# Electric current induced by an external magnetic field in the presence of electroweak matter

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**Abstract.** We study the generation of an electric current, along the external magnetic field, of fermions, interacting by parity violating electroweak forces with background matter. First, we discuss the situation of massive particles with nonzero anomalous magnetic moments. We show that the induced current is vanishing for such particles in the state of equilibrium. Then, the case of massless fermions is studied. We demonstrate that the contribution of the electroweak interaction is washed out from the expression for the current, which turns out to coincide with the prediction of the chiral magnetic effect. Our results are compared with findings of other authors.

## 1 Introduction

The evolution of chiral charged particles in external fields reveal multiple quantum phenomena. First, we mention the Adler-Bell-Jackiw anomaly, which consists in the non-conservation of the axial current in the presence of an external electromagnetic field. This anomaly was shown in Refs. [1, 2] to be closely related to the chiral magnetic effect (the CME), which is the excitation of the electric current of chiral particles $\mathbf{J}_{\text{CME}} = \alpha_{\text{em}}(\mu_{\text{R}} - \mu_{\text{L}})\mathbf{B}/\pi$ along the external magnetic field $\mathbf{B}$. Here $\alpha_{\text{em}} \approx 1/137$ is the fine structure constant and $\mu_{\text{R,L}}$ are the chemical potentials of right and left chiral fermions. One can also mention the chiral vortical effect, which is the generation of the anomalous current in a rotating matter. There are active searches for manifestations of the CME in astrophysics and cosmology, as well as in accelerator physics.

The main feature of the CME is the unbroken chiral symmetry of charged particles. It means that any nonzero mass makes $\mathbf{J}_{\text{CME}}$ to vanish [3, 4]. The majority of known elementary particles acquire masses through the electroweak mechanism. However, it is likely to be an electroweak crossover rather than a first order phase transition. Therefore, charged particles will remain massive at any pressure and chemical potential unless a new physics beyond the standard model is accounted for. There are indications that a chiral phase transition can happen in dense matter owing to the QCD effects. Some astrophysical applications for the magnetic fields generation in compact stars due to the CME and the electroweak interaction between quarks in dense matter are considered in Ref. [5].

In this connection, there is a particular interest in searching for the possibility of the generation of the current $\mathbf{J} \parallel \mathbf{B}$ in the system of massive charged particles. In this situation, one

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would have an instability of the magnetic field without necessity to demand the restoration of the chiral symmetry. One of the examples of such a system was studied in Ref. [6], where the dynamo amplification of magnetic fields in an inhomogeneous electroweak matter was discussed.

There is an additional open question on the influence of the external axial-vector field $V_5^\mu$ on the magnitude of the anomalous current in the CME. If a homogeneous and isotropic $V_5^\mu$ is present, the Lagrangian of the interaction of the fermion field $\psi$ with $V_5^\mu$ can be represented as $\mathcal{L}_5 \sim \bar{\psi}\gamma^\mu\gamma^5\psi V_{5\mu} \to \psi^\dagger\gamma^5\psi V_5$, which shows that the chiral imbalance $\mu_5 = (\mu_R - \mu_L)/2$ could be shifted by $V_5 \equiv V_5^0$. Thus the CME could contain the contribution of $V_5$ [7–9].

In this work, we summarize our recent results in Refs. [10, 11] on the studies of the generation of the current $\mathbf{J} \parallel \mathbf{B}$ in the presence of an axial-vector field, which can be the electroweak interaction with background matter. In Sec. 2, we discuss the situation of massive particles with anomalous magnetic moments. The case of massless fermions is considered in Sec. 3. We discuss our results in Sec. 4.

## 2 Induced electric current of massive fermions with anomalous magnetic moments

In this section, we study the generation of an equilibrium electric current of massive fermions, e.g., electrons, with anomalous magnetic moments induced by the electroweak interaction with background matter under the influence of the external magnetic field [10]. The Lagrangian for an electron, described by the bispinor $\psi_e$, has the form,

$$\mathcal{L} = \bar{\psi}_e\left[\gamma_\mu(\mathrm{i}\partial^\mu + eA^\mu) - m + \frac{\mu}{2}\sigma^{\mu\nu}F_{\mu\nu} - \gamma_\mu(V_L^\mu P_L + V_R^\mu P_R)\right]\psi_e, \tag{1}$$

