Functional Approach for the Description of Vacuum Influence on Electron States

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Abstract. In the path integral approach we describe evolution of interacting electromagnetic and fermionic fields by the use of density matrix formalism. The equation for density matrix and transitions probability for fermionic field is obtained as average of electromagnetic field influence functional. We obtain a formula for electromagnetic field influence functional calculating for its various initial and final state. We derive electromagnetic field influence functional when its initial and final states are vacuum. We represent the equations of the movement for the fermionic field and expression for energy of an electron taking into account influence of a vacuum of the electromagnetic field.

1 Introduction

At the present time the development of non-perturbative methods of non-linear dynamics description is actual for quantum systems interacting with strong electromagnetic field [1–5]. One of these methods is Feynman-Vernon influence functional approach in the path integral formalism [6]. This method development was presented in many papers [7–9]. In the Feynman-Vernon influence functional approach the main point is its calculation for the model of investigated system. In this paper we calculate Feynman-Vernon influence functional of quantum electromagnetic field vacuum on fermionic field. He does possible to write down the quasiclassical equation of the movement for the fermionny field taking into account interaction him with a vacuum of the electromagnetic field. The offered equation allows to define expression for energy of interaction of an electron with a vacuum.

2 Description of quantum transitions of the system in the holomorphic representation by the method of functional integration

The QED Lagrangian for a spin-1/2 field interacting with the electromagnetic field is given by

$$\mathcal{L}_{full}(x) = \overline{\psi}(x)(\imath\gamma^{\mu}\partial_{\mu} - m)\psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) - j_{\mu}(x)A^{\mu}(x) \tag{1}$$

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where γ^μ are Dirac matrices, $\psi(x)$ is a bispinor field of spin-1/2 particles (e.g. electron–positron field), $A_\mu(x)$ is electromagnetic four-potential;

$$F_{\mu\nu}(x) = \partial_\mu A_\mu(x) - \partial_\nu A_\mu(x) \tag{2}$$

is the electromagnetic field tensor,

$$j_\mu(x) = e\overline{\psi}(x)\gamma_\mu\psi(x) \tag{3}$$

is four-current. We use natural units system $c = \hbar = 1$ in formulas (1)–(3) and further.

By the use of the second quantization formalism we present fields as field operators $\hat{\psi}(\mathbf{x}, t)$, $\hat{\overline{\psi}}(\mathbf{x}, t)$, $\hat{j}_\mu(\mathbf{x}, t)$, $\hat{A}^\mu(\mathbf{x}, t)$ where $\mathbf{x}$ and t describe coordinate of 3D space and time moment. Here we introduce the creation and annihilation operators:

$$\hat{A}^\mu(\mathbf{x}, t) = \sum_{\mathbf{k},\lambda=\pm1} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}^{(b)}}} \left(\varepsilon_\lambda^\mu \hat{a}_{\mathbf{k}\lambda} e^{\imath kx} + \varepsilon_\lambda^{\mu*} \hat{a}_{\mathbf{k}\lambda}^\dagger e^{-\imath kx}\right) \tag{4}$$

$$\hat{\psi}(\mathbf{x}, t) = \sum_{\mathbf{p},\sigma=\pm\frac{1}{2}} \frac{1}{\sqrt{2V\omega_{\mathbf{p}}^{(f)}}} \left(\hat{b}_{\mathbf{p}\sigma} u_\sigma(p) e^{\imath px} + \hat{c}_{\mathbf{p}\sigma}^\dagger u_\sigma(-p) e^{-\imath px}\right), \tag{5}$$

$$\hat{\psi}(\mathbf{x}, t) = \sum_{\mathbf{p},\sigma=\pm\frac{1}{2}} \frac{1}{\sqrt{2V\omega_{\mathbf{p}}^{(f)}}} \left(\hat{b}_{\mathbf{p}\sigma}^\dagger u_\sigma(p) e^{-\imath px} + \hat{c}_{\mathbf{p}\sigma} u_\sigma(-p) e^{\imath px}\right), \tag{6}$$

$$\hat{j}_\mu(\mathbf{x}) = \hat{j}_\mu(\mathbf{x}, \hat{b}_{\mathbf{p}\sigma}, \hat{b}_{\mathbf{p}\sigma}^\dagger, \hat{c}_{\mathbf{p}\sigma}, \hat{c}_{\mathbf{p}\sigma}^\dagger), \;\; \hat{j}_\mu^+(\mathbf{k}, t) = \int \hat{j}_\mu(\mathbf{x}, t) e^{\imath \mathbf{kx}} d\mathbf{x}, \;\; \hat{j}_\mu^-(\mathbf{k}, t) = \int \hat{j}_\mu(\mathbf{x}, t) e^{-\imath \mathbf{kx}} d\mathbf{x}, \tag{7}$$

