

# The linear radial spectrum of scalar mesons from QCD sum rules in the planar limit

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**Abstract.** We discuss calculations of large- $N_c$  masses of light scalar mesons from the QCD sum rules. Two methods based on the use of linear radial Regge trajectories are presented. We point out the appearance of state near 0.5 GeV in the scalar isoscalar channel which emerges in both methods and presumably corresponds to the lightest scalar particle — the sigma-meson.

## 1 Introduction

It is widely known that the physics of non-perturbative strong interactions is enciphered in the values of hadron masses. This intricate physics is especially pronounced in the hadrons consisting of  $u$ - and  $d$ -quarks as their masses  $m_{u,d}$  are much less than the non-perturbative scale  $\Lambda_{\text{QCD}}$ . At the same time, the given hadrons shape the surrounding world. Aside from the nucleons and pions, an important role is played by the scalar sigma-meson which is responsible for the main part of the nucleon attraction potential. In the particle physics, the given resonance is identified as  $f_0(500)$  [1] and is indispensable for description of the chiral symmetry breaking in many phenomenological field models describing the strong interactions. The scalar sector below and near 1 GeV is perhaps the most difficult for traditional approaches in the hadron spectroscopy. The usual quark model faces serious problems in explaining the existence and properties of light scalar mesons, perhaps due to a strong admixture of glueball component. Despite the recent progress in description of these states by dispersive methods [1, 2] the scalar sector still remains puzzling.

The physical characteristics of hadrons are encoded in various correlation functions of corresponding hadron currents. Perhaps the most important characteristics is the hadron mass. The calculation of a hadron mass from first principles consists in finding the relevant pole of two-point correlator  $\langle JJ \rangle$ , where the current  $J$  is built from the quark and gluon fields and interpolates the given hadron. For instance, if the scalar isoscalar state  $f_0$  represents an ordinary light non-strange quark-antiquark meson, its current should be interpolated by the quark bilinear  $J = \bar{q}q$ , where  $q$  stays for the  $u$  or  $d$  quark. In the real QCD, the straightforward calculations of correlators are possible only in the framework of lattice simulations which are still rather restricted.

It is usually believed that confinement in QCD leads to approximately linear radial Regge trajectories (see, e.g., [3]). The most important quantity in this picture is the slope of trajectories. The slope is expected to be nearly universal as arising from flavor-independent non-perturbative gluodynamics which thereby sets a mass scale for light hadrons.

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Among the phenomenological approaches to the hadron spectroscopy, the method of spectral sum rules [4] is perhaps the most related with QCD. In many cases, it permits to calculate reliably the masses of ground states on the radial trajectories. This method exploits some information from QCD via the Operator Product Expansion (OPE) of correlation functions [4]. On the other hand, one assumes a certain spectral representation for a correlator in question. Typically the representation is given by the ansatz "one infinitely narrow resonance + perturbative continuum". Such an approximation is very rough but works well phenomenologically in many cases. Theoretically the zero-width approximation arises in the large- $N_c$  limit of QCD [5]. In this limit, the only singularities of the two-point correlation function of a hadron current  $J$  are one-hadron states. For instance, the two-point correlator for  $J = \bar{q}q$  has the following form to lowest order in  $1/N_c$  (in the momentum space),

$$\Pi_S(q^2) = \langle J^S(q)J^S(-q) \rangle = \sum_n \frac{G_n^2 M_S^2(n)}{q^2 - M_S^2(n)}, \quad (1)$$

where the residues appear from the definition of the matrix element  $\langle 0|J^S|n \rangle = G_n M_S(n)$ . The OPE of the correlator (1) in the large- $N_c$  limit and to the lowest order in the perturbation theory reads [6]

$$\Pi_S(Q^2) = \frac{3Q^2}{16\pi^2} \log \frac{Q^2}{\mu^2} + \frac{3}{2Q^2} m_q \langle \bar{q}q \rangle - \frac{\alpha_s}{16\pi} \frac{\langle G^2 \rangle}{Q^2} - \frac{11}{3} \pi \alpha_s \frac{\langle \bar{q}q \rangle^2}{Q^4} + \dots, \quad (2)$$

where  $\langle G^2 \rangle$  and  $\langle \bar{q}q \rangle$  denote the gluon and quark vacuum condensate, respectively. According to the main assumption of classical QCD sum rules [4], these vacuum characteristics are universal, i.e., their values do not depend on the quantum numbers of a hadron current  $J$  (the method is not applicable otherwise).

In the present talk we will demonstrate how all these ideas can be used for calculation of large- $N_c$  masses of light scalar mesons.