where $A^\mu = (0, 0, Bx, 0)$ is the vector potential corresponding to the constant and homogeneous magnetic field, directed along the $z$-axis, $e > 0$ is the elementary charge, $P_{L,R} = (1 \mp \gamma^5)/2$ are the chiral projectors, $\gamma^\mu = (\gamma^0, \boldsymbol{\gamma})$, $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu]_-$, and $\gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ are the Dirac matrices, $m$ is the electron mass, $\mu$ is the anomalous magnetic moment, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\mathbf{E}, \mathbf{B})$ is the electromagnetic field tensor (with $\mathbf{E} = 0$), and $V_{L,R}^\mu = (V_{L,R}^0, \mathbf{V}_{L,R})$ are the effective potentials of the electroweak interaction of the electron chiral projections with background matter. We shall suppose that the background matter is macroscopically at rest and unpolarized. In this situation, $\mathbf{V}_{L,R} = 0$ and $V_{L,R}^0 \equiv V_{L,R} \neq 0$. The explicit form of $V_{L,R}$ is given in Ref. [7] for the case of background matter consisting of neutrons and protons.

Eq. (1) was solved in Refs. [10, 12]. Using the wave functions of electrons and positrons, one gets the following expression for the total current $\mathbf{J} = \mathbf{J}_e + \mathbf{J}_{\bar{e}}$ along the magnetic field $\mathbf{B}$:

$$\mathbf{J} = \Pi\mathbf{B}, \quad \Pi = -\frac{\alpha_{\mathrm{em}}}{\pi}\sum_{n=1}^{\infty}\sum_{s=\pm 1}\int_{-\infty}^{+\infty}\frac{\mathrm{d}p_z}{\mathcal{E}}\left[p_z\left(1 + s\frac{V_5^2}{R^2}\right) - s\frac{\mu Bm V_5}{R^2}\right]\Delta f, \tag{2}$$

where $\alpha_{\mathrm{em}} = e^2/4\pi$ is the fine structure constant, $V_5 = (V_L - V_R)/2$, $\Delta f = f(\mathcal{E} - \chi_{\mathrm{eff}}) - f(\mathcal{E} + \chi_{\mathrm{eff}})$, $f(E) = [\exp(\beta E) + 1]^{-1}$ is the Fermi-Dirac distribution function, $\chi_{\mathrm{eff}} = \chi - \bar{V}$, $\chi$ is the chemical potential of the electron-positron plasma, $\beta = 1/T$ is the reciprocal temperature, $\bar{V} = (V_L + V_R)/2$, $n = 1, 2, \ldots$ and $s = \pm 1$ are discrete quantum numbers, which the energy levels depend on, and

$$\mathcal{E} = \sqrt{p_z^2 + 2eBn + m^2 + (\mu B)^2 + V_5^2 + 2sR^2},$$

$$R^2 = \sqrt{(p_z V_5 - \mu Bm)^2 + 2eBn\left[(\mu B)^2 + V_5^2\right]}. \tag{3}$$

It is interesting to mention that the total current in Eq. (2) is proportional to the averaged group velocity of a charged particle along the magnetic field, $v_z = \partial \mathcal{E}/\partial p_z$: $\mathbf{J} = -\alpha_{\mathrm{em}} \langle v_z \rangle \mathbf{B}/\pi$.

Considering Eq. (2) in case of a degenerate electron gas, it was claimed in Ref. [13] that $\Pi \neq 0$. Analogous result was obtained in Ref. [9] on the basis of the analysis of the effective Lagrangians in one-loop approximation. The instability of the magnetic field, driven by the anomalous current $\mathbf{J} = \Pi\mathbf{B}$, and some astrophysical applications were studied in Ref. [13]. The claim of Refs. [9, 13] that there is $\mathbf{J} = \Pi\mathbf{B} \neq 0$ in the considered system is based on the fact that the energy levels at $n > 0$ in Eq. (3) are neither symmetric nor antisymmetric functions of $p_z$, which is the momentum projection along the magnetic field. Thus, the integration over $p_z$ in the symmetric limits in Eq. (2) could give a nonzero result. The asymmetry coefficient of the energy levels in Eq. (3) is $\mu B m V_5$. It is this term, which $\mathbf{J} \parallel \mathbf{B}$ in Refs. [9, 13] is proportional to.