where $\hat{a}_{\mathbf{k}\lambda}^\dagger$ and $\hat{a}_{\mathbf{k}\lambda}$ are creation and annihilation operators of photon with wave vector $\mathbf{k}$ and polarization (the eigenvalue of the z-component of the photon spin) $\lambda = \pm 1$, ε_λ^μ is unitary 4-vector of polarization and $\varepsilon_\lambda^{\mu*}\varepsilon_{\lambda\mu} = -1$; $\hat{b}_{\mathbf{p}\sigma}^\dagger$ and $\hat{b}_{\mathbf{p}\sigma}$ are creation and annihilation operators of electron with wave vector $\mathbf{p}$ and polarization $\sigma = \pm\frac{1}{2}$; $\hat{c}_{\mathbf{p}\sigma}^\dagger$ and $\hat{c}_{\mathbf{p}\sigma}$ are creation and annihilation operators of positron with wave vector $\mathbf{p}$ and polarization $\sigma = \pm\frac{1}{2}$, V is volume of space.

For evolution of the interacting systems we construct Hamiltonian of full system $\hat{H}_{full}$ on the base of operators $\hat{a}_{\mathbf{k}\lambda}^\dagger$, $\hat{a}_{\mathbf{k}\lambda}$, $\hat{b}_{\mathbf{p}\sigma}^\dagger$, $\hat{b}_{\mathbf{p}\sigma}$, $\hat{c}_{\mathbf{p}\sigma}^\dagger$, $\hat{c}_{\mathbf{p}\sigma}$.

$$\hat{H}_{full} = \hat{H}_{Dirac} + \hat{H}_{field} + \hat{H}_{int} \tag{8}$$

where

$$\hat{H}_{Dirac} = \sum_{\mathbf{p},\sigma} \omega_{\mathbf{p}}^{(f)} \left(\hat{b}_{\mathbf{p}\sigma}^\dagger \hat{b}_{\mathbf{p}\sigma} + \hat{c}_{\mathbf{p}\sigma}^\dagger \hat{c}_{\mathbf{p}\sigma}\right) \quad \hat{H}_{field} = \sum_{\mathbf{k},\lambda} \omega_{\mathbf{k}}^{(b)} \hat{a}_{\mathbf{k}\lambda}^\dagger \hat{a}_{\mathbf{k}\lambda} \tag{9}$$

— Hamiltonians of Dirac field, of electromagnetic field;

$$\hat{H}_{int} = \sum_{\mathbf{k},\lambda} \frac{1}{\sqrt{2\omega_{\mathbf{k}}^{(b)} V}} \left(\varepsilon_\lambda^\mu \hat{j}_\mu^+(\mathbf{k}, t) \hat{a}_{\mathbf{k}\lambda} + \varepsilon_\lambda^{*\mu} \hat{j}_\mu^-(\mathbf{k}, t) \hat{a}_{\mathbf{k}\lambda}^\dagger\right) \tag{10}$$

— interaction part of Hamiltonian.

The evolution equation of statistical operator $\hat{\rho}$ is the following

$$\hat{\rho}(t_f) = \hat{U}(t_f, t_{in})\hat{\rho}(t_{in})\hat{U}^\dagger(t_f, t_{in}) \tag{11}$$

where $\hat{\rho}(t_{in})$ is statistical operator, describing initial state at moment t_{in} and $\hat{U}(t_f, t_{in})$ is operator of evolution from initial time moment t_{in} to final time moment t_f

$$\hat{U}(t_f, t_{in}) = \hat{T}\exp[-\frac{\imath}{\hbar}\int_{t_{in}}^{t_f}\hat{H}_{full}(\tau)d\tau]. \tag{12}$$

For density matrix calculation we need to formulate equation (13) in holomorphic representation.

We introduce coherent representation $|\alpha_{\mathbf{k}\lambda}\rangle$ for electromagnetic field by the following way

$$\hat{a}_{\mathbf{k}\lambda}|a_{\mathbf{k}\lambda}\rangle = a_{\mathbf{k}\lambda}|a_{\mathbf{k}\lambda}\rangle, \qquad \langle a_{\mathbf{k}\lambda}|\hat{a}^\dagger_{\mathbf{k}\lambda} = \langle a_{\mathbf{k}\lambda}|a^*_{\mathbf{k}\lambda}, \tag{13}$$