## 2 Scalar sum rules: Some results

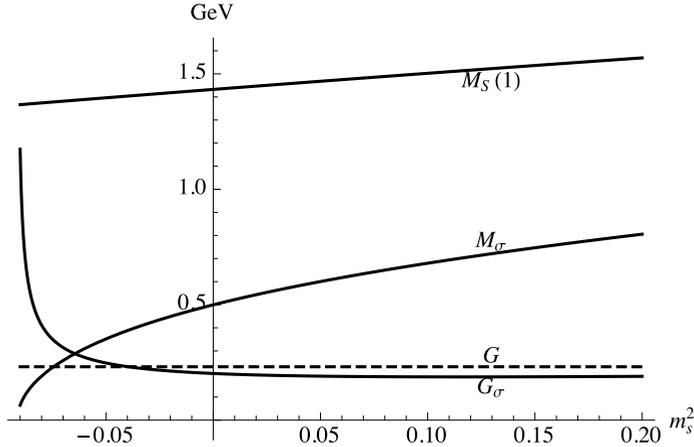
We will assume the linear radial spectrum with universal slope

$$M_S^2(n) = \Lambda^2(n + m_s^2), \quad n = 0, 1, 2, \dots, \quad (3)$$

and (for consistency with the OPE):  $G_n = G$ . With the linear ansatz (3) for the radial mass spectrum, the expression (1) can be summed analytically, expanded at large  $Q^2 = -q^2$  and compared with the corresponding OPE in QCD. Thus one obtains a set of sum rules. Similar large- $N_c$  sum rules were considered many times in the past for vector, axial, scalar and pseudoscalar channels (see, e.g., Refs. in [7]).

As *a priori* we do not know reliably the radial Regge behavior of scalar masses, two simple possibilities can be considered: (I) The ground  $n = 0$  state lies on the linear trajectory (3); (II) The state  $n = 0$ , below called  $\sigma$ , is not described by the linear spectrum (3). The second assumption looks more physical. Within the latter assumption, the mass of  $\sigma$ -meson can be derived as a function of the intercept parameter  $m_s^2$  (we refer to Ref. [8] for details, the chiral limit is considered),

$$M_\sigma^2 = \frac{\frac{1}{16\pi^2} \Lambda^6 m_s^2 \left(m_s^2 + \frac{1}{2}\right) \left(m_s^2 + 1\right) + \frac{11}{3} \pi \alpha_s \langle \bar{q}q \rangle^2}{\frac{3}{32\pi^2} \Lambda^4 \left(m_s^4 + m_s^2 + \frac{1}{6}\right) + \frac{\alpha_s}{16\pi} \langle G^2 \rangle}. \quad (4)$$



**Figure 1.** The values of  $M_\sigma$ ,  $G_\sigma$ ,  $G$ , and the first state on the scalar trajectory  $M_S(1)$  as a function of dimensionless intercept  $m_s^2$ .

Substituting the physical values of vacuum condensates and numerical value for slope  $\Lambda^2$  obtained from a solution of QCD sum rules, the mass function (4) is displayed in Fig. 1 [8].

The mass of the first radially excited state  $M_S(1)$  is rather stable and seems to reproduce the mass of  $a_0(1450)$ -meson,  $M_{a_0(1450)} = 1474 \pm 19$  MeV [1]. Its isosinglet partner (the candidates is  $f_0(1370)$ ) should be degenerate with  $a_0(1450)$  in the planar limit.

The plot in Fig. 1 demonstrates that the actual prediction for  $M_\sigma$  is rather sensitive to the intercept of scalar linear trajectory, though initially  $M_\sigma$  is not described by the linear spectrum (3). And *vice versa*, the expected value of  $M_\sigma$  (around 0.5 GeV [1]) imposes a strong bound on the allowed values of intercept  $m_s^2$ . The plot in Fig. 1 shows that  $m_s^2$  is likely close to zero.

Thus, interpolating the scalar states by the simplest quark bilinear current, we predict a light scalar resonance with mass about  $500 \pm 100$  MeV which is a reasonable candidate for the scalar sigma-meson  $f_0(500)$  [1].

### 3 Borelized scalar sum rules: Some results

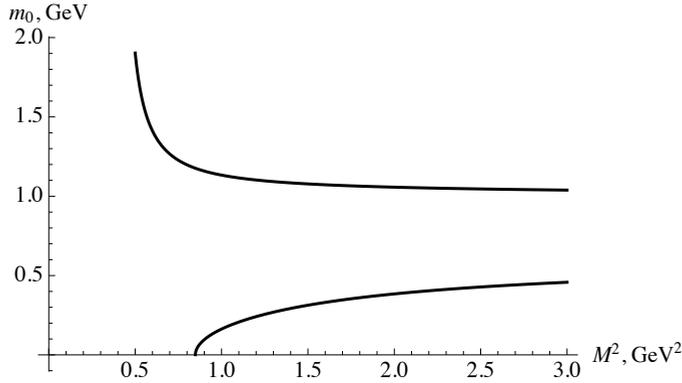
The original QCD sum rules made use of the Borel transformation [4],

$$L_M \Pi(Q^2) = \lim_{\substack{Q^2, n \rightarrow \infty \\ Q^2/n = M^2}} \frac{1}{(n-1)!} (Q^2)^n \left( -\frac{d}{dQ^2} \right)^n \Pi(Q^2), \quad (5)$$

