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Short- and long-range rapidity correlations in the model with a lattice in transverse plane

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Abstract. In the framework of the quark-gluon string model we consider the various fluctuation and correlation observables used in the analysis of the multi-particle production in hadronic interactions at high energy. We express these observables through the fundamental string characteristics and analyze their resulting properties: the dependence on the width of observation window(s), the range of the correlation in rapidity, the intensive or strongly intensive behavior. To take into account the influence of a string fusion processes on the string characteristics and on the behavior of the observables. we use the version of string model with a lattice (grid) in the impact parameter plane. In particular we show that the observable between multiplicities in two acceptance windows separated in rapidity, which is a strongly intensive in the case with independent identical strings, loses this property, when we take into account the string fusion and the formation of strings of a few different types takes place in a collision.

1 Introduction

At present the rapidity correlations between yields of charged particles in high-energy hadronic interactions are intensively studied both theoretically and experimentally. In particular it is expected that the investigations of long-range rapidity correlations can give the information about the initial stage of hadron interactions [1].

For the description of the soft part of a multi-particle production at high energy, which saturates the inelastic cross section and which at present can not be described quantitatively directly by QCD, the quark-gluon string (color flux-tubes) model is usually applied [2, 3]. This picture is confirmed by lattice QCD simulations [4]. It also has support through the interpretation of the pomeron as a cylindrical structure in the limit of large number of the colors [5, 6]. In this case the cut pomeron corresponds to the formation of two strings, connecting quarks of a target and a projectile (the color reconnection process), which give after their fragmentation two chains of observed hadrons [3, 7]. In particular this model is exploited in all present MC event generators: PYTHIA, VENUS, HIJNG, AMPT, EPOS etc., for the description of the soft pat of a strong interaction.

Usually one distinguishes the short- and long-range parts of rapidity correlations. Since the strong interaction is considered as a local one in a space of rapidities, it gives rise directly only to short-range rapidity correlations (SR), upto $1\div2$ units of rapidity. The long-range rapidity correlations (LR) arise due to event-by-event fluctuation in the number of cut pomerons (i.e. in the number of strings), as it was demonstrated in [8].

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When describing the soft part of the multi-particle production in high-energy hadronic interactions by a string model, it assumes that the SR correlations arise only between particles produced from the decay of the same string, whereas the LR correlations arise due to fluctuation in the number and types of emerging sources (strings). Therefore, to find a signature of the string fusion and percolation phenomenon [9–11] in ultrarelativistic heavy ion collisions the study of the LR correlations between multiplicities in two separated rapidity intervals, known as the forward-backward (FB) multiplicity correlations, was proposed [12].

In present work we chose as a basic quantities the characteristics of a string: the single and double distributions of particles produced from a fragmentation of a single string,

$$\lambda(\eta) \equiv \frac{dN_{ch}}{d\eta} , \qquad \lambda_2(\eta_1, \eta_2) \equiv \frac{d^2 N_{ch}}{d\eta_1\, d\eta_2} , \tag{1}$$

and the number of the strings, N, produced in a given event. We introduce in the standard way [13] the two-particle correlation function of a string:

$$\Lambda(\eta_1, \eta_2) \equiv \frac{\lambda_2(\eta_1, \eta_2)}{\lambda(\eta_1)\lambda(\eta_2)} - 1 . \tag{2}$$

In mid-rapidity region at LHC energies we assume the translation invariance in rapidity for the string characteristics. Then

$$\lambda(\eta) = \mu_0 , \qquad \lambda_2(\eta_1, \eta_2) = \lambda_2(\eta_1 - \eta_2) , \qquad \Lambda(\eta_1, \eta_2) = \Lambda(\eta_1 - \eta_2) . \tag{3}$$

We express the observable quantities through these fundamental string characteristics and analyze their resulting properties. We also discuss the influence of the process of string fusion on the string characteristics and on the behavior of the observables.

Studying fluctuation and correlation phenomena one tries to suppress the contribution of the so-called "volume" fluctuations originating from trivial fluctuations in the number of initial sources (strings), for example, to obtain more clear signal on the process of string fusion. In this connection one defines the intensive and strongly intensive observables, which do not depend on the number of sources and on the fluctuations of this number, respectively. We analyze these properties for various observables in the framework of the string model too.