$$\langle a'_{\mathbf{k}'\lambda'}|a_{\mathbf{k}\lambda}\rangle = \delta_{\mathbf{k}'\mathbf{k}}\delta_{\lambda'\lambda}\exp\left\{-\frac{1}{2}\left(a'^*_{\mathbf{k}'\lambda'}a'_{\mathbf{k}\lambda} + a^*_{\mathbf{k}\lambda}a_{\mathbf{k}\lambda} - a'^*_{\mathbf{k}'\lambda'}a_{\mathbf{k}\lambda} - a^*_{\mathbf{k}\lambda}a_{\mathbf{k}'\lambda'}\right)\right\}. \tag{14}$$

$$\int |a_{\mathbf{k}\lambda}\rangle\langle a_{\mathbf{k}\lambda}|da^*_{\mathbf{k}\lambda}da_{\mathbf{k}\lambda} = \hat{1}. \tag{15}$$

where $\alpha_{\mathbf{k}\lambda}$ — complex value, which describe one mode of quantum electromagnetic field with wave vector **k** and polarization λ.

We introduce grassman representation $|b_{\mathbf{p},\sigma}\rangle$ and $|c_{\mathbf{p},\sigma}\rangle$ for Dirac field by the following way

$$\hat{b}_{\mathbf{p},\sigma}|b_{\mathbf{p},\sigma}\rangle = b_{\mathbf{p},\sigma}|b_{\mathbf{p},\sigma}\rangle, \qquad \langle b_{\mathbf{p},\sigma}|\hat{b}^\dagger_{\mathbf{p},\sigma} = \langle b_{\mathbf{p},\sigma}|b^*_{\mathbf{p},\sigma}, \qquad \int |b_{\mathbf{p}\sigma}\rangle\langle b_{\mathbf{p}\sigma}|db^*_{\mathbf{p}\sigma}db_{\mathbf{p}\sigma} = \hat{1}; \tag{16}$$

$$\hat{c}_{\mathbf{p},\sigma}|c_{\mathbf{p},\sigma}\rangle = c_{\mathbf{p},\sigma}|c_{\mathbf{p},\sigma}\rangle, \qquad \langle c_{\mathbf{p},\sigma}|\hat{c}^\dagger_{\mathbf{p},\sigma} = \langle c_{\mathbf{p},\sigma}|c^*_{\mathbf{p},\sigma}, \qquad \int |c_{\mathbf{p}\sigma}\rangle\langle c_{\mathbf{p}\sigma}|dc^*_{\mathbf{p}\sigma}dc_{\mathbf{p}\sigma} = \hat{1}; \tag{17}$$

$$\langle b'_{\mathbf{p}'\sigma'}|b_{\mathbf{p}\sigma}\rangle = \delta_{\mathbf{p}'\mathbf{p}}\delta_{\sigma'\sigma}\exp\left\{-\frac{1}{2}\left(b'^*_{\mathbf{p}'\sigma'}b'_{\mathbf{p}'\sigma'} + b^*_{\mathbf{p}\sigma}b_{\mathbf{p}\sigma} - b'^*_{\mathbf{p}'\sigma'}b_{\mathbf{p}\sigma} - b^*_{\mathbf{p}\sigma}b'_{\mathbf{p}'\sigma'}\right)\right\}. \tag{18}$$

$$\langle c'_{\mathbf{p}'\sigma'}|c_{\mathbf{p}\sigma}\rangle = \delta_{\mathbf{p}'\mathbf{p}}\delta_{\sigma'\sigma}\exp\left\{-\frac{1}{2}\left(c'^*_{\mathbf{p}'\sigma'}c'_{\mathbf{p}'\sigma'} + c^*_{\mathbf{p}\sigma}c_{\mathbf{p}\sigma} - c'^*_{\mathbf{p}'\sigma'}c_{\mathbf{p}\sigma} - c^*_{\mathbf{p}\sigma}c'_{\mathbf{p}'\sigma'}\right)\right\}. \tag{19}$$

where $b_{\mathbf{p},\sigma}$ and $c_{\mathbf{p},\sigma}$ — grassman variable.