The borelized version has a number of advantages and can be applied to our large- $N_c$  case. The details are contained in Ref. [9]. In short, the mass of ground scalar meson  $m_0 \equiv M_S(0)$  as a function of Borel parameter is shown in Fig. 2. It is seen that there are two solutions with "Borel window" extending to infinity. The corresponding asymptotic values are given by

$$M_S^2(0) = \frac{\Lambda^2}{2} \pm \frac{1}{2} \sqrt{\frac{\Lambda^4}{3} - 64\pi^2 \left( m_q \langle \bar{q}q \rangle + \frac{\alpha_s}{24\pi} \langle G^2 \rangle \right)}. \quad (6)$$

The heavier state corresponds to the ground scalar mass in the standard QCD sum rules. Normalizing this mass to the value  $m_{f_0} = 1.00 \pm 0.03$  GeV extracted from these canonical



**Figure 2.** The mass of ground scalar meson  $m_0 \equiv M_S(0)$  as a function of Borel parameter at  $\Lambda^2 = 1.38 \text{ GeV}^2$  [9].

sum rules [6] we predict the value of slope for the scalar trajectory,  $\Lambda_{f_0}^2 = 1.38 \pm 0.07 \text{ GeV}^2$ , which is used in Fig. 2. We obtain then the mass of the lightest scalar state,  $M_\sigma \approx 0.62 \text{ GeV}$ .

We arrive thus at the conclusion that our method predicts two parallel scalar trajectories. The ground state on the first trajectory can be identified with  $f_0(980)$  and on the second one with  $f_0(500)$  [1]. The existence of two parallel radial scalar trajectories seems to agree with the experimental data [3]. The masses of predicted radial states and a tentative comparison with the observed scalar mesons for two trajectories are displayed in Tables 1 and 2, correspondingly.

**Table 1.** The radial spectrum of the first  $f_0$ -trajectory for the slope  $\Lambda^2 = 1.38 \pm 0.07 \text{ GeV}^2$ . The first 5 predicted states are tentatively assigned to the resonances  $f_0(980)$ ,  $f_0(1500)$ ,  $f_0(2020)$ ,  $f_0(2200)$ , and  $X(2540)$  [1].

$n$	0	1	2	3	4
$m_{f_0}$ (th 1)	$1000 \pm 30$	$1540 \pm 20$	$1940 \pm 40$	$2270 \pm 50$	$2560 \pm 50$
$m_{f_0}$ (exp 1)	$990 \pm 20$	$1504 \pm 6$	$1992 \pm 16$	$2189 \pm 13$	$2539 \pm 14^{+38}_{-14}$

**Table 2.** The radial spectrum of the second  $f_0$ -trajectory for the slope  $\Lambda^2 = 1.38 \pm 0.07 \text{ GeV}^2$ . The first 5 predicted states are tentatively assigned to the resonances  $f_0(500)$ ,  $f_0(1370)$ ,  $f_0(1710)$ ,  $f_0(2100)$ , and  $f_0(2330)$  [1].

$n$	0	1	2	3	4
$m_{f_0}$ (th 2)	620	$1330 \pm 30$	$1780 \pm 40$	$2130 \pm 50$	$2430 \pm 60$
$m_{f_0}$ (exp 2)	400–550	1200–1500	$1723^{+6}_{-5}$	$2101 \pm 7$	2300–2350

## 4 Discussions and conclusions

We have put forward new extensions of SVZ sum rules making use of the large- $N_c$  (planar) limit and assuming for the radial excitations a linear Regge spectrum with universal slope. The choice of spectrum is motivated by hadron string models and related approaches and also

by the meson spectroscopy. The considered ansatz allows to solve the arising sum rules with a minimal number of inputs.

The prediction of the second scalar trajectory is a rather surprising feature of borelized planar sum rules [9]. The ground state on the second radial trajectory turns out to be significantly lighter than on the first trajectory. It looks tempting to identify this state with the elusive  $\sigma$  (called also  $f_0(500)$ ) meson [1]. The lightest scalar state in the standard SVZ sum rules lies near 1 GeV [6] and cannot be made significantly lighter within this method [10]. Our extension of the SVZ method leads thus to a new result. It is interesting to check whether a similar result appears in the framework of unborelized planar sum rules. Our analysis in the first part gives a positive answer. A light scalar state near 0.5 GeV, however, emerges in a different way [8].

