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Compact Exotic Tetraquark Mesons in Large-N_c QCD

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Abstract. We embark on systematic explorations of the behaviour of tetraquark mesons, *i.e.*, colour-singlet bound states of two quarks and two antiquarks, in the (idealized) limit of a large number of colour degrees of freedom, N_c, of quantum chromodynamics, QCD. Considering the scattering of two ordinary mesons into two ordinary mesons, we start off with formulating a set of selection criteria that should enable us to *unambiguously* single out precisely those contributions to all encountered scattering amplitudes that potentially will develop tetraquark poles. Assuming that tetraquark mesons do exist and, if so, emerge in the contributions compatible with our criteria at largest admissible order of N_c, we deduce, for the categories of tetraquarks that exhibit either four or only two different open quark flavours, that the decay rates of these tetraquark types are, at least, of order $1/N_c^2$ and that internal consistency requires all the members of the first species to exist pairwise, distinguishable by their favoured two-ordinary-meson decay channels.

1 Stimulus: Decay Rates of Tetraquark States in the $1/N_c$ Expansion

The multiquark hadrons, such as tetraquarks (mesonic bound states of two antiquarks and two quarks) and pentaquarks (baryonic bound states of four quarks and one antiquark), form, from both the theoretical and the experimental points of view, one of the most challenging riddles in hadron phenomenology. Starting, for obvious reasons, with the mesonic and thus simpler case of tetraquarks, we approach this issue by investigating the manifestation of tetraquark mesons as intermediate states in scattering reactions of two ordinary mesons into two ordinary mesons in form of contributions of (narrow-width) poles to the associated scattering amplitudes [1–6].

We tackle this task in two steps: First, we attempt to formulate rigorous criteria that allow us to isolate the contributions of Feynman diagrams capable of developing a pole interpretable as being related to a compact (understood in contrast to a more loosely bound molecular-type) tetraquark meson; we shall refer to the members of this class of Feynman diagrams potentially of interest to us as the *tetraquark-friendly*, or *tetraquark-phile*, ones. Then, we invoke QCD in some well-defined disguise, to gain qualitative access to principal features of selected variants of tetraquarks discriminated basically by the flavour of their quark and antiquark constituents.

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1.1 Identification of Tetraquark-phile Feynman Diagrams: Set of Selection Criteria

In order to establish the contribution of a tetraquark T of mass m_T (assumed not to rise with N_c in the limit $N_c \to \infty$) viewed as bound state of four (anti-) quarks $\overset{(-)}{q}_i$ of masses m_i, $i = 1, \dots, 4$, to the s channel of the scattering of two appropriate ordinary mesons, of momenta p and q, we search for a corresponding pole in four-point correlation functions of quark-bilinear operators $j_{ij} \equiv \bar{q}_i\, q_j$ (dropping parity and spin) interpolating the incoming and outgoing mesons M_{ij} (by having nonzero matrix elements between vacuum and meson state), with decay constants $f_{M_{ij}}$:

$$\langle 0|j_{ij}|M_{ij}\rangle \equiv f_{M_{ij}} \neq 0 \ . \tag{1}$$

We characterize a correlator contribution potentially supporting a tetraquark pole (tagged by a subscript T) by the demand that the corresponding tetraquark-friendly Feynman diagram must

- involve the Mandelstam variable $s \equiv (p+q)^2$ in a nontrivial (*viz.*, non-polynomial) way, and
- admit an intermediate four-quark state, with a branch cut starting at $s = (m_1+m_2+m_3+m_4)^2$.

The existence of the latter threshold may be established by means of the Landau equations [7].