In holomorphic representation $|a_{\mathbf{k}\lambda}, b_{\mathbf{p}\sigma}, c_{\mathbf{p}\sigma}\rangle = |a_{\mathbf{k}\lambda}\rangle \otimes |b_{\mathbf{p}\sigma}\rangle \otimes |c_{\mathbf{p}\sigma}\rangle$ we present the evolution equation in the following

$$\begin{aligned}\rho(a^*_f, b^*_f, c^*_f, a'_f, b'_f, c'_f; t_f) = \int da'^*_{in}da'_{in}db'^*_{in}db'_{in}dc'^*_{in}dc'_{in}da^*_{in}da_{in}db^*_{in}db_{in}dc^*_{in}dc_{in}\times \\ \times\langle a_f, b_f, c_f|\hat{U}(t_f, t_{in})|a_{in}, b_{in}, c_{in}\rangle\rho(a^*_{in}, b^*_{in}, c^*_{in}, a'_{in}, b'_{in}, c'_{in}; t_{in})\times \\ \times\langle a'_{in}, b'_{in}, c'_{in}|\hat{U}^\dagger(t_f, t_{in})|a'_f, b'_f, c'_f\rangle\end{aligned} \tag{20}$$

where the density matrix:

$$\rho(a_f^*, b_f^*, c_f^*, a_f', b_f', c_f'; t_f) = \langle a_f, b_f, c_f|\hat{\rho}(t_f)|a_f', b_f', c_f'\rangle \tag{21}$$

For simplification we relabel variables $\alpha_{\mathbf{k}\lambda}$ and $\theta_{\mathbf{p}\sigma}$ to α and θ.

Be the use of amplitude transition properties [10], we present the kernel of evolution operator $\langle a_f, b_f, c_f|\hat{U}(t_f, t_{in})|a_{in}, b_{in}, c_{in}\rangle$ as a path integral:

$$\begin{aligned}\langle a_f, b_f, c_f|\hat{U}(t_f, t_{in})|a_{in}, b_{in}, c_{in}\rangle = \int \mathfrak{D}a^*(\tau)\mathfrak{D}a(\tau)\mathfrak{D}b^*(\tau)\mathfrak{D}b(\tau)\mathfrak{D}c^*(\tau)\mathfrak{D}c(\tau)\times \\ \times \exp\left\{\imath S_{full}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right]\right\},\end{aligned} \tag{22}$$

where full system action is the following

$$\begin{aligned}S_{full}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] = \\ = S_f\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] + S_b\left[a^*(\tau), a(\tau)\right] + \\ + S_{int}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right],\end{aligned} \tag{23}$$

where

$$\begin{aligned}S_f\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] = \int_{t_{in}}^{t_f}\left(\frac{\dot{b}^*(\tau)b(\tau) - b^*(\tau)\dot{b}(\tau)}{2\imath} + \frac{\dot{c}^*(\tau)c(\tau) - c^*(\tau)\dot{c}(\tau)}{2\imath} - \right. \\ \left. - \omega^{(f)}\left(b^*(\tau)b(\tau) + c^*(\tau)c(\tau)\right)\right)d\tau\end{aligned} \tag{24}$$

is the action S_f of fermionic field,

$$S_b\left[a^*(\tau), a(\tau)\right] = \int_{t_{in}}^{t_f}\left(\frac{\dot{a}^*(\tau)a(\tau) - a^*(\tau)\dot{a}(\tau)}{2\imath} - \omega^{(b)}a^*(\tau)a(\tau)\right)d\tau \tag{25}$$

is the action S_b of bosonic field,

$$\begin{aligned}S_{int}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] = \\ = \int_{t_{in}}^{t_f}\left[\varepsilon_\mu \tilde{j}^{\mu+}\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right]a(\tau) + \varepsilon_\mu^* \tilde{j}^{\mu-}\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right]a^*(\tau)\right]d\tau;\end{aligned} \tag{26}$$

is the action S_{int} of interaction part.

Using (20)–(22) we present equation (20) for density matrix evolution in the following form:

$$\begin{aligned}\rho(a_f^*, b_f^*, c_f^*, a_f', b_f', c_f'; t_f) = \int da_{in}'^* da_{in}' db_{in}'^* db_{in}' dc_{in}'^* dc_{in}' da_{in}^* da_{in} db_{in}^* db_{in} dc_{in}^* dc_{in}\times \\ \times \mathfrak{D}a^*(\tau)\mathfrak{D}a(\tau)\mathfrak{D}b^*(\tau)\mathfrak{D}b(\tau)\mathfrak{D}c^*(\tau)\mathfrak{D}c(\tau)\mathfrak{D}a'^*(\tau)\mathfrak{D}a'(\tau)\mathfrak{D}b'^*(\tau)\mathfrak{D}b'(\tau)\mathfrak{D}c'^*(\tau)\mathfrak{D}c'(\tau)\times \\ \times\exp\left\{\imath\left(S_{full}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] - S_{full}\left[a'^*(\tau), a'(\tau), b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau)\right]\right)\right\} \\ \times\rho(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in}).\end{aligned} \tag{27}$$