2 Independent identical strings

We start our consideration from a simple case of the model with independent identical strings [14]. In this model we suppose that the number of strings, N, fluctuates event by event around some mean value, $\langle N \rangle$, with some scaled variance, $\omega_N = D_N / \langle N \rangle$.

As the first observable we consider the two-particle correlation function $C_2(\eta_1, \eta_2)$, defined similarly to the two-particle correlation function $\Lambda(\eta_1, \eta_2)$ of a single string (see the formulae (1-3)), but for all produced particles. In [15] it was shown, that in this model

$$C_2(\eta_1, \eta_2) = \frac{\Lambda(\eta_1, \eta_2) + \omega_N}{\langle N \rangle} . \tag{4}$$

We see that this observable is not nor strongly intensive, due to dependence on ω_N, nor even simply intensive, due to dependence on $\langle N \rangle$. It naturally splits into the sum of SR and LR contributions:

$$C_2^{SR}(\Delta\eta) = \left. \frac{\Lambda(\Delta\eta)}{\langle N \rangle} \right|_{\Delta\eta \gg \eta_{corr}} \to 0 , \qquad C_2^{LR}(\Delta\eta) = \frac{\omega_N}{\langle N \rangle} = const , \tag{5}$$

where the η_{corr} is a typical correlation length in rapidity between particles produced from the same string. Note that one often looses the LR part, using di-hadron correlation approach for the extraction of C_2 from experimental data [15].

Another correlation observable is the correlation coefficient, b_{FB}, of the FB multiplicity correlations, studied e.g. in [16]:

$$b_{FB} = \frac{\langle n_F n_B\rangle - \langle n_F\rangle\langle n_B\rangle}{\langle n_F{}^2\rangle - \langle n_F\rangle^2} = \frac{\mathrm{cov}(n_F, n_B)}{D_{n_F}} , \tag{6}$$

which characterizes the strength of the correlation between the event multiplicities n_F and n_B measured in the separated forward, $\delta\eta_F$, and backward, $\delta\eta_B$, rapidity windows with the distance $\Delta\eta$ between their centers.

In this note we confine ourselves to a simple case of symmetric reactions and symmetric observation windows, for which

$$\langle n_F\rangle = \langle n_B\rangle \equiv \langle n\rangle , \qquad D_{n_F} = D_{n_B} \equiv D_n , \qquad \omega_{n_F} = \omega_{n_B} \equiv \omega_n = D_n/\langle n\rangle . \tag{7}$$

We'll assume also that the width of the observation windows is smaller than the typical correlation length:

$$\delta\eta_F = \delta\eta_B \equiv \delta\eta \ll \eta_{corr} . \tag{8}$$

For such small windows we have the model independent connection between the two-particle correlation function and the FB multiplicity correlation coefficient [15]:

$$C_2(\Delta\eta) = \frac{\mathrm{cov}(n_F, n_B)}{\langle n_F\rangle\langle n_B\rangle} = \frac{D_{n_F}}{\langle n_F\rangle\langle n_B\rangle} b_{FB} = \frac{\omega_{n_F}}{\langle n_B\rangle} b_{FB} = \frac{\omega_n}{\langle n\rangle} b_{FB} , \tag{9}$$

where $\Delta\eta$ is a rapidity distance between the forward and backward observation windows.

The scaled variance of the multiplicity, ω_n, in a given rapidity interval, $\delta\eta$, in formula (9), also can be expressed by the model independent way through the two-particle correlation function [8, 15]:

$$\omega_n = 1 + \frac{\langle n\rangle}{\delta\eta^2} \int_{\delta\eta} d\eta_1 \int_{\delta\eta} d\eta_2 \, C_2(\eta_1 - \eta_2) \approx 1 + \langle n\rangle \, C_2(0) . \tag{10}$$

The last transition is valid for the small rapidity window, as given by (8). Combining (9) and (10) we find the model independent connection:

$$b_{FB} = \frac{\langle n\rangle \, C_2(\Delta\eta)}{1 + \langle n\rangle \, C_2(0)} . \tag{11}$$

Now in the framework of the model using the formula (4) we can express the observables b_{FB} and ω_n through the fundamental string characteristics:

$$b_{FB} = \frac{\mu_0 \, \delta\eta \, [\omega_N + \Lambda(\Delta\eta)]}{1 + \mu_0 \, \delta\eta \, [\omega_N + \Lambda(0)]} . \tag{12}$$

$$\omega_n = 1 + \langle\mu\rangle[\omega_N + \frac{1}{\delta\eta^2} \int_{\delta\eta} d\eta_1 \int_{\delta\eta} d\eta_2 \, \Lambda(\eta_1 - \eta_2)] \approx 1 + \mu_0 \, \delta\eta \, [\omega_N + \Lambda(0)] . \tag{13}$$

We have used that in this model $\langle n\rangle = \langle N\rangle\langle\mu\rangle = \langle N\rangle\mu_0 \, \delta\eta$.

