian or else appear, say, due to spontaneous breaking of global scale symmetry. The latter option points towards a nice scenario in which one starts with the classically scale-invariant theory, generates classically the Planck scale, and then semiclassically — the EW scale in a natural way.

To endow eq. (2) with the physical meaning, notice that e^{-B} can be viewed as resulting from a saddle-point calculation of some functional integral. This observation leads to the following construction. Consider a theory containing the real scalar field φ, the metric field $g_{\mu\nu}$, and, possibly, other fields whose presence we ignore for the moment.[2] Let M_P be the only classical scale in the theory. Then, in the path integral formalism, the (spatially-homogeneous time-independent) vacuum expectation value (vev) of φ is given by

$$\langle\varphi\rangle \sim \int \mathcal{D}\varphi \mathcal{D}g_{\mu\nu}\, \varphi(0) e^{-S} \ , \tag{3}$$

where we switched to the euclidean signature for simplicity, and S is the euclidean action of the theory.[3] Suppose now that in the large-field limit the scalar field degree of freedom is reorganized according to

$$\varphi \to M_P e^{\bar{\varphi}/M_P} \ , \quad \varphi \gtrsim M_P \ , \tag{4}$$

where we put M_P in front of the exponent as it is the single classical scale at hand. Then, the expression in the path integral in this limit becomes

$$\varphi(0) e^{-S} \to M_P e^{-\mathcal{B}} \ , \quad \mathcal{B} = -\bar{\varphi}(0)/M_P + S \ . \tag{5}$$

Assuming that the saddle-point approximation (SPA) can be applied to the functional $\mathcal{B}$ and identifying φ with the low-energy Higgs field degree of freedom, one arrives at eq. (2), where B is the value of $\mathcal{B}$ taken at an appropriate saddle-point configuration.

The scheme outlined above must be justified by addressing the following questions:

- What are the saddle points of the functional $\mathcal{B}$, through which the modified path integral can be evaluated?
- Is there a semiclassical parameter that would justify the SPA made in obtaining eq. (2)?
- What is the physics behind the change of variable (4)?

The next section is dedicated to an explicit toy-model example, in which we demonstrate how the first two questions are resolved. The third question represents a more serious issue. Indeed, the value of $\langle\varphi\rangle$ is expected to be independent of any transformations of the variables used in the intermediate computations. In the example below we will see that what appears to be the canonical scalar field degree of freedom in the low-energy (or low-field) limit may not be so in the limit of large fields and energies. One may notice here an analogy with the gauge theories, the valid description of the confinement phase of which is performed in terms of Wilson loops, not the gauge field itself [16]. One may also suggest that although

[2] Other fields are expected to be irrelevant for the leading-order estimation of the effect in which only φ and $g_{\mu\nu}$ are involved.

[3] Because of the presence of gravity, the euclidean path integral must be treated with caution [14]. We assume that quantum gravity resolves possible issues arising when using this formulation; see also [15].

the full calculation will indeed produce the same answer regardless the choice of variables, the *partial* answer that includes only a finite amount of loops is sensitive to the background configuration upon which we build the perturbation theory. Hence, eq. (4) may result from a certain resummation procedure. We leave for future the further discussion of this issue.

2 One example

In choosing a particular model for quantitative tests of the mechanism discussed in the previous section, one must deal with the fact that we are not aware of an explicit UV completed theory comprising the SM and General Relativity. Therefore, we cannot say if the chosen model is indeed embeddable into the quantum gravity framework. However, our analysis remains reasonable as it helps to uncover those general properties of the theory in the regime of strong gravity, that favor the generation of the hierarchy of scales in the suggested way.

As was discussed above, we choose M_P to be the only classical scale in the model. We demand further that no new degrees of freedom interfere between the weak the the Planck scales. In the low-energy limit the model should match the CI SM and General Relativity. At high energies, operators of higher dimensions suppressed by a proper cutoff must also be considered. It is our assumption about the UV theory, that among such operators those are present, that we find useful for generating the hierarchy of scales.