We construct an expression for the probability of a quantum transition of a system $P(m, \chi, t_f; n, \zeta, t_{in})$ from a state at the initial instant of time $\rho_{n\zeta}(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in})$ to a state at a finite time $\rho_{m\chi}(a_{in}^*, b_{in}^*, c_{in}^*, a_{in}', b_{in}', c_{in}'; t_{in})$, using formula (28)

$$P(m,\chi,t_f;n,\zeta,t_{in}) = \int da'^*_{in}da'_{in}db'^*_{in}db'_{in}dc'^*_{in}dc'_{in}da^*_{in}da_{in}db^*_{in}db_{in}dc^*_{in}dc_{in}$$
$$\times da'^*_f da'_f db'^*_f db'_f dc'^*_f dc'_f da^*_f da_f db^*_f db_f dc^*_f dc_f$$
$$\times \mathfrak{D}a^*(\tau)\mathfrak{D}a(\tau)\mathfrak{D}b^*(\tau)\mathfrak{D}b(\tau)\mathfrak{D}c^*(\tau)\mathfrak{D}c(\tau)\mathfrak{D}a'^*(\tau)\mathfrak{D}a'(\tau)\mathfrak{D}b'^*(\tau)\mathfrak{D}b'(\tau)\mathfrak{D}c'^*(\tau)\mathfrak{D}c'(\tau)$$
$$\times \rho_{m\chi}(a^*_f, b^*_f, c^*_f, a'_f, b'_f, c'_f; t_f)$$
$$\times\exp\left\{\imath\left(S_{full}\left[a^*(\tau),a(\tau),b^*(\tau),b(\tau),c^*(\tau),c(\tau)\right] - S_{full}\left[a'^*(\tau),a'(\tau),b'^*(\tau),b'(\tau),c'^*(\tau),c'(\tau)\right]\right)\right\}$$
$$\times \rho_{n\zeta}(a^*_{in}, b^*_{in}, c^*_{in}, a'_{in}, b'_{in}, c'_{in}; t_{in}). \quad (28)$$

Where m, n - the symbols correspond to the states of the fermion field; χ, ζ - correspond to the states of the electromagnetic field.

3 The Feynman-Vernon Influence Functional of quantum electromagnetic field on fermionic field

We can choose at the initial moment t_{in}

$$\rho_{n\zeta}(a^*_{in}, b^*_{in}, c^*_{in}, a'_{in}, b'_{in}, c'_{in}; t_{in}) = \rho_n(b^*_{in}, c^*_{in}, b'_{in}, c'_{in}; t_{in}) \times \rho_\zeta(a^*_{in}, a'_{in}; t_{in}), \quad (29)$$

at the final moment t_f

$$\rho_{m\chi}(a^*_{in}, b^*_{in}, c^*_{in}, a'_{in}, b'_{in}, c'_{in}; t_{in}) = \rho_m(b^*_{in}, c^*_{in}, b'_{in}, c'_{in}; t_{in}) \times \rho_\chi(a^*_{in}, a'_{in}; t_{in}), \quad (30)$$

This is true for many models.

By the use of (30),(31) we present the transition probability in the path integral representation

$$P(m,\chi;t_f;n,\zeta;t_{in}) = \int \mathcal{D}b'^*(\tau)\mathcal{D}b'(\tau)\mathcal{D}b^*(\tau)\mathcal{D}b(\tau)\mathcal{D}c'^*(\tau)\mathcal{D}c'(\tau)\mathcal{D}c^*(\tau)\mathcal{D}c(\tau)\times$$
$$\times da^*_f da_f da'^*_f da'_f db^*_f db_f db'^*_f db'_f dc^*_f dc_f dc'^*_f dc'_f\times$$
$$\times\rho_m(b_f, c_f, b'^*_f, c'^*_f)\exp\left\{\imath\left(S_f[b^*(\tau),b(\tau),c^*(\tau),c(\tau)] - S_f[b'^*(\tau),b'(\tau),c'^*(\tau),c'(\tau)]\right)\right\}\times$$
$$\times F_{\chi\zeta}[b'^*(\tau),b'(\tau),b^*(\tau),b(\tau),c'^*(\tau),c'(\tau),c^*(\tau),c(\tau)]\rho_m(b_{in}c_{in}, b'^*_{in}, c'^*_{in}), \quad (31)$$