1.2 Large-N_c QCD: Quantum Chromodynamics' Limit $N_c \to \infty$ & $1/N_c$ Expansion

Large-N_c QCD [8, 9] constitutes a rather extreme generalization of QCD defined by letting the number N_c of the colour degrees of freedom of QCD grow beyond bounds, *i.e.*, by considering the limit $N_c \to \infty$ (and expansions thereabout), and simultaneously demanding that the strong coupling g_s decreases for rising N_c such that the strong fine-structure coupling α_s behaves like

$$\alpha_s \equiv \frac{g_s^2}{4\pi} \propto \frac{1}{N_c} \ .$$

Conventionally — but not necessarily — all quarks are required to still transform according to the fundamental representation, of dimension N_c, of the then underlying gauge group SU(N_c). Large-N_c QCD provides an, at least, qualitative understanding of crucial aspects of the hadron spectrum within a, compared to QCD, simpler theoretical environment and therefore has been applied [10–14] also to the issues of both existence and characterization of tetraquark mesons. Among the first insights arising thereof is the large-N_c behaviour of the decay constants $f_{M_{ij}}$ of ordinary mesons, defined in Eq. (1), which prove to exhibit a square-root increase with N_c [9]:

$$f_{M_{ij}} \propto \sqrt{N_c} \ .$$

1.3 Actual Target: Large-N_c Behaviour of Total Decay Width of Tetraquark Mesons

The crucial feature of any tetraquark meson is the N_c dependence of its total decay width [11]. Experimental observability of a tetraquark meson forbids its decay rate to grow with rising N_c, that is to say, in the limit $N_c \to \infty$; consequently, detectable tetraquarks must be narrow states.

Since the two quarks and two antiquarks building up a given tetraquark may be grouped to ordinary-meson states in two different ways, in each case we analyze in parallel two scattering channels discriminated according to whether the assignment of quark flavours in the incoming and outgoing ordinary mesons is identical (in that case dubbed as "flavour-preserving") or not (in that case dubbed as "flavour-rearranging," that is, has undergone some flavour reshuffling).

Since, according to the above, we will have to deal with two scattering channels, we better should be aware of the fact that it might easily happen that the obtained consistency conditions cannot be satisfied by a single tetraquark meson but inevitably require the contributions of two tetraquarks (generally called T_A and T_B) with, of course, *identical* quark-flavour composition. We will take into account the possibility of involuntary doubling of the tetraquark spectrum by suitable formulation of the tetraquark-friendly four-point correlators from the very beginning.

2 Flavour-Exotic Tetraquark Mesons ≡ Four-Different-Flavour States

Genuinely flavour-exotic tetraquarks, carrying not less than four different quark flavours, may induce poles in two classes of four-point correlators related to two-ordinary-meson scattering,

$$\text{flavour preserving:} \qquad \langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle\,, \qquad \langle j_{14}^\dagger\, j_{32}^\dagger\, j_{14}\, j_{32}\rangle\,; \tag{2a}$$

$$\text{flavour reshuffling:} \qquad \langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle\,. \tag{2b}$$

2.1 Flavour-Preserving Four-Current Correlation Functions: N_c-Leading Diagrams

Figure 1 depicts a few typical examples of Feynman diagrams of high order in N_c contributing to the flavour-preserving correlator $\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle$. Diagrams of the type Fig. 1(a) or 1(b) may be shown not to satisfy the criteria of Subsect. 1.1 and thus not to lead to four-quark cuts [1, 4]. In contrast to that, a Feynman diagram of the type shown in Fig. 1(c) might develop tetraquark poles. Accordingly, we find, as upper bounds to the large-N_c behaviour of the tetraquark-phile contributions to both flavour-preserving four-point correlation functions of Eq. (2a) [1, 4], that

$$\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle_{\rm T} = O(N_c^0)\,, \qquad \langle j_{14}^\dagger\, j_{32}^\dagger\, j_{14}\, j_{32}\rangle_{\rm T} = O(N_c^0)\,. \tag{3}$$