To describe the Higgs-gravity sector of the model, we take the following Lagrangian,

$$\frac{\mathcal{L}_{\varphi,g}}{\sqrt{g}} = -\frac{1}{2}a(\varphi^2/M_P^2)M_P^2R+\frac{1}{2}b(\varphi^2/M_P^2)(\partial\varphi)^2 + \sum_{n=1}^{\infty}\delta_n(\varphi^2/M_P^2)\frac{(\partial\varphi)^{2n+2}}{a(\varphi^2/M_P^2)^{2n}M_P^{4n}} + \frac{\lambda(\varphi^2/M_P^2)}{4}\varphi^4 \ . \tag{6}$$

Here φ is the real scalar field to be associated with the Higgs field degree of freedom. a, b and δ_n are fractional-rational functions of φ^2/M_P^2 chosen as follows,

$$a = 1+\xi\varphi^2/M_P^2\ , \quad b = \frac{1+\kappa\varphi^2/M_P^2}{1+\varphi^2/M_P^2}\ , \quad \delta_2 \equiv \delta\xi^2\ , \quad \delta_n = 0\ , \quad n>1\ , \tag{7}$$

where ξ, δ and κ are constants and ξ, $\delta > 0$, $\kappa > -1/\xi$. Finally, the φ-dependence of the quartic coupling λ can be taken to mimic the RG running of the Higgs self-coupling in the SM setting.

As we will see, despite its apparent complexity, the theory (6) serves good in illustrating the main features of the mechanism. Furthermore, its form can be simplified by eliminating the non-minimal coupling of the scalar field to gravity. To this end, one performs the Weyl rescaling of the metric:

$$\tilde{g}_{\mu\nu} = a(\varphi^2/M_P^2)g_{\mu\nu}\ . \tag{8}$$

The Lagrangian is rewritten as[4]

$$\frac{\tilde{\mathcal{L}}}{\sqrt{\tilde{g}}} = -\frac{1}{2}M_P^2\tilde{R} + \frac{1}{2}\tilde{a}(\varphi^2/M_P^2)(\tilde{\partial}\varphi)^2 + \delta\xi^2\frac{(\tilde{\partial}\varphi)^4}{a(\varphi^2/M_P^2)^2M_P^4} + \frac{\lambda}{4}\varphi^4 a(\varphi^2/M_P^2)^{-2}\ , \tag{9}$$

[4]Transformation of different quantities under the Weyl rescaling can be found, e.g., in [17].

where

$$\tilde{a} = \frac{1}{a}\left(b + \frac{3M_P^2}{2a}\left(\frac{da}{d\varphi}\right)^2\right) , \tag{10}$$

and by $(\tilde{\partial}\varphi)^2$ we understand the kinetic term in which the partial derivatives are contracted with the transformed metric $\tilde{g}^{\mu\nu}$. In the limit $\varphi \ll M_P$, $\partial\varphi \ll M_P^2$, the scalar part of the Lagrangian (9) describes the field theory with the canonical kinetic term and the potential term, while in the opposite limit it describes the scale-invariant theory with the kinetic term

$$M_P^2 a_{HE}\frac{(\tilde{\partial}\varphi)^2}{\varphi^2} + \delta\frac{(\tilde{\partial}\varphi)^4}{\varphi^4} , \quad a_{HE} = 6 + \frac{\kappa}{\xi} . \tag{11}$$

From here we see that in the high-energy regime the canonical degree of freedom[5] is carried not by φ, but by $\bar{\varphi}$ related to φ via eq. (4). This is one step towards the justification of rewriting the large-field part of the path integral in terms of $\bar{\varphi}$ when evaluating the vev of φ. Hence, the quadratic coupling of φ to the Ricci scalar is an important ingredient of the mechanism.

Let us now look for saddle points of the functional $\mathcal{B}$ in the theory (6). Clearly, they solve (euclidean) equations of motion for the variable $\bar{\varphi}$ everywhere except the origin. At the origin, they solve the equations provided that the latter are accompanied with the point source of the field $\bar{\varphi}$,

$$\bar{\varphi}(0) = \int d^4x j(x)\bar{\varphi}(x) , \quad j(x) = \delta^{(4)}(x) . \tag{12}$$

The solutions of the equations with the source are expected to be singular at the source point. Because of this, we will refer to them as singular instantons.[6]

We adopt the spherically-symmetric ansatz for the metric in the form

$$d\tilde{s}^2 = f^2(r)dr^2 + r^2 d\Omega_3^2 . \tag{13}$$

Next, we demand the singular instanton to obey vacuum boundary conditions at infinity,

$$f^2 \to 1 , \quad \varphi \to 0 , \quad r \to \infty . \tag{14}$$

Note that the point source of $\bar{\varphi}$ can be viewed as an additional boundary condition, imposed on the solution of the usual equations of motion. Hence, the singular instanton is a unique solution of these equations.