where

$$F_{\chi\zeta}[b'^*(\tau),b'(\tau),b^*(\tau),b(\tau),c'^*(\tau),c'(\tau),c^*(\tau),c(\tau)] =$$
$$= \int da'^*_{in}da'_{in}da^*_{in}da_{in}da'^*_f da'_f da^*_f da_f \mathfrak{D}a^*(\tau)\mathfrak{D}a(\tau)\mathfrak{D}a'^*(\tau)\mathfrak{D}a'(\tau)\times$$
$$\times\rho_\chi(a^*_f, a'_f, t_f)\exp\left\{\imath S_{infl}\left[a^*(\tau),a(\tau),b^*(\tau),b(\tau),c^*(\tau),c(\tau)\right]\right\}\times$$
$$\times\exp\left\{-\imath S_{infl}\left[a'^*(\tau),a'(\tau),b'^*(\tau),b'(\tau),c'^*(\tau),c'(\tau)\right]\right\}\rho_\zeta(a^*_{in}, a'_{in}, t_{in}). \quad (32)$$

is influence functional of electromagnetic field on fermionic subsystems. It is represented in the form

$$F_{\chi\zeta}[b'^*(\tau),b'(\tau),b^*(\tau),b(\tau),c'^*(\tau),c'(\tau),c^*(\tau),c(\tau)] = \int da^*_{in}da_{in}da'^*_{in}da'_{in}da^*_f da_f da'^*_f da'_f\times$$
$$\times\rho_\chi(a^*_f, a'_f, t_f)U_{infl}[a^*_f, b^*_f, c^*_f, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in}]\times$$
$$\times U^*_{infl}[a'^*_{in}, b'^*_{in}, c'^*_{in}, t_{in}, b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau), a_f, b_f, c_f, t_f]\ \ \rho_\zeta(a^*_{in}, a'_{in}, t_{in}). \quad (33)$$

where

$$U_{infl}[a^*_f, b^*_f, c^*_f, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in}] =$$
$$= \int \mathfrak{D}a^*(\tau)\mathfrak{D}a(\tau) \exp\left\{\imath S_{infl}[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)]\right\} \qquad (34)$$

is electromagnetic field transition amplitude from initial state $|a_{in}\rangle$ to final state $|a_f\rangle$.

$$S_{infl}[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)] =$$
$$= S_b\left[a^*(\tau), a(\tau)\right] + S_{int}\left[a^*(\tau), a(\tau), b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right]. \qquad (35)$$

is action of fermionic field under electromagnetic field influence. It describe electromagnetic field influence on fermionic field.

The influence functional (34) present the influence of one mode of electromagnetic field with wave vector $\mathbf{k}$ and polarization λ. We note it by index $F_{\mathbf{k},\lambda}[b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)]$.

The influence functional of all electromagnetic field modes calculate as production of one mode influence functionals:

$$F[b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)] =$$
$$= \prod_{\mathbf{k}\lambda} F_{\mathbf{k},\lambda}[b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)]. \qquad (36)$$

For calculating influence functional by eq. (34) we need to calculate amplitude $U_{infl}[a^*_f, b^*_f, c^*_f, t_f, b^*(\tau), b(\tau), c^*(\tau), c(\tau), a_{in}, b_{in}, c_{in}, t_{in}]$ and to specify $\rho_\zeta(a^*_{in}, a'_{in}, t_{in})$, $\rho_\chi(a^*_f, a'_f, t_f)$.

4 Electromagnetic field influence functional calculation

In this section we calculate functional $U_{infl}[a^*_f, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau), a_{in}, b_{in}, t_{in}]$. For this we present equation (35) as finite-multiple integral [9].

$$U_{infl}[a^*_f, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau), a_{in}, b_{in}, t_{in}] =$$
$$= \lim_{n\longrightarrow\infty} \int\limits_C \dots \int\limits_C da^*_n da_n \cdots da^*_1 da_1 exp\left[\imath(\Delta S^{(infl)}_n + ... + \Delta S^{(infl)}_1 + \Delta S^{(infl)}_0)\right], \qquad (37)$$

where

$$\Delta S^{(infl)}_k = -\frac{1}{2\imath}(|a_{k+1}|^2 + |a_k|^2) + (1 - \imath\Delta\tau\omega^{(b)})a^*_{k+1}a_k + \Delta\tau e j^{\mu+}_k \varepsilon^*_\mu a^*_{k+1} + \Delta\tau e j^{\mu-}_k \varepsilon_\mu a_k, \qquad (38)$$

time interval t is spliting on equal intervals $\Delta\tau$. So $n\Delta\tau = t_f$ and $a^*_{n+1} = a^*_f$, $a_0 = a_{in}$.