The contributions of the poles at $p^2 = m^2_{T_{A,B}}$ to these correlators involve ordinary-meson decay constants, f_M, and tetraquark–two-ordinary-meson transition amplitudes, $A(M_{ij}\, M_{kl} \leftrightarrow T_{A,B})$:

$$\langle j_{12}^\dagger\, j_{34}^\dagger\, j_{12}\, j_{34}\rangle_{\rm T} = f_M^4 \left(\frac{|A(M_{12}\, M_{34} \leftrightarrow T_A)|^2}{p^2 - m^2_{T_A}} + \frac{|A(M_{12}\, M_{34} \leftrightarrow T_B)|^2}{p^2 - m^2_{T_B}}\right) + \cdots\,,$$

$$\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{14}\, j_{32}\rangle_{\rm T} = f_M^4 \left(\frac{|A(M_{14}\, M_{32} \leftrightarrow T_A)|^2}{p^2 - m^2_{T_A}} + \frac{|A(M_{14}\, M_{32} \leftrightarrow T_B)|^2}{p^2 - m^2_{T_B}}\right) + \cdots\,.$$

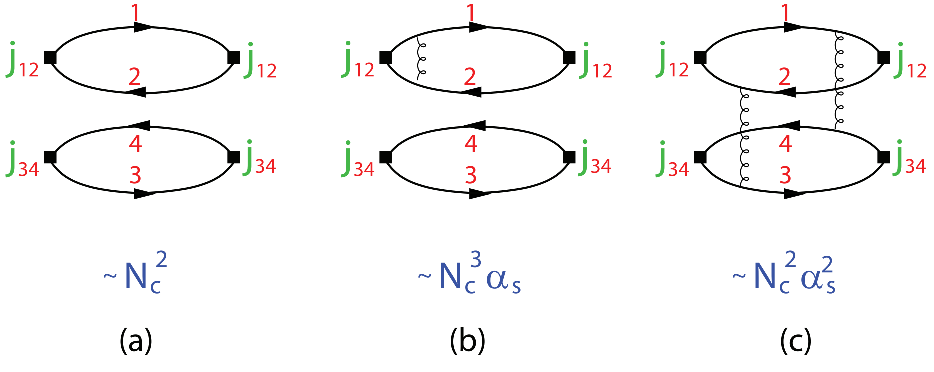

Figure 1. Some of the contributions to the flavour-preserving four-point Green functions (2a) [1, Fig. 1].

2.2 Flavour-Rearranging Four-Current Correlation Functions: N_c-Leading Graphs

Figure 2 depicts some representatives of Feynman diagrams of high order in N_c that contribute to the flavour-rearranging correlator $\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle$. The diagrams of the type Fig. 2(a) or 2(b) turn out to be not compatible with the requirements proposed in Subsect. 1.1 [1, 4]. This result may be understood by unfolding the quark lines to their respective box shape, as illustrated by Fig. 2 [3]. The decisive aspect in any such quest for a tetraquark pole is the flow of colour [13].

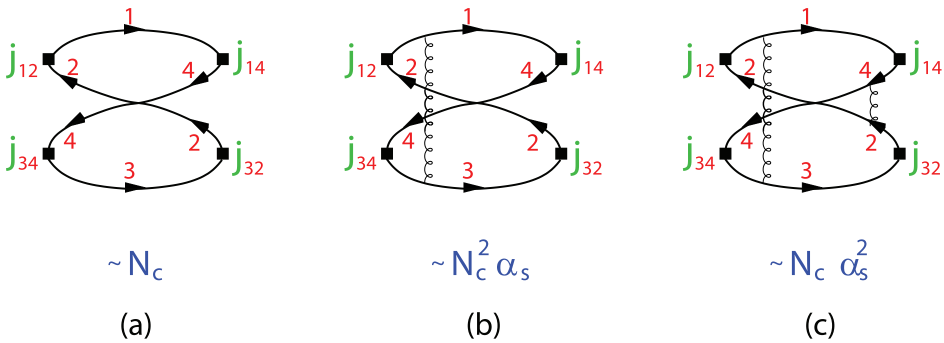

Figure 2. Some of the contributions to the flavour-rearranging four-point Green function (2b) [1, Fig. 2].