Let us study the asymptotics of the instanton in different regimes of the theory. At large distances (the low-field limit), the configuration exhibits the massless power-like fall-off, $\varphi \sim r^{-2}$. Near the origin (the large-field limit), the quartic derivative operator in eq. (11) starts dominating and the asymptotics become

$$\bar{\varphi}' \sim -M_P^2\delta^{-1/6} , \quad f \sim rM_P\delta^{1/6} , \quad r \to 0 . \tag{15}$$

We observe that, contrary to what one can expect from a singular configuration, the magnitude of $\bar{\varphi}$ in the center of the instanton is finite. It is the higher-degree derivative part of the kinetic term, which regularizes the solution, as in the limit of zero δ the latter would have the logarithmic divergence,

$$\bar{\varphi}' \sim -\frac{M_P}{r} , \quad f \sim M_P^2 r^2 , \quad r \to 0 , \quad \delta = 0 . \tag{16}$$

[5] Up to a constant multiplier.

[6] The same name was adopted in [18] for singular configurations of a similar kind.

It is easy to make sure that the regularization of the instanton is not a specific feature of the derivative term of the 4th degree considered here; in fact, any higher-dimensional derivative operator of the form given in the 2nd line of eq. (6) would do the same job. Note also that, despite being finite, the solution experiences the physical singularity at the source point. Indeed, from eqs. (13) and (15) it follows that the scalar curvature behaves as

$$\tilde{R} \sim \frac{1}{r^2} \;, \quad r \to 0 \;. \tag{17}$$

Let us now estimate the value B of $\mathcal{B}$ computed on the singular instanton in the theory (6). It is reasonable to divide the total contribution to B in two parts,

$$B = B_{LE} + B_{HE} \;. \tag{18}$$

The term B_{LE} collects contributions from the low-field regime, $r \gtrsim M_P^{-1}$, while the term B_{HE} is saturated in the large-field domain where the higher-dimensional operators come into play. Using the Einstein equations, we obtain

$$B_{LE} \approx -2\pi^2 \int dr\, \mathcal{L}_V \;, \quad \mathcal{L}_V = r^3 f\, \frac{\lambda}{4} \varphi^4 a^{-2} \;. \tag{19}$$

Hence, the low-field contribution is determined by the properties of the potential for the field φ, that is, by the properties of the theory at the energies below the Planck scale. The large-field term, on the other hand, is given by

$$B_{HE} \approx -\frac{\bar{\varphi}(0)}{M_P} + 2\pi^2 \int dr\, \mathcal{L}_\delta \;, \quad \mathcal{L}_\delta = r^3 \delta \frac{\bar{\varphi}'^4}{f^3 M_P^4} \;. \tag{20}$$

The two options are now possible:

- Both B_{LE} and B_{HE} contribute significantly to B. In this case, one would generally expect a certain amount of fine-tuning taking place between the low-energy and high-energy regimes of the theory in order to reproduce the hierarchy of scales.
- The contribution from the sub-Planckian regime of the theory is negligible and $B \approx B_{HE}$.

As we will see shortly, in the theory (6) the second possibility is realized. The first option remains an interesting alternative that will be studied elsewhere.

The computation of B for the different values of the parameters of the theory is performed numerically. However, it is instructive to provide an analytic estimate of the dependence of B_{HE} on the parameter a_{HE}, defined in eq. (11):

$$B_{HE} \sim \frac{1}{\sqrt{a_{HE}}} \;. \tag{21}$$

Figure 1 demonstrates this dependence explicitly. As for B_{LE}, from figure 2 we see that the contribution of the scalar potential is virtually negligible compared to the contribution coming from the higher-dimensional term. This result holds regardless the particular form of the potential; for illustrative purposes, we take the latter to coincide with the effective Higgs potential in the SM setting and for the central values of the top quark and the Higgs masses. Hence, the estimation (21) is applicable to B itself. Choosing a_{HE} properly, one can achieve the value $B \approx 37$, in which case eq. (2) is reproduced. Note that a_{HE} can be viewed as a semiclassical parameter whose smallness ensures that $B \gg 1$ and justifies the SPA made in evaluating the path integral. The possible interpretation of this fact is that for a_{HE} sufficiently small, the effects of the strong-gravity regime of the theory on the vev of φ are suppressed, and the hierarchy of scales emerges.