We integrate through intermediate variables $a_1, \dots, a_n$ and obtain the following result in continual limit $\Delta t \to 0$, $n \to \infty$:

$$U_{infl}[a^*_f, b_f, t_f, b^*(\tau)b(\tau), c^*(\tau)c(\tau)a_{in}, b_{in}, t_{in}] =$$
$$= \exp\left[-\frac{1}{2}(|a_f|^2 + |a_{in}|^2) + Aa^*_f a_{in} + Ba^*_f + Ca_{in} + S_{infvac}\right]. \qquad (39)$$

where function $A = \exp\left[-i\omega^{(b)}t\right]$ and functional B, C, S_{inf}

$$B[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] = -ie\int_{t_in}^{t_f} j^{\mu+}(b^*(\tau), b(\tau), c^*(\tau), c(\tau))\varepsilon^*_\mu e^{-i\omega^{(b)}(t-\tau)}d\tau, \qquad (40)$$

$$C[b^*(\tau), b(\tau)c^*(\tau), c(\tau)] = -ie\int_{t_in}^{t_f} j^{\mu-}(b^*(\tau), b(\tau), c^*(\tau), c(\tau))\varepsilon_\mu e^{-i\omega^{(b)}\tau}d\tau, \tag{41}$$

$$S_{infvac}[b^*(\tau), b(\tau), c^*(\tau), c(\tau)] =$$
$$= -\int_{t_{in}}^{t_f}\int_{t_in}^{\tau} j^{\mu+}(b^*(\tau), b(\tau), c^*(\tau), c(\tau))\varepsilon^*_\mu j^{\nu-}(b^*(\tau), b(\tau, c^*(\tau), c(\tau))\varepsilon_\nu e^{-i\omega^{(b)}(\tau-\tau')}d\tau d\tau'. \tag{42}$$

Now we calculate the influence functional of electromagnetic vacuum. For this we use equation (42) and specify initial and final state of electromagnetic field as vacuum:

$$\rho(a^*_{in}, a'_{in}; t_{in}) = \zeta(a^*_{in})\zeta^*(a'_{in}), \quad \rho(a_f, a'_f) = \chi(a_f)\chi^*(a'_f) \tag{43}$$

$$\zeta_v(a_{in}) = \exp[-\frac{|a_{in}|^2}{2}], \qquad \chi^*_v(a_f) = \exp[-\frac{|a_f|^2}{2}]. \tag{44}$$

By the use of (34) and (40),(45) we calculate the influence functional of one mode **k** and λ electromagnetic field on one mode **p** and σ fermionic field:

$$F_{vac\mathbf{k},\lambda} = \exp[S_{infvac\mathbf{k}\lambda} + S'_{infvac\mathbf{k}\lambda}], \tag{45}$$

where functionals $S_{infvac\mathbf{k}\lambda}, S'_{infvac\mathbf{k}\lambda}$ is determined by (43).

We define influence functional of miltimode electromagnetic vacuum using (37), (43) and (46).

$$F_{vacall}[b'^*(\tau), b'(\tau), b^*(\tau), b(\tau), c'^*(\tau), c'(\tau), c^*(\tau), c(\tau)] = \exp[S_{infvac} + S'_{infvac}], \tag{46}$$

where

$$S_{infvac} = -\sum_{\mathbf{k},\lambda}\frac{1}{2\omega_{\mathbf{k}}V}\int\limits_{t_{in}}^{t_f}\int\limits_{t_{in}}^{\tau}\left(\varepsilon^\mu_\lambda j^+_\mu(\mathbf{k},\tau)\varepsilon^{*\nu}_\lambda j^-_\nu(\mathbf{k},\tau')e^{\imath\omega_{\mathbf{k}}(\tau-\tau')}d\tau d\tau'\right), \tag{47}$$

functional S'_{infvac} has the same structure as S_{infvac} with variables $b'^*(\tau), b'(\tau), c'^*(\tau), c'(\tau)$ i.e. j'^+_μ, j'^-_ν.

5 The fermionic field evolution under electromagnetic vacuum influence

We describe the fermionic field evolution by the use of probability transition (32). The initial conditions and the final conditions are given in the form

$$\begin{gathered}\rho_n(b^*_{in}, c^*_{in}, b'_{in}, c'_{ind}; t_{in}) = \psi_n(b^*_{in}, c^*_{in})\psi^*_n(b'_{in}, c'_{in}),\\ \rho_m(b^*_f, c^*_f, b'_f, c'_f; t_f) = \psi_m(b^*_f, c^*_f)\psi^*_m(b'_f, c'_f).\end{gathered} \tag{48}$$

Using equation (47) for influence functional of multimode electromagnetic vacuum, we obtain equation for probability transition

$$P(m; t_f|n; t_{in}) = K^*(m; t_f|n; t_{in})K(m; t_f|n; t_{in}), \tag{49}$$

where

$$K(m;t_f|n;t_{in}) = \int \mathcal{D}b^*(\tau)\mathcal{D}b(\tau)\mathcal{D}c^*(\tau)\mathcal{D}c(\tau)db^*_f db_f dc^*_f dc_f \times$$
$$\times \psi^*_m(b_f, c_f)\exp\left\{\imath\left(S_{fullvac}[b^*(\tau), b(\tau), c^*(\tau), c(\tau)]\right)\right\}\psi_n(b^*_{in}, c^*_{in}), \quad (50)$$