We see that the b_{FB}, like the C_2, is not nor strongly intensive nor intensive. Since the string two-particle correlation function decreases to zero, $\Lambda(\Delta\eta) \to 0$, with increase of $\Delta\eta$, The FB correlation coefficient splits into a sum of SR and LR contributions, $b_{FB} = b_{SR} + b_{LR}$, where

$$b_{SR} = \frac{\mu_0\, \delta\eta\, \Lambda(\Delta\eta)}{1 + \mu_0\, \delta\eta\, [\omega_N + \Lambda(0)]}\,, \qquad b_{LR} = \frac{\mu_0\, \delta\eta\, \omega_N}{1 + \mu_0\, \delta\eta\, [\omega_N + \Lambda(0)]}\,. \tag{14}$$

We see also that the value of the FB correlation coefficient is proportional to the width of the observation windows $\delta\eta$.

By formula (13) we see that the ω_n is intensive, but is not strongly intensive observable, since it is independent of the mean number of strings, $\langle N\rangle$, but depends on the fluctuations in their number, ω_N. From the physical point of view it means that we expect strong dependence of this observable on the choice of the width of collision centrality classes.

It is important to note that the scaled variance of the number of particles, ω_μ, produced in a given rapidity interval $\delta\eta$ from fragmentation of a single string, are also completely determined by the string two-particle correlation function:

$$\omega_\mu = 1 + \frac{\langle\mu\rangle}{\delta\eta^2} \int_{\delta\eta} d\eta_1 \int_{\delta\eta} d\eta_2\, \Lambda(\eta_1 - \eta_2) \approx 1 + \mu_0\, \delta\eta\, \Lambda(0)\,. \tag{15}$$

This corresponds to a general fact that in the absence of correlations between particles produced from a given source the multiplicity distribution from such source will be poissonian ($\omega_\mu = 1$). The quantities (13) and (15) are fulfilled the oblivious connection: $\omega_n = \omega_\mu + \langle\mu\rangle\omega_N$.

Basing on the multiplicities n_F and n_B in two separated rapidity windows, we can also construct the observable

$$\Sigma(n_F, n_B) \equiv \frac{\langle n_F\rangle\, \omega_{n_B} + \langle n_B\rangle\, \omega_{n_F} - 2\,\mathrm{cov}(n_F\, n_B)}{\langle n_F\rangle + \langle n_B\rangle}\,, \tag{16}$$

introduced in accordance with [17] to suppress the contribution of the "volume" fluctuations. For symmetric reactions and symmetric observation windows it can be simplified to

$$\Sigma(n_F, n_B) = \frac{D_n - \mathrm{cov}(n_F, n_B)}{\langle n\rangle} = \frac{\langle n^2\rangle - \langle n_F n_B\rangle}{\langle n\rangle} = \omega_n(1 - b_{FB})\,. \tag{17}$$

Recalling (12) and (13) we can express this observable through the fundamental string characteristics:

$$\Sigma(n_F, n_B) = 1 + \mu_0\delta\eta\, [\Lambda(0) - \Lambda(\Delta\eta)] = \Sigma(\mu_F, \mu_B)\,, \tag{18}$$

where $\Delta\eta$ is a distance between the observation windows. Here we also introduced the strongly intensive observable, $\Sigma(\mu_F, \mu_B)$, for multiplicities μ_F and μ_B produced from decay of a single string by formulas similar to (16) and (17). By this formula we really see that in the framework of this model the observable $\Sigma(n_F, n_B)$ is a strongly intensive, it is independent of both the mean number of string $\langle N\rangle$ and its fluctuation ω_N. It depends only on the string characteristics μ_0, $\Lambda(\Delta\eta)$ and the width of observation windows, $\delta\eta$.