To finish the analysis, one should supplement the Higgs-gravity Lagrangian (6) with the rest of the low-energy content. As soon as the leading order of the SPA is concerned, the fields which do not participate in building the instanton configuration can be ignored. Fluctuations of the fields affect the prefactor which in eq. (2) is set to be equal to M_P. We expect that the higher-order corrections in the SPA do not spoil the leading-order calculation.

3 Discussion and Conclusion

Let us summarize our findings. In this report, we attempted to look at the hierarchy of scales (1) as emerging due to the non-perturbative effect relating the low-energy and the Planck-scale physics. We argued that the weak scale can appear as a result of the exponential suppression of the Planck mass due to instantons. By construction, the mechanism does not require a fine-tuning among the parameters of the theory.

The effect we considered depends strongly on the features of the theory in the strong-gravity regime. The example of the previous section demonstrates some of the properties that are important in order for the mechanism to work successfully:

- The non-minimal coupling of the scalar field to gravity, controlled by the parameter ξ. It allowed us to change the scalar field variable according to eq. (4).
- In the high-energy regime, the non-minimal coupling lies close to the conformal limit. Indeed, in this regime the theory is written in terms of the original variables as follows,

$$\frac{\mathcal{L}_{\varphi,g,HE}}{\sqrt{g}} = \frac{1}{2}\frac{1}{6-a_{HE}}\varphi^2 R + \frac{1}{2}(\partial\varphi)^2 + \delta\frac{(\partial\varphi)^4}{\varphi^4} \ . \tag{22}$$

  According to eq. (21), the large suppression rate of the Planck mass is achieved if $\sqrt{a_{HE}} \ll 1$, which implies the near-conformal coupling of φ to the Ricci scalar.
- In the high-energy regime, the theory contains the scale-invariant higher-dimensional operators. They regularize the otherwise divergent instanton and provide the dominant contribution to the instanton action.

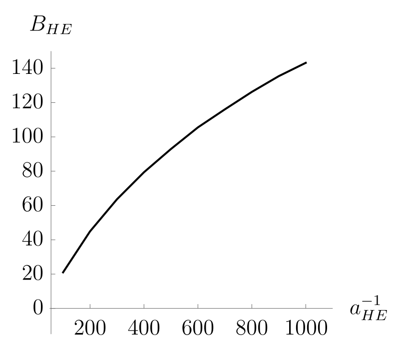

Figure 1: The high-energy part of the suppression rate B plotted against the coefficient a_{HE}^{-1}. An agreement with eq. (21) is observed. Here we take $\delta = 5 \cdot 10^{-8}$ in order to ensure the separation of scales at which $\tilde{a}$ changes and at which the quartic derivative operator in eq. (11) starts dominating.

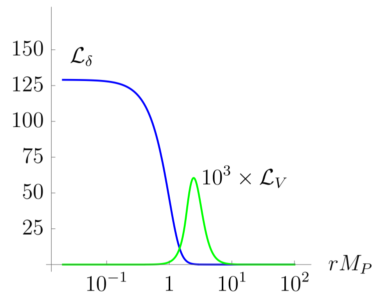

Figure 2: The functions $\mathcal{L}_V$ and $\mathcal{L}_\delta$ contributing to B_{LE} and B_{HE} according to eqs. (19) and (20). We take $a_{HE} = 0.005$, $\delta = 5 \cdot 10^{-8}$ and $\xi = 5 \cdot 10^2$. The self-coupling λ is taken as if it underwent the RG running in the SM with $m_H = 125.09$ GeV [19], $m_t = 172.25$ GeV [20] and the field-dependent normalization point $\mu = \bar{\varphi}$.

Scale invariance plays an important role in our analysis. Combining it with the absence of heavy degrees of freedom can protect the Higgs mass from large radiative corrections. Therefore, it provides a useful framework to address the Hierarchy problem. As was mentioned in section 1, it is appealing to implement a no-scale scenario in which all mass parameters appear due to the violation of the scale (or conformal, as in the pure SM case) symmetry. An example of such theory and its phenomenology were considered, e.g., in [21, 22], and the mechanism of the non-perturbative generation of the EW scale out of the Planck scale in such theories was studied in [23].

Despite some arguments provided above, the crucial ingredient of the mechanism — the exponential change of the field variable (4) — waits for a more rigorous justification. The same is true for the singular instanton whose physical implications are yet to be uncovered. Here, a careful treatment of fluctuations arising on top of these configurations may be fruitful. As one interesting direction in this research, one can mention possible applications of singular configurations, composed of many instantons of the type considered here, to evaluation of many-point correlation functions in the scalar sector of a theory.

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