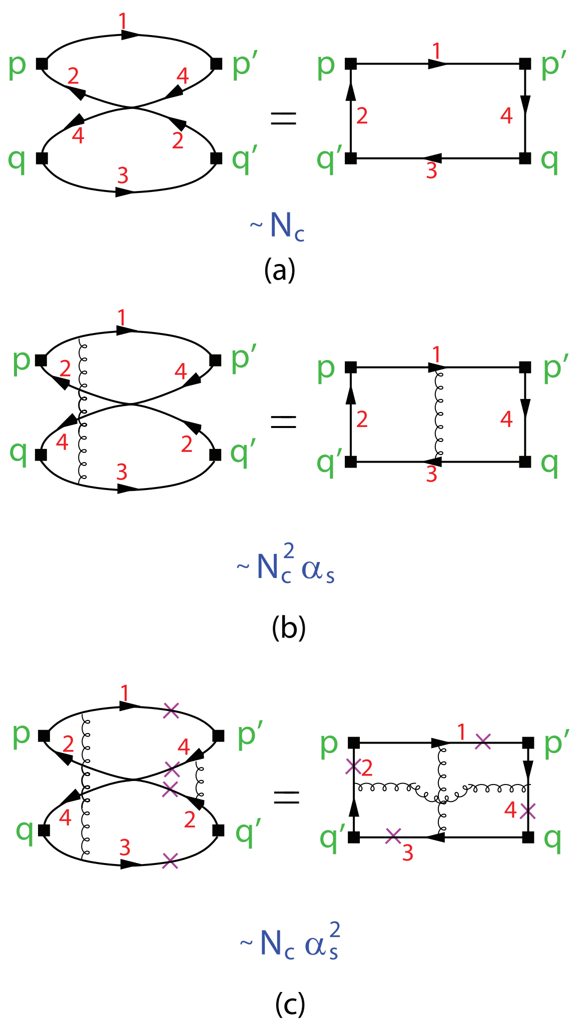

Figure 3. Unfolding of quark lines to box shapes of all illustrative Feynman diagrams depicted in Fig. 2.

As a consequence, we infer, for the large-N_c behaviour of the tetraquark-phile contributions to the flavour-rearranging four-point correlation function of type Eq. (2b), the upper bound [1, 4]

$$\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle_{\rm T} = O(N_c^{-1}) \,. \tag{4}$$

Here, the required tetraquark-induced redistribution of quark flavour at the poles at $p^2 = m_{T_{A,B}}^2$ combines different tetraquark–two-ordinary-meson transition amplitudes $A(M_{ij}\, M_{kl} \leftrightarrow T_{A,B})$:

$$\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle_{\rm T} = f_M^4 \left(\frac{A(M_{12}\, M_{34} \leftrightarrow T_A)\, A(T_A \leftrightarrow M_{14}\, M_{32})}{p^2 - m_{T_A}^2} \right. \\ \left. + \frac{A(M_{12}\, M_{34} \leftrightarrow T_B)\, A(T_B \leftrightarrow M_{14}\, M_{32})}{p^2 - m_{T_B}^2} \right) + \cdots \,.$$

2.3 Large-N_c Amplitudes of Tetraquark–Two-Meson Transitions and Decay Widths

The *upper bounds* on the large-N_c dependence of the tetraquark-friendly contributions to both categories of four-point correlation function arising, on the one hand, from flavour-preserving reactions, given by Eq. (3), but, on the other hand, from flavour-reshuffling reactions, given by Eq. (4), are certainly different. Let us, for definiteness, assume that tetraquark poles contribute to such scattering amplitude at the largest possible order of N_c, being tantamount to the lowest conceivable order of the perturbation expansion of the scattering amplitude in powers of $1/N_c$.