From the formula (18) we see the main properties of the $\Sigma(n_F, n_B)$, which we expect in this model. Starting from the value 1 at $\Delta\eta = 0$ it increases with a distance between the centers of the observation windows, $\Delta\eta$, since the two-particle correlation function of a string $\Lambda(\Delta\eta)$ decrease to zero with $\Delta\eta$. The extent of the $\Sigma(\Delta\eta)$ increase with $\Delta\eta$ is proportional to the width of the observation windows $\delta\eta$.

More detailed description of the $\Sigma(n_F, n_B)$ needs the knowledge of the two-particle correlation function of a string $\Lambda(\Delta\eta)$. In paper [15] in the framework of the model with independent identical strings this function was fitted using the experimental pp ALICE data on

Table 1. The value of the parameters in formula (19) for the two-particle correlation function of a string $\Lambda(\Delta\eta)$ (integrated over azimuth), obtained by a fitting [15] of the experimental pp ALICE data [16] on forward-backward correlations between multiplicities at three initial energies

$\sqrt{s}$, TeV	0.9	2.76	7.0
$\mu_0\Lambda_0$	0.73	0.83	0.93
η_{corr}	1.52	1.43	1.33

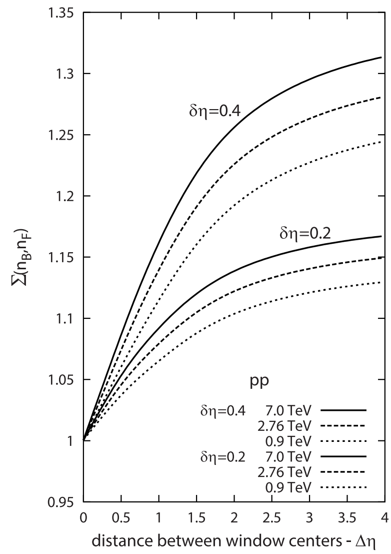

Figure 1. The strongly intensive observable, $\Sigma(n_F, n_B)$, between multiplicities in two small pseudorapidity windows (of the width $\delta\eta = 0.2$ and 0.4) as a function of the distance between window centers, $\Delta\eta$, calculated in the model with independent identical strings using the two-particle correlation function of a string $\Lambda(\Delta\eta)$ (see formula (19) and Table 1).

FB correlations between multiplicities in windows separated in rapidity and azimuth at three initial energies [16] together with the value of scaled variance of the number of strings ω_N.

Integrating the fits from the paper [15] over azimuth we find the $\Lambda(\Delta\eta)$ for three initial energies: 0.9, 2.76 and 7 TeV, which can be well approximated by the exponent

$$\Lambda(\Delta\eta) = \Lambda_0 \exp(-|\Delta\eta|/\eta_{corr}) \,, \tag{19}$$

with the parameters presented in Table 1. We see that the correlation length, η_{corr}, decreases with the increase of collision energy. In Sec.3 we'll see that this can be explained by an increase with energy of the admixture of fused strings in pp collisions.

The results of the calculation of the strongly intensive observable $\Sigma(n_F, n_B)$ by formulae (18) with this two-particle correlation function for two width of the observation windows $\delta\eta = 0.2$ and 0.4 are presented in Fig. 1. It illustrates the properties of $\Sigma(n_F, n_B)$, considered above, based on formula (18). At large distances between observation windows $\Delta\eta \gg \eta_{corr}$ the two-particle correlation function of a string, $\Lambda(\Delta\eta)$, (19), goes to zero and the $\Sigma(n_F, n_B)$ saturates to $\omega_\mu = 1 + \mu_0\delta\eta\Lambda(0)$. So we have

$$\Sigma(n_F, n_B) \to 1 \quad \text{at} \quad \Delta\eta \ll \eta_{corr} \qquad \text{and} \qquad \Sigma(n_F, n_B) \to \omega_\mu \quad \text{at} \quad \Delta\eta \gg \eta_{corr} \,.$$

Note that ω_μ increases with the width, $\delta\eta$, of the observation windows, (15).

In Fig. 1 we see also some general increase of the $\Sigma(n_F, n_B)$ with initial energy, below in Sec.3 we will show that in the framework of the string model it can be also interpreted as a signal of an increase with energy of the admixture of the fused strings.

3 String fusion effects

In this section we consider the influence of processes of interaction between strings on the strongly intensive observable $\Sigma(n_F, n_B)$. This influence increases with initial energy and with going from pp to heavy ion collisions. One of the possible ways to take these processes into account is to pass from the model with independent identical strings to the model with string fusion and percolation [9–11].

To account the string fusion processes we used approach with the finite lattice (the grid) in the impact parameter plane, suggested in [18] and later successfully exploited for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in ultra relativistic nuclear collisions. In this approach one splits the impact parameter plane into cells, which area is equal to the transverse area of single string and supposes the fusion of all strings with the centers in a given cell.

In this model the definite set of strings of different types corresponds to given event. Each such string, originating from a fusion of k primary strings, is characterized by its own parameters: the mean multiplicity per unit of rapidity, $\mu_0^{(k)}$, and the string correlation function, $\Lambda_k(\Delta\eta)$. These parameters uniquely determine the strongly intensive observable, $\Sigma_k(\mu_F, \mu_B)$, between multiplicities, produced from decay of a string of a given kind k, defined by formulas similar to (16) and (17). For example, for small observation windows, $\delta\eta \ll \eta_{corr}^{(k)}$, separated by the rapidity distance $\Delta\eta$, similarly to (18), we have

$$\Sigma_k(\mu_F, \mu_B) = 1 + \mu_0^{(k)} \delta\eta \, [\Lambda_k(0) - \Lambda_k(\Delta\eta)] \, . \tag{20}$$

In the model with k string types the direct calculation gives for the observable $\Sigma(n_F, n_B)$:

$$\Sigma(n_F, n_B) = \sum_{k=1} \alpha_k \, \Sigma_k(\mu_F, \mu_B) \, , \qquad \alpha_k = \langle n^{(k)} \rangle / \langle n \rangle \, , \tag{21}$$

where $\langle n^{(k)} \rangle$ is a mean number of particles produced from all sells with k fused strings in the rapidity observation window $\delta\eta$. Note that the same result was obtained in the model with two types of strings in [19] for the long-range part of $\Sigma(n_F, n_B)$. Substituting (20) in formula (21) we find

$$\Sigma(n_F, n_B) = 1 + \delta\eta \sum_{k=1} \alpha_k \, \mu_0^{(k)} \, [\Lambda_k(0) - \Lambda_k(\Delta\eta)] \, . \tag{22}$$

If we use else the simple exponential parametrization similar to (19) for $\Lambda_k(\Delta\eta)$:

$$\Lambda_k(\Delta\eta) = \Lambda_0^{(k)} \exp\left(-|\Delta\eta| / \eta_{corr}^{(k)}\right) , \tag{23}$$

then we can rewrite (22) as

$$\Sigma(n_F, n_B) = 1 + \delta\eta \sum_{k=1} \alpha_k \, \mu_0^{(k)} \Lambda_0^{(k)} [1 - \exp\left(-|\Delta\eta| / \eta_{corr}^{(k)}\right)] \, . \tag{24}$$

We see that in this case each string of the type k is characterized by two parameters: the product $\mu_0^{(k)} \Lambda_0^{(k)}$, where the $\mu_0^{(k)}$ is the mean multiplicity per unit of rapidity from a decay of such string, and its two-particle correlation length $\eta_{corr}^{(k)}$, which determines the correlations between particles, produced from a fragmentation of the string.

In the framework of the string fusion model [9–11] one usually supposes that the mean multiplicity per unit of rapidity for fused string, $\mu_0^{(k)}$, increase as $\sqrt{k}$ with k. The dependence of the correlation length $\eta_{corr}^{(k)}$ on k is not so obvious. Basing on a simple geometrical picture of string fragmentation (see, e.g., [7, 20–22]) one can expect the decrease of the correlation length, $\eta_{corr}^{(k)}$, with increase of k. In this picture with a growth of string tension the fragmentation process is finished at smaller string segments in rapidity. The correlation takes place only between particles originating from a fragmentation of neighbour string segments and hence the correlation length $\eta_{corr}^{(k)}$ will decrease with k for fused strings.

Indirectly this fact is confirmed by the analysis [23] of the experimental STAR [24] and ALICE [25] data on net-charge fluctuations in pp and AA collisions. The dependence of net-charge fluctuations on the rapidity width of the observation window can be well described in a string model if one supposes the decrease of the correlation length with the transition to collisions of heavier nuclei and/or to higher energies, i.e. to collisions in which the proportion of fused strings is increasing.