Nevertheless, a careful analysis reveals that $\Pi = 0$ in Eq. (2). This fact is not quite obvious. To demonstrate it, we introduce the notation in Eq. (2),

$$\frac{\Delta f}{\mathcal{E}} = F(Q^2 + 2sR^2), \quad Q^2 = p_z^2 + 2eBn + m^2 + (\mu B)^2 + V_5^2. \tag{4}$$

Then we decompose $F(Q^2 + 2sR^2)$,

$$F(Q^2 + 2sR^2) = \sum_{k=0}^{\infty} 2^{2k} R^{4k} \left[ \frac{F^{(2k)}(Q^2)}{(2k)!} + 2sR^2 \frac{F^{(2k+1)}(Q^2)}{(2k+1)!} \right], \quad F^{(k)}(Q^2) = \frac{\mathrm{d}^k F(Q^2)}{\mathrm{d}(Q^2)^k}, \tag{5}$$

in a formal series.

The sum over $s$ of the integrand in Eq. (2) gives

$$
\begin{aligned}
I &= \sum_{s=\pm 1} \left[ p_z + s(p_z V_5 - \mu B m) \frac{V_5}{R^2} \right] F(Q^2 + 2sR^2) \\
&= 2 \sum_{k=0}^{\infty} 2^{2k} R^{4k} \left[ p_z \frac{F^{(2k)}(Q^2)}{(2k)!} + 2V_5(p_z V_5 - \mu B m) \frac{F^{(2k+1)}(Q^2)}{(2k+1)!} \right] \\
&= 2 \sum_{k=0}^{\infty} 2^{2k} \left\{ p_z R^{4k} \frac{F^{(2k)}(Q^2)}{(2k)!} + \frac{1}{k+1} \frac{\partial}{\partial p_z} \left[ R^{4(k+1)} \right] \frac{F^{(2k+1)}(Q^2)}{(2k+1)!} \right\},
\end{aligned}
\tag{6}
$$

where we use the identities,

$$\frac{\partial R^4}{\partial p_z} = 2V_5(p_z V_5 - \mu B m), \quad \frac{1}{k+1} \frac{\partial}{\partial p_z} \left[ R^{4(k+1)} \right] = R^{4k} \frac{\partial R^4}{\partial p_z}, \tag{7}$$

and take into account Eq. (3).

Integrating Eq. (6) over $p_z$ and then by parts, one gets

$$
\begin{aligned}
\int_{-\infty}^{+\infty} I \mathrm{d}p_z = & 2 \sum_{k=0}^{\infty} 2^{2k} \left[ \int_{-\infty}^{+\infty} \mathrm{d}p_z \left\{ R^{4k} p_z \frac{F^{(2k)}(Q^2)}{(2k)!} - \frac{1}{k+1} \frac{\partial}{\partial p_z} \left[ F^{(2k+1)}(Q^2) \right] \frac{R^{4(k+1)}}{(2k+1)!} \right\} \right. \\
& \left. + \frac{F^{(2k+1)}(Q^2) R^{4(k+1)}}{(k+1)(2k+1)!} \Bigg|_{-\infty}^{+\infty} \right].
\end{aligned}
\tag{8}
$$

The function $F(Q^2)$ is proportional to the Fermi-Dirac distribution functions, which are vanishing at great values of the argument. The same property has any derivative of $F(Q^2)$ in Eq. (5). Thus, the last term in Eq. (8) disappears at $p_z \to \pm\infty$.

Taking into account that

$$\frac{\partial}{\partial p_z}\left[F^{(2k+1)}(Q^2)\right] = 2p_z\frac{\mathrm{d}}{\mathrm{d}Q^2}\left[F^{(2k+1)}(Q^2)\right] = 2p_z F^{(2k+2)}(Q^2), \tag{9}$$

and changing the summation index $k \to k - 1$ in the second term in Eq. (8), one obtains

$$\int_{-\infty}^{+\infty} I\mathrm{d}p_z = 2\sum_{k=0}^{\infty} 2^{2k}\int_{-\infty}^{+\infty}\mathrm{d}p_z R^{4k}p_z\frac{F^{(2k)}(Q^2)}{(2k)!} - 2\sum_{k=1}^{\infty} 2^{2k}\int_{-\infty}^{+\infty}\mathrm{d}p_z R^{4k}p_z\frac{F^{(2k)}(Q^2)}{(2k)!}$$

$$= 2\int_{-\infty}^{+\infty}\mathrm{d}p_z\, p_z F(Q^2) = 0, \tag{10}$$

where we use the fact that $F^{(0)}(Q^2) = F(Q^2)$. To get the vanishing result of the integration in the symmetric limits in Eq. (10), we account for that $Q^2$ is the even function of $p_z$ (see Eq. (4)), i.e. the integrand in Eq. (10) is the odd function.