It is easy to convince oneself that in this case the arising self-consistency conditions on the large-N_c dependence of all the tetraquark–two-ordinary-meson transition amplitudes entering in the involved pole terms cannot be satisfied by the contribution of merely a single tetraquark. However, since the N_c order of the flavour-rearranging scattering processes is smaller than the N_c order of the flavour-preserving scattering processes, the self-consistency conditions can be satisfied by postulating the existence of two tetraquarks with, clearly, identical flavour content but different couplings to that two ordinary mesons with different assignment of quark flavour:

$$A(T_A \leftrightarrow M_{12}\, M_{34}) = O(N_c^{-1}) \,, \qquad A(T_A \leftrightarrow M_{14}\, M_{32}) = O(N_c^{-2}) \,, \tag{5a}$$

$$A(T_B \leftrightarrow M_{12}\, M_{34}) = O(N_c^{-2}) \,, \qquad A(T_B \leftrightarrow M_{14}\, M_{32}) = O(N_c^{-1}) \,. \tag{5b}$$

Hence, the total decay rates of these two tetraquarks $T_{A,B}$ exhibit the same large-N_c behaviour:

$$\Gamma(T_A) = O(N_c^{-2}) \,, \qquad \Gamma(T_B) = O(N_c^{-2}) \,.$$

2.4 Mixing of Flavour-Exotic Tetraquark States of Identical Quark-Flavour Content

By construction, our two tetraquark states T_A and T_B introduced above are bound states of one and the same set of (anti-) quarks and thus trivially exhibit the same flavour quantum numbers. An upper bound to their mixing strength g_{AB} is provided by the flavour-rearranging correlator:

$$\langle j_{14}^\dagger\, j_{32}^\dagger\, j_{12}\, j_{34}\rangle_{\rm T} = f_M^4 \left(\frac{A(M_{12}\, M_{34} \leftrightarrow T_A)}{p^2 - m_{T_A}^2}\, g_{AB}\, \frac{A(T_B \leftrightarrow M_{14}\, M_{32})}{p^2 - m_{T_B}^2} \right) + \cdots \,.$$

Our upper bounds (5a) and (5b) to any N_c-*leading* among the tetraquark–two-ordinary-meson transition amplitudes $A(M_{ij}\, M_{kl} \leftrightarrow T_{A,B})$ translate into an upper bound to the mixing strength:

$$g_{AB} \le O(N_c^{-1}) \,.$$

3 Flavour-Cryptoexotic Tetraquark Meson = Just Two Open Flavours

For cryptoexotic tetraquarks $T = (\bar q_1\, q_2\, \bar q_2\, q_3)$, with one quark–antiquark pair having the same flavour, the following two classes of four-point Green functions have to be taken into account:

$$\text{flavour preserving:} \qquad \langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle\,, \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{13}\, j_{22}\rangle\,; \tag{6a}$$
$$\text{flavour reshuffling:} \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle\,. \tag{6b}$$

3.1 Flavour-Preserving Four-Current Correlation Functions: N_c-Leading Diagrams

Figure 4 depicts two examples of N_c-leading tetraquark-phile Feynman diagrams contributing to the flavour-preserving correlator $\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle$, with their tetraquark friendliness revealed by purple crosses. The implied upper bounds to the large-N_c behaviour of these correlators are

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = O(N_c^0)\,, \qquad \langle j_{13}^\dagger\, j_{22}^\dagger\, j_{13}\, j_{22}\rangle_{\rm T} = O(N_c^0)\,. \tag{7}$$

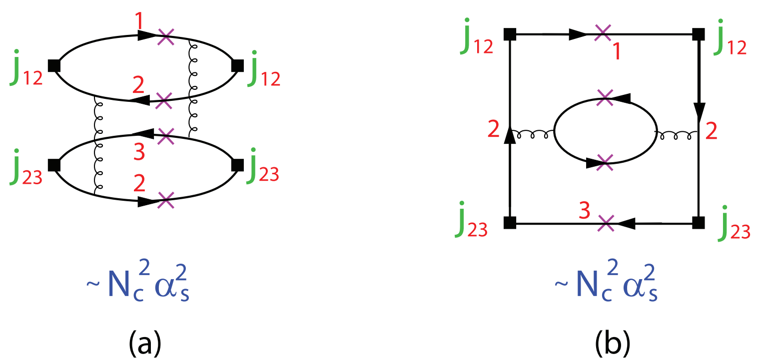

Figure 4. Some of the contributions to the flavour-preserving four-point Green functions (6a) [1, Fig. 3].