When one predicts some quark-antiquark state it is important to indicate its place on the angular Regge trajectory as well. In other words, what are  $f_2, f_4, \dots$  companions of  $f_0(500)$  on this trajectory? In order to answer this question we must know the slope of the trajectory under consideration. According to the analysis of the first paper in Ref. [3], the slope of  $f_0$  trajectory, most likely, lies in the interval  $1.1 \div 1.2 \text{ GeV}^2$ . Several independent estimations made in some further papers of Ref. [3] seem to confirm this value. Consider for example the estimate on the  $\sigma$ -meson mass,  $m_\sigma \approx 390 \text{ MeV}$  [8]. Then we obtain  $m_{f_2} \approx 1.53 \div 1.60 \text{ GeV}$ . The PDG contains a well established resonance  $f_2(1565)$  [1] with mass  $m_{f_2(1565)} = 1562 \pm 13 \text{ MeV}$ . It is a natural companion of  $\sigma$ -meson on the corresponding angular Regge trajectory. The next state would have the mass  $m_{f_4} \approx 2.13 \div 2.23 \text{ GeV}$ . The discovery of the predicted tensor meson  $f_4$  (and perhaps the next companion  $f_6$  with  $m_{f_6} \approx 2.60 \div 2.71 \text{ GeV}$ ) would confirm our conjecture about the form of Regge trajectory with the  $\sigma$ -meson on the top. A tentative candidate for our  $f_4$  in the Particle Data is the resonance  $f_4(2220)$  having still undetermined spin — its value is either  $J = 2$  or  $J = 4$  [1]. Our model would favor the second possibility.

It is interesting to note that the predicted trajectory is drawn in the first paper of Ref. [3] among numerous angular Regge trajectories for isosinglet  $P$ -wave states of even spin. But the resonance  $f_2(1565)$  is replaced there by  $f_2(1525)$  (and is absent on other trajectories). As a result,  $m_{f_0}^2$  has a very small negative value leading to disappearance of a scalar state from this trajectory. The predicted  $f_4$ -companion is labelled as  $f_4(2150)$ . The modern PDG contains the state  $f_2'(1525)$  but this resonance is typically produced in reactions with  $K$ -mesons that evidently indicates on the dominant strange component. For this reason we should exclude it from our estimates.

Our prediction of the Regge trajectory containing the  $\sigma$ -meson on the top seems to contradict to studies of the  $\sigma$ -state on the complex Regge trajectory which claim that because of very large width the corresponding state cannot belong to usual Regge trajectories [2]. It is not excluded, however, that this observation may simply indicate on limitations of the usual methods which are applied to description of the  $\pi\pi$ -scattering. These methods are based on analyticity and unitarity of  $S$ -matrix and do not contain serious dynamical inputs. The generation of a huge width for  $f_0(500)$  represents, most likely, some dynamical effect. For this reason uncovering the genuine nature of  $\sigma$ -meson requires dynamical approaches.

Since the used sum rule method is based on the narrow-width approximation, a direct translation of our predictions to the physical parameters of a broad resonance looks questionable. As a matter of fact, we claim only that a scalar isoscalar pole in the range 400–600 MeV can naturally exist in the large- $N_c$  limit.

Another pertinent question is why the  $\sigma$ -meson lies below the linear radial Regge trajectory like the ground vector states? In the latter case, one can propose a simple qualitative explanation. The ground vector states are  $S$ -wave, so they represent relatively compact hadrons. In this case, a contribution from the Coulomb part of the Cornell confinement po-

tential,  $V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r$ , is not small, effectively "decreasing" the tension  $\sigma$  at smaller distances and, hence, masses of the ground  $S$ -wave states. In the case of  $\sigma$ -meson, one can imagine the following situation: This state represents a tetraquark but the admixture of additional  $q\bar{q}$ -pair is small and gives a small direct contribution to the mass. For this reason we may use the large- $N_c$  limit as a first approximation. However, due to the extra  $q\bar{q}$ -pair, the  $\sigma$ -meson (originally representing a scalar  $P$ -wave state) can exist as a  $S$ -wave state. Due to this phenomenon, on the one hand, the decay of this state becomes OZI-superallowed, explaining thereby its abnormally large width, on the other hand, its mass decreases similarly to the masses of ground  $S$ -wave vector mesons.

In conclusion, our analysis demonstrates that the existence of a light scalar state is well compatible with the structure of the planar sum rules in the scalar channel and may follow in a natural way from the Regge phenomenology.

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