Thus, we have demonstrated that in Eq. (2)

$$\mathbf{J} = \Pi\mathbf{B} = 0, \quad \text{since} \quad \Pi = 0, \tag{11}$$

for arbitrary characteristics of the external fields and charged particles. Accounting for the fact that the lowest energy level with $n = 0$ does not contribute to the anomalous current either [10, 13], we get that there is no electric current $\mathbf{J} \parallel \mathbf{B}$ in the system of massive electrons with anomalous magnetic moments, electroweakly interacting with background matter, in the state of equilibrium.

## 3 Induced electric current of massless fermions electroweakly interacting with background matter

In this section, we analyze the influence of the electroweak interaction of background matter on the generation of an electric current of massless fermions along the external magnetic field. [11]. The Lagrangian for such a fermion, e.g., an electron, can be obtained on the basis of Eq. (1) if we set $\mu = 0$ there. Before the consideration of a massless electron, we obtain the solution of the corresponding Dirac equation in the situation $m \neq 0$ in Eq. (1). Then, we approach to the chiral limit $m \to 0$.

Let us look for the solution of the Dirac equation, which results from Eq. (1) with $\mu = 0$, in the form,

$$\psi_e = \exp\left(-\mathrm{i}Et + \mathrm{i}p_y y + \mathrm{i}p_z z\right)\psi_x, \tag{12}$$

where $\psi_x = \psi(x)$ is the bispinor which depends on $x$ and $p_{y,z}$ are the momentum projections along the $y$- and $z$-axes. We shall choose the chiral representation of the Dirac matrices [14]. We can represent $\psi_x$ in the form [4],

$$\psi_x^{\mathrm{T}} = \left(C_1 u_{n-1}, \mathrm{i}C_2 u_n, C_3 u_{n-1}, \mathrm{i}C_4 u_n\right), \tag{13}$$

where $C_i$, $i = 1, \ldots, 4$, are the spin coefficients,

$$u_n(\eta) = \left(\frac{eB}{\pi}\right)^{1/4}\exp\left(-\frac{\eta^2}{2}\right)\frac{H_n(\eta)}{\sqrt{2^n n!}}, \quad n = 0, 1, \ldots, \tag{14}$$

are the Hermite functions, $H_n(\eta)$ are the Hermite polynomials, and $\eta = \sqrt{eB}x + p_y/\sqrt{eB}$.

The energy spectrum for $n > 0$ reads [4, 15]

$$E = \bar{V} + \lambda\mathcal{E}, \quad \mathcal{E} = \sqrt{(\mathcal{E}_0 + sV_5)^2 + m^2}, \quad \mathcal{E}_0 = \sqrt{p_z^2 + 2eBn}, \tag{15}$$

where $s = \pm1$ is the discrete quantum number, dealing with the spin operator [15], and $\lambda = \pm1$ is the sign of the energy, i.e. the electron energy reads $E_e = E(\lambda = +1) = \mathcal{E} + \bar{V}$, and the positron energy has the form, $E_{\bar{e}} = -E(\lambda = -1) = \mathcal{E} - \bar{V}$. For $n = 0$, one has [4, 11]

$$E = \bar{V} + \lambda\mathcal{E}, \quad \mathcal{E} = \sqrt{(p_z + V_5)^2 + m^2}. \tag{16}$$

Note that, at $n = 0$, there is only one spin state of the electron.