3.2 Flavour-Rearranging Four-Current Correlation Functions: N_c-Leading Graphs

Figure 4(b) demonstrates that the flavour-reshuffling Green function, $\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle$, receives contributions of topologically new, and even N_c-leading, tetraquark-phile Feynman diagrams:

$$\langle j_{13}^\dagger\, j_{22}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = O(N_c^0)\,. \tag{8}$$

3.3 Large-N_c Amplitudes of Tetraquark–Two-Meson Transitions and Decay Widths

The *upper bounds* (7) and (8) to the large-N_c dependence of the tetraquark-phile contributions to the flavour-preserving and flavour-reshuffling four-point correlation functions are identical. Thus, a single cryptoexotic tetraquark T can satisfy the emerging self-consistency conditions:

$$A(T \leftrightarrow M_{12}\, M_{23}) = O(N_c^{-1})\,, \qquad A(T \leftrightarrow M_{13}\, M_{22}) = O(N_c^{-1})$$
$$\Longrightarrow \qquad \Gamma(T) = O(N_c^{-2})\,.$$

3.4 Mixing of Equal-Flavour Ordinary and Flavour-Cryptoexotic Tetraquark Meson

Needless to say, each cryptoexotic tetraquark $T = (\bar q_1\, q_2\, \bar q_2\, q_3)$ and associated ordinary meson M_{13} enjoy the same net quark-flavour quantum numbers and can mix. An upper bound to their mixing strength $g_{TM_{13}}$ can be estimated from, *e.g.*, the correlator $\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T}$ according to

$$\langle j_{12}^\dagger\, j_{23}^\dagger\, j_{12}\, j_{23}\rangle_{\rm T} = f_M^4 \left(\frac{A(M_{12}\, M_{23} \leftrightarrow T)}{p^2 - m_T^2}\, g_{TM_{13}}\, \frac{A(M_{13} \leftrightarrow M_{12}\, M_{23})}{p^2 - m_{M_{13}}^2}\right) + \cdots\,.$$

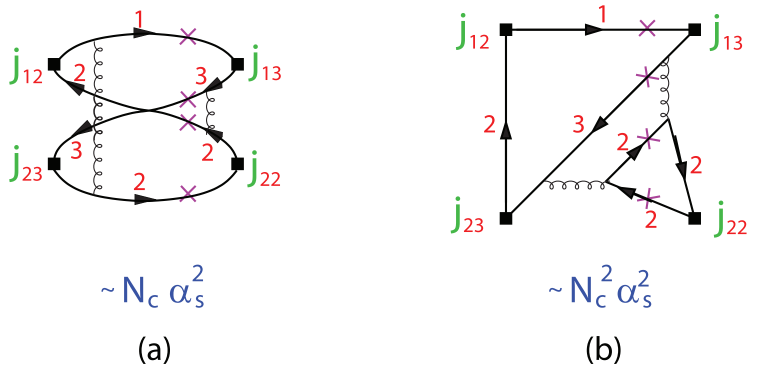

Figure 5. Some of the contributions to the flavour-rearranging four-point Green function (6b) [1, Fig. 4].

The three-ordinary-meson coupling, $A(M_{13} \leftrightarrow M_{12}\,M_{23})$, has been found [8, 9] to behave like

$$A(M_{13} \leftrightarrow M_{12}\,M_{23}) \propto \frac{1}{\sqrt{N_c}} \ .$$

Thus, the large-N_c behaviour of any cryptoexotic-tetraquark–ordinary-meson mixing satisfies

$$g_{T M_{13}} \leq O(N_c^{-1/2}) \ ,$$

if, for definiteness, we choose to adhere to the upper bound $A(T \leftrightarrow M_{12}\,M_{23}) = O(N_c^{-1})$ to the large-N_c behaviour of the cryptoexotic-tetraquark–two-ordinary-meson transition amplitudes.

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