$K^*(m;t_f|n;t_{in})$ is conjugated transition amplitude,

$$S_{full}\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] =$$
$$= S_f\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right] + S_{infvac}\left[b^*(\tau), b(\tau), c^*(\tau), c(\tau)\right], \quad (51)$$

In equations (51), (52) we going from grassman variables $b^*(\tau), b(\tau), c^*(\tau), c(\tau)$ to functions of spinor field $\psi(x), \overline{\psi}(x)$. In equation (48) we convert summation over discrete $\mathbf{k}$ to integral over their continuum values and Fourier transform $j^+_\mu(\mathbf{k},\tau)$, $j^-_\nu(\mathbf{k},\tau')$ to function $j_\mu(\mathbf{x},\tau)$, $j_\nu(\mathbf{x}',\tau')$ by the use of equations (7). Then we consider the limit $t_f \to \infty$, $t_{in} \to -\infty$. So the effective action (52) for action of fermionic field under electromagnetic field vacuum influence is presenting in the following

$$S_{fullvac}\left[\overline{\psi}(x), \psi(x)\right] = \int \mathcal{L}_{fullvac}(\gamma^\mu\partial_\mu\psi(x), \psi(x))dx, \quad (52)$$

where

$$\mathcal{L}_{fullvac} = \overline{\psi}(x)(\imath\gamma^\mu\partial_\mu - m)\psi(x) + \frac{1}{2\pi}j_\mu(x)\int D^{\mu\nu}(x-x')j_\nu(x')dx', \quad (53)$$

where

$$D^{\mu\nu}(x-x') = \frac{1}{(2\pi)^3}\int\frac{2\pi\imath d\mathbf{k}}{\omega_\mathbf{k}}\left(\sum_\lambda \varepsilon^\mu_\lambda\varepsilon^{*\nu}_\lambda\right)e^{-\imath\mathbf{k}(\mathbf{x}-\mathbf{x}')}e^{\imath\omega(\tau-\tau')} \quad (54)$$

— is the photon propagator [11], [12].

The transition amplitude (51) is equivalent to S-matrix. In addition it contain contribution of electromagnetic vacuum influence on fermionic field dynamics:

$$K(m|n) = \int \mathcal{D}\overline{\psi}(\tau)\mathcal{D}\psi(\tau)\overline{\psi}_m(x)\exp\left\{\imath\left(S_{fullvac}[\overline{\psi}(\tau), \psi(\tau)]\right)\right\}\psi_n(x), \quad (55)$$

where action $S_{fullvac}[\overline{\psi}(\tau), \psi(\tau)]$ is determined by equation (53).

We construct the equations for the evolution of the fermion field in the proposed model in the semiclassical approximation. The equation is found from the principle of stationary action (), $\delta S_{fullvac}[\overline{\psi}(x), \psi(x)] = 0$

$$(\imath\gamma^\mu\partial_\mu - m)\psi(x) + \frac{1}{2\pi}\gamma^\mu\psi(x)\int D^{\mu\nu}(x-x')j_\nu(x')dx' = 0, \quad (56)$$

The Dirac equation for a free electron, taking into account its interaction with a vacuum, is integro-differential. It is non-linear and non-local.

The Lagrangian density, the equation of motion for the fermion field, allow us to determine the expression for the interaction energy of the electron and the electromagnetic field vacuum

$$\triangle E_{vac} = \frac{1}{2\pi}\int j_\mu(x)D^{\mu\nu}(x-x')j_\nu(x')dx'dx, \quad (57)$$

. The energy $\triangle E_{vac}$ is determined by the quantum state of the electron, the bispinor $\psi(x)$.

6 Summary

We consider the evolution of fermionic field interacting with electromagnetic field in path integral approach by the use of electromagnetic field influence functional inroduction. We obtain formula for influence functional calculation in specific model of electromagnetic field, interaction with fermionic fields and initial and final states. We calculate the electromagnetic vacuum influence functional. This lead to addition term in Lagrangian of spinor field and modify fermionic field dynamics. The Dirac equation for a free electron, taking into account its interaction with a vacuum, is integro-differential. It is non-linear and non-local. The expression for the interaction energy of the electron and the electromagnetic field vacuum is determined by the quantum state of the electron, the bispinor $\psi(x)$. The developed method allow to describe interacting fermionic system without perturbation expansions.

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