The spin coefficients obey the system [4, 11],

$$(\mathcal{E} \mp p_z \pm V_5)\,C_{1,3} \mp \sqrt{2eBn}\,C_{2,4} + mC_{3,1} = 0,$$
$$(\mathcal{E} \pm p_z \pm V_5)\,C_{2,4} \mp \sqrt{2eBn}\,C_{1,3} + mC_{4,2} = 0, \tag{17}$$

where we consider the particle (electron) degrees of freedom, $\lambda = 1$. Since we are mainly interested in the dynamics of electrons at the lowest energy level, we should set $n = 0$ in Eq. (17). It results from Eq. (13) that, in this situation, $C_1 = C_3 = 0$ to avoid the appearance of Hermite functions with negative indexes.

If, besides setting $n = 0$ in Eq. (17), we approach to the limit $m \to 0$ there, one gets

$$(\mathcal{E} + p_z + V_5)\,C_2 = 0, \quad \text{or} \quad \begin{cases} \mathcal{E} = -p_z - V_5, \\ C_2 \neq 0, \quad \text{and} \quad C_4 = 0, \end{cases} \tag{18}$$

$$(\mathcal{E} - p_z - V_5)\,C_4 = 0, \quad \text{or} \quad \begin{cases} \mathcal{E} = p_z + V_5, \\ C_4 \neq 0, \quad \text{and} \quad C_2 = 0. \end{cases} \tag{19}$$

We can see that Eq. (18) corresponds to a right electron and Eq. (19) to a left one.

The energy spectrum in Eq. (16) in the limit $m \to 0$ reads

$$\mathcal{E} = |p_z + V_5|. \tag{20}$$

Comparing Eq. (20) with Eqs. (18) and (19), we obtain that for a right electron

$$|p_z + V_5| = -p_z - V_5, \quad \text{or} \quad p_z < -V_5, \tag{21}$$

and

$$|p_z + V_5| = p_z + V_5, \quad \text{or} \quad p_z > -V_5, \tag{22}$$

for a left particle.

Therefore the total energy of a left electron at the lowest energy level has the form,

$$E_{eL}^{(n=0)} = V_L + p_z, \quad -V_5 < p_z < +\infty, \tag{23}$$

and

$$E_{eR}^{(n=0)} = V_R - p_z, \quad -\infty < p_z < -V_5, \tag{24}$$

of a right particle. Comparing Eqs. (23) and (24) with analogous expressions obtained in Refs. [7, 8], one can see that that the form of the spectrum at $n = 0$ formally coincides with that in Refs. [7, 8]. However, the range of the $p_z$ variation is different.

To complete the solution of the Dirac equation at $n = 0$ and $m \rightarrow 0$ we should fix the remaining spin coefficients. One gets that

$$C_2^{(R)} = C_4^{(L)} = \frac{1}{2\pi}, \quad C_2^{(L)} = C_4^{(R)} = 0, \tag{25}$$

which results from the normalization condition

$$\int \mathrm{d}^3 x \psi_{p_y p_z n}^\dagger \psi_{p_y' p_z' n'} = \delta \left( p_y - p_y' \right) \delta \left( p_z - p_z' \right) \delta_{nn'}, \tag{26}$$

of the total wave function.

The wave function of a positron can be obtained from Eqs. (12) and (13) by applying the charge conjugation $\psi_{\bar{e}} = \mathrm{i}\gamma^2 \psi_e^*$ and setting $\lambda = -1$ in Eq. (15). Finally one has

$$\psi_{\bar{e}}^{\mathrm{T}} = \exp(-\mathrm{i}E_{\bar{e}}t - \mathrm{i}p_y y - \mathrm{i}p_z z) \times (-\mathrm{i}C_4 u_n, -C_3 u_{n-1}, \mathrm{i}C_2 u_n, C_1 u_{n-1}), \tag{27}$$

where the coefficients $C_i$ obey the system in Eq. (17).

If $n = 0$, we obtain on the basis of Eqs. (27) and (16) that

$$\psi_{\bar{e}\mathrm{R}}^{(n=0)} = \exp(-\mathrm{i}E_{\bar{e}\mathrm{R}}t - \mathrm{i}p_y y - \mathrm{i}p_z z) \times \frac{\mathrm{i}u_0}{2\pi} (-1, 0, 0, 0)^{\mathrm{T}}, \tag{28}$$

where

$$E_{\bar{e}\mathrm{R}}^{(n=0)} = p_z - V_{\mathrm{R}}, \quad -V_5 < p_z < +\infty, \tag{29}$$