By (24) both these factors, the increase of $\mu_0^{(k)}$ and the decrease of $\eta_{corr}^{(k)}$ for fused string, lead to the steeper increase of $\Sigma_k(\mu_F, \mu_B)$, (20), with $\Delta\eta$ and to its saturation at a higher level $\omega_\mu^{(k)} = 1 + \delta\eta\,\mu_0^{(k)}\Lambda_k(0)$. Due to (21) this behaviour transmits to the observable $\Sigma(n_F, n_B)$, as the last is a weighted average of $\Sigma_k(\mu_F, \mu_B)$ with the weights $\alpha_k = \langle n^{(k)}\rangle/\langle n\rangle$, which are the mean portions of the particles produced from a given type of strings.

In real experiment we have always a mixture of fused and single strings. So with the transition to pp collisions at higher energy or/and to collisions of nuclei the proportion of fused strings will increase and we will observe the steeper increase of $\Sigma(n_F, n_B)$, with $\Delta\eta$ and its saturation at a higher level. Really, in Fig.1 we see such behaviour of $\Sigma(n_F, n_B)$, when we compare $\Sigma(n_F, n_B)$ for pp collisions at three initial energies: 0.9, 2.76 and 7 TeV, obtained through fitting [15] of the experimental pp ALICE data [16] on FB correlations between multiplicities.

Table 1 illustrates the increase of $\mu_0\Lambda_0$ and the decrease of the correlation length η_{corr} with energy for this data. Note that these values are the some effective ones, because in the model at each energy we had supposed that all strings are identical. So they only indirectly reflects the influence of the increase of the proportion of fused strings with energy in pp collisions.

For studies of the $\Sigma(n_F, n_B)$ dependence on multiplicity classes we can predict the behaviour similar to the one in Fig.1. For more central pp collisions due to the increase of the proportion of fused strings in such collisions we also have to observe the steeper increase of $\Sigma(n_F, n_B)$, with $\Delta\eta$ and its saturation at a higher level.

Note that from a general point of view, this simultaneously means that the observable $\Sigma(n_F, n_B)$, strictly speaking, can not be considered any more as strongly intensive. Through the weight factors, $\alpha_k = \langle n^{(k)}\rangle/\langle n\rangle$, entering the formula (21), the observable $\Sigma(n_F, n_B)$ becomes dependent on collision conditions (e.g., on the collision centrality).

4 Conclusion

In the framework of the quark-gluon string model we have considered the various fluctuation and correlation observables: the two-particle correlation function, $C_2(\eta_1, \eta_2)$, the FB correlation coefficient b_{FB}, the scaled variance of the multiplicity ω_n in given rapidity interval $\delta\eta$ and the $\Sigma(n_F, n_B)$ observable for multiplicities n_F and n_B in two acceptance windows, $\delta\eta_F$ and $\delta\eta_B$, separated by some rapidity interval $\Delta\eta$, which usually used in the analysis of the multi-particle production in hadronic interactions at high energy.

We expressed these observables through the basic quantities of the model: the fundamental string characteristics (the multiplicity per unit of rapidity, produced from decay of a single string, μ_0, and its two-particle correlation function, $\Lambda(\eta_1, \eta_2)$) and the parameters (the mean number, $\langle N \rangle$, and the scaled variance, ω_N), describing event-by-event fluctuation of number of the strings, N, produced in a given event.

Based on these formulas, we have determined the properties of considered observables within the framework of the model. We have found its dependencies on the width of the observation window(s) and on the distance (gap) between these windows, which enabled us to separate the contributions of SR and LR correlations. We analyzed their intensive or strongly intensive behavior.

We also discuss the influence of the process of string fusion on the string characteristics and on the behavior of the observables. To take into account the string fusion we use the version of string model with a lattice (grid) in the impact parameter plane.

We show that the observable $\Sigma(n_F, n_B)$, which is a strongly intensive in the case with independent identical strings, loses this property, when we take into account the string fusion processes and a formation of strings of a few different types takes place in a collision. In this situation the observable $\Sigma(n_F, n_B)$ is proved to be equal to a weighted average of its values for different string types. Unfortunately, in this case through the weight factors this observable becomes dependent on collision conditions. We predict the changes in the behaviour of $\Sigma(n_F, n_B)$ with energy and collision centrality.

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