is the energy of right positrons at the lowest energy level. For left positrons one has

$$\psi_{\bar{e}\mathrm{L}}^{(n=0)} = \exp(-\mathrm{i}E_{\bar{e}\mathrm{L}}t - \mathrm{i}p_y y - \mathrm{i}p_z z) \times \frac{\mathrm{i}u_0}{2\pi} (0, 0, 1, 0)^{\mathrm{T}}, \tag{30}$$

where

$$E_{\bar{e}\mathrm{L}}^{(n=0)} = -p_z - V_{\mathrm{L}}, \quad -\infty < p_z < -V_5, \tag{31}$$

is the energy of left positrons at the lowest energy level. The positron wave functions in Eqs. (28) and (30) satisfy the normalization condition in Eq. (26).

The contributions of left and right electrons at the lowest energy level to the current are

$$\mathbf{J}_{e\mathrm{L,R}}^{(n=0)} = -e \int \mathrm{d}p_y \mathrm{d}p_z \bar{\psi}_{e\mathrm{L,R}} \gamma \psi_{e\mathrm{L,R}} f(E_{e\mathrm{L,R}}^{(n=0)} - \mu_{\mathrm{L,R}}), \tag{32}$$

where $\mu_{\mathrm{L,R}}$ are the chemical potentials of left and right particles. First we notice that the components of the current, transverse with respect to $\mathbf{B}$, are vanishing. Performing the integration over $-\infty < p_y < +\infty$ and accounting for Eqs. (23)-(25), on the basis of Eq. (32) we obtain the expression for the total current of electrons $\mathbf{J}_e^{(n=0)} = \mathbf{J}_{\mathrm{L}}^{(n=0)} + \mathbf{J}_{\mathrm{R}}^{(n=0)}$ at $n = 0$,

$$\begin{aligned}
\mathbf{J}_e^{(n=0)} &= \frac{e^2 \mathbf{B}}{(2\pi)^2} \left[ \int_{-\infty}^{-V_5} \mathrm{d}p_z f(-p_z + V_{\mathrm{R}} - \mu_{\mathrm{R}}) - \int_{-V_5}^{+\infty} \mathrm{d}p_z f(p_z + V_{\mathrm{L}} - \mu_{\mathrm{L}}) \right] \\
&= \frac{e^2 \mathbf{B}}{(2\pi)^2} \int_0^{+\infty} \mathrm{d}p \left[ f(p + \bar{V} - \mu_{\mathrm{R}}) - f(p + \bar{V} - \mu_{\mathrm{L}}) \right].
\end{aligned} \tag{33}$$

Analogously to Refs. [7, 8] one can show that higher energy levels with $n > 0$ do not contribute to the current. Thus we shall omit the superscript in Eq. (33) for brevity.

The positron contribution to the current $\mathbf{J}_{\bar{e}}$ can be obtained analogously to Eq. (32) as

$$\mathbf{J}_{\bar{e}\mathrm{L,R}}^{(n=0)} = e \int \mathrm{d}p_y \mathrm{d}p_z \bar{\psi}_{\bar{e}\mathrm{L,R}} \gamma \psi_{\bar{e}\mathrm{L,R}} f(E_{\bar{e}\mathrm{L,R}}^{(n=0)} + \mu_{\mathrm{L,R}}). \tag{34}$$

Using Eqs. (28)-(31), we obtain on the basis of Eq. (34) the total contribution of positrons at the lowest energy level to the current in the form,

$$
\begin{aligned}
\mathbf{J}_{\bar{e}}^{(n=0)} &= \frac{e^2 \mathbf{B}}{(2\pi)^2} \left[ \int_{-\infty}^{-V_5} dp_z f(-p_z - V_L + \mu_L) - \int_{-V_5}^{+\infty} dp_z f(p_z - V_R + \mu_R) \right] \\
&= \frac{e^2 \mathbf{B}}{(2\pi)^2} \int_0^{+\infty} dp \left[ f(p - \bar{V} + \mu_L) - f(p - \bar{V} + \mu_R) \right].
\end{aligned}
\tag{35}
$$

It should be noted that higher energy levels do not contribute to the current.

Using Eqs. (33) and (35), we obtain that the total current $\mathbf{J} = \mathbf{J}_e + \mathbf{J}_{\bar{e}}$ reads

$$
\begin{aligned}
\mathbf{J} &= \frac{e^2 \mathbf{B}}{(2\pi)^2} \int_0^{+\infty} dp \left[ f(p + \bar{V} - \mu_R) - f(p - \bar{V} + \mu_R) - f(p + \bar{V} - \mu_L) + f(p - \bar{V} + \mu_L) \right] \\
&= \frac{2\alpha_{\text{em}}}{\pi} \mu_5 \mathbf{B} \equiv \mathbf{J}_{\text{CME}},
\end{aligned}
\tag{36}
$$

which is in agreement with the predictions of the CME at the absence of the electroweak interaction.

# 4 Conclusion

In this work, we have analyzed the possibility of the existence of the electric current induced along the external magnetic field in the system of massive charged electrons, having anomalous magnetic moments and electroweakly interacting with background matter, which was supposed to be nonmoving and unpolarized. We have obtained that both the lowest, with $n = 0$ (see also Refs. [10, 13]), and higher, with $n > 0$, energy levels do not contribute to this current.

The analysis of the contribution of the higher energy levels with $n > 0$ to the induced current is not trivial. Nevertheless, in Sec. 2, we have revealed that this contribution is vanishing; cf. Eq. (11). This result is valid at any characteristics of the electron-positron field, such as $m$, $\mu$, etc., and any parameters of the external fields, such $B$ and $V_5$.

The cancellation of the induced current $\mathbf{J} \parallel \mathbf{B}$ for $n > 0$ in the considered system, which was supposed to be in the equilibrium, corrects the recent claims in Refs. [9, 13] that such a current can be nonzero. The incorrect nonzero expression for the current, obtained Ref. [13], was because of the error in the integration over the longitudinal momentum in the case of the degenerate electron gas. The discrepancy between our results and the findings of Ref. [9] can be explained by the consideration of a nonequilibrium state of the system in Ref. [9]. Therefore the current $\mathbf{J} \parallel \mathbf{B} \neq 0$, derived in Ref. [9], will tend to zero very rapidly in a realistic medium.

The main reason for the cancellation of the current consists in the fact that the longitudinal momentum $p_z$ can vary from $-\infty$ to $+\infty$ for a particle with a nonzero mass. Even the feature of the energy spectrum for $n > 0$ that it is not symmetric with respect to the transformation $p_z \rightarrow -p_z$ (see Eq. (3)), which Ref. [13] appealed to in order to justify the existence of the nonzero induced current, does not help to generate $\mathbf{J} \parallel \mathbf{B} \neq 0$. Thus the cases of $m \neq 0$ and $m = 0$ are different generically. In the latter situation, the induced current can exist owing to the CME, which is based on the asymmetric motion of charged massless particles at the lowest energy level with respect to the external magnetic field. The difference between the systems of massive and massless particles consists in the chiral symmetry: it is broken in the former case and restored in the latter one.

In the present work, we have also elaborated the improved derivation of the anomalous current of massless charged fermions, interacting with an axial-vector field under the influence of the external magnetic field, flowing along the magnetic field. We have chosen a particular example of the axial-vector field as the electroweak interaction of an electron with nonmoving and unpolarized background matter. Unlike Refs. [16, 17], here we have used the method of the relativistic quantum mechanics, originally proposed in Ref. [3] to describe the CME.

Using the exact solution of the Dirac equation, found in Refs. [4, 11, 15], we have shown in Sec. 3 that the axial-vector field does not contribute to the current $\mathbf{J} \parallel \mathbf{B}$; cf. Eq. (36). The value of the current coincides with the prediction of the CME even in the case when chiral fermions electroweakly interact with background matter, confirming the findings of Refs. [16, 17].

To obtain this result in frames of the relativistic quantum mechanics one has to consider the solution of the Dirac equation for a massive electron in the external fields and then approach to the limit $m \to 0$. If one sets $m = 0$ in the Dirac equation from the very beginning, i.e. if one considers the chiral Lagrangian in Eq. (1) with $\mu = 0$ and $m = 0$, one obtains the anomalous current coinciding with that in Refs. [7, 8], which is inconsistent with the results of Refs. [16, 17]. Thus we conclude that the system of chiral fermions, where the external axial-vector field is present, can be prepared in two non-equivalent ways [11].